Onset of transition in the flow of polymer solutions through deformable tubes

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Experiments are performed to investigate laminar-turbulent transition in the flow of Newtonian and viscoelastic fluids in soft-walled micro-tubes of diameter $\sim 400\mu$m by using the micro particle image velocimetry (micro-PIV) technique. The Newtonian fluids used are water and water-glycerine mixtures, while the polymer solutions used are prepared by dissolving polyacrylamide in water. Using different tube diameters, elastic modulus of the tube wall, and polymer concentrations, we probe a wide range of dimensionless wall elasticity parameter Σ and dimensionless fluid elasticity number E. Here, $\Sigma = (\rho G R^2)/\eta^2$, ρ is the fluid density, G is the shear modulus of the soft wall, R is the radius of the tube and η is the solution viscosity. The elasticity of the polymer solution is characterized by $E = (\lambda \eta_0)/R^2 \rho$, where λ is the zero-shear relaxation time, η_0 is the zero-shear viscosity, ρ is the solution density and R is the tube radius. The onset of transition is detected by a shift in the ratio of center-line-peak to average velocity. A jump in the normalized center-line velocity fluctuations and the flattening of velocity profile are also used to corroborate the onset of instability. Transition for the flow of Newtonian fluid through deformable tubes (of shear modulus ~ 50kPa) is observed at a transition Reynolds number $Re_t \sim 700$, which is much lower than $Re_t \sim 2000$ for a rigid tube. For tubes of lowest shear modulus ~ 30kPa, Re_t for Newtonian fluid is as low as 250. For the flow of polymer solutions in a deformable tube (of shear modulus ~ 50kPa), $Re_t \sim 100$, which is much lower than that for Newtonian flow in a deformable tube with the same shear modulus, indicating a destabilizing effect of polymer elasticity on the transition already present for Newtonian fluids. Conversely, we also find instances where flow of a polymer solution in a rigid tube is stable, but wall elasticity destabilizes the flow in a deformable tube. The jump in normalized velocity fluctuations for the flow of both Newtonian and polymer solutions in soft-walled tubes is much gentler compared to that for Newtonian transition in rigid tubes. Hence, mechanism underlying the soft-wall transition for the flow of both Newtonian fluids and polymer solutions could be very different as compared to transition of Newtonian flows in rigid pipes. When Re_t is plotted with the wall elasticity parameter Σ for different moduli of the tube wall, by taking Newtonian fluids of different viscosities and polymer solutions of different concentrations, we observed a data collapse, with Re_t following a scaling relation of $Re_t \sim \Sigma^{0.7}$. Thus, both fluid and wall elasticity combine to trigger a transition at Re as low as 100 in the flow of polymer solutions through deformable tubes.

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I. INTRODUCTION

Flow of Newtonian fluids through rigid tubes has been well-known to undergo a transition usually at $Re \sim 2000$[1–6], although, if sufficient care is taken, the transition can be delayed upto $Re \sim 10^5$. Here the Reynolds number is defined as $Re = DV\rho/\eta$, where D is the tube diameter, V is the cross-sectional average velocity, ρ is the fluid density and η is the fluid viscosity. However, in the microfluidic context, it is highly desirable for the flow to be unstable at lower Re for improved mixing[7]. In the case of polymer solutions, instabilities at very low Re were first observed for torsional geometries, wherein the presence of 'hoop' stress is responsible for triggering such an instability[8–13]. However, these instabilities were only observed for viscoelastic flows with curved streamlines. The instability at very low inertia was observed to be stabilised by the presence of a soft surface at the bottom of the torsional flow configuration[14]. It was first observed by Krindel and Silberberg[15,16] that a dynamical interaction between the tube wall and the fluid flow causes an instability in flow of Newtonian fluids through deformable tubes at Re much lower than that for rigid tubes. We will henceforth refer such transitions in a tube that occur at Re much lower than 2000 as 'early transition'. Flow through soft tubes and channels is relevant to a wide range of practical applications. Blood flow through veins and arteries are mimicked very closely by flow through soft tubes and channels[17]. Mixing in microfluidic applications is achieved by using long microchannels which involve very high pressure drop[18]. Further, for microscale reactors, mixing time of the reactors is a critical parameter for the reaction to occur[7]. Early transition caused by using a soft wall can potentially aid in efficient mixing at low pumping cost in micro-channels which can be exploited in the design of various micro-fluidic devices[19,20]. Verma and Kumaran[21] investigated the flow of Newtonian fluid through deformable tubes sized ∼1mm. The tubes fabricated consisted of a rigid development section and a soft test section. Pressure difference was measured across the test section by the introduction of a pressure tap just before the entry into the test section. Friction factor data for flow through deformable tubes showed a transition at Re much lower than the transition Re for Newtonian fluid through rigid tubes. Further, it was observed that the transition Re is lowered by the use of tubes made of lower shear modulus. Wall oscillations and dye stream visualization experiments further corroborated the observed phenomenon. A plot of transition Re with wall elasticity number Σ showed a reasonable data collapse. The data followed the scaling law $Re_t \propto \Sigma^{5/8}$. Neelamegam and Shankar[22] performed similar experiments for flow through deformable tubes of diameter 1.65mm. A water tank was placed before the inlet to minimize inlet noise. The transition Re data followed the scaling law of $Re_t \propto \Sigma^{3/2}$, which was very different from the scaling law obtained for Verma et al[21]. However, a more in-depth understanding of the soft wall transition would be possible with the use of micro particle image velocimetry (micro-PIV) to obtain velocity profiles and time-dependent velocity fluctuations in the post-transition regime. A summary of the experimental work carried out in this area can be found in the review by Kumaran[23].

It has also been well-established recently that addition of polymer to a Newtonian solvent causes early transition in the flow through rigid tubes and channels [24–26]. This instability is believed to be a result of a combination of elastic and inertial forces and is referred to as 'elasto-inertial' instability[27,28]. The post-transition characteristics of 'elasto-inertial' instability are found to be very different as compared to that for Newtonian transition [29]. It has been recently suggested that 'elasto-inertial' instability is a linear instability[30], implying that the instability is independent of the amplitude of inlet perturbations. There is also another class of instability in the flow of highly concentrated polymer solutions[29,31,32] in rigid tubes, but in this work we do not probe the concentrated regime. Srinivas et al[25] showed that introduction of small amounts of polymer to an otherwise Newtonian solvent decreased the transition Re significantly for flow through a channel, especially when the bottom wall of the channel was made soft. While there have been many theoretical studies[33–35] that explored non-Newtonian effects such as viscoelasticity and shear thinning on flow past deformable surfaces, thus far, there has not been any experimental study carried out to understand the transition in the flow of polymer solutions through a deformable tube.

In this article, we experimentally investigate the flow of Newtonian and polymer solutions through deformable micro-tubes using micro-PIV measurements. Specifically we focus on: (i) post-transition characteristics for the flow of a Newtonian fluid through deformable micro-tubes. (ii) Onset of transition and post-transition behaviour for the flow of polymer solutions through deformable micro-tubes. Our objective is to investigate whether the introduction of polymer to soft wall transition could lead to an instability at very low Re for flow through deformable tubes, similar to the observations on channel flows by Srinivas and Kumaran[25], (iii) Condensing the data for transition Re for tubes of different shear modulus, tubes of different diameters and different polymer concentrations using suitable dimensionless groups.

Theoretical studies by Kumaran and co-workers (summarized in the review by Shankar[36]) show that, at sufficiently high Re, there are two different types of instability possible for flow through deformable tubes. One is called the 'inviscid mode' in which the viscous stresses in a thin 'critical layer' (where the phase speed of disturbances equals the base velocity) play an important role. For this class of modes, $Re_t \sim \Sigma^{1/2}$. There is another class of modes termed 'wall modes' where viscous stresses are confined near the fluid-wall interface where $Re_t \sim \Sigma^{3/4}$. Thus, for a given Σ, Re_t is greater for wall modes compared to inviscid modes. Gaurav and Shankar[37] performed linear stability analysis for flow of a Newtonian fluid through new-Hookean tubes. At very low Re, instability was predicted which was stabilized with increasing Re. However, an instability at such low Re has not been observed in experiments. Linear stability analysis predicts 'wall mode' instabilities at much higher Re compared to experiments[37,38]. This discrepancy was reconciled by considering the deformed shape of the tube and then performing linear stability analysis which predicted instabilities at similar Re as compared to in experiments[39].

There has been recent progress in the experimental study of flow past soft surfaces on account of its immense practical applications. A material that is frequently used for fabricating microchannels has been polydimethyl siloxane (PDMS)[40]. A major advantage of using PDMS is the control over the shear modulus of the fabricated micro-channel. Verma and Kumaran[21] and Neelamegam and Shankar[22] further performed experiments on soft-walled PDMS tubes and observed transition at much lower Re as compared to a rigid tube. Verma and Kumaran[21] observed a scaling relation as $Re_t \sim \Sigma^{5/8}$, which is very close to the wall mode scaling exponent of $3/4$. Further, a jump in the scattering intensity of the tube walls corresponded with the transition Re obtained from friction factor charts, thereby providing further evidence of presence of wall modes. Verma and Kumaran[41] observed ultrafast mixing by the introduction of a soft bottom wall in a microchannel fabricated using PDMS. Using break-up of dye stream and wall fluctuation data, they obtained the scaling relation $Re_t \sim \Sigma^{5/8}$ which was similar to the scaling observed in Verma and Kumaran[21]. This further indicated the presence of wall modes in the instability observed for deformable channel flow. Srinivas and Kumaran[42] showed that the transition Re for micro-channels in which the bottom wall is made of soft PDMS could be as low as $Re \sim 250$. Srinivas and Kumaran[42] observed that the near-wall velocity fluctuations are much higher as compared to the fluctuations observed at the center line. The velocity was flatter for the post-transition regime. Srinivas and Kumaran[25] investigated the flow of polymer solution through a soft-walled micro-channel. It was observed that addition of small amounts of polymer (~ 50ppm) caused the transition Re to reach as low as $Re \sim 30$. Transition was detected by the flattening of velocity profile and by a sharp increase in the streamwise velocity fluctuations. Further, dye-stream visualization experiments showed increased lateral mixing in the post-transition regime. Velocity fluctuations were observed to be higher near the deformable wall, thereby indicating the presence of wall modes. Flow of polymer solutions through deformable tubes can serve as a simple model for flow of blood through veins and arteries[43]. Instabilities arising in such flow conditions could be relevant in biomedical applications. Hence, in this paper, we focus mainly on three objectives: (i) Investigate the onset of transition and the post-transition regime for the flow of polymer solutions through deformable micro-tubes, (ii) Examine the effect of addition of polymer on soft-wall transition, and (iii) Collate data for transition Re vs fluid elasticity and wall elasticity parameters to investigate the effect of addition of polymer on soft wall transition.

The rest of this paper is organized as follows: Section 2 describes the experimental protocol, Section 3 provides the micro-PIV analysis for the flow of Newtonian solvent through deformable tubes, Section 4 contains the micro-PIV analysis for the flow of polymer solutions through deformable tubes. Section 5 provides a discussion related to the analysis of onset of instability and Section 6 provides the conclusions.

II. EXPERIMENTAL PROTOCOL

A. Fabrication of Deformable Microtubes

Deformable microtubes are fabricated using PDMS (SylGuard©) procured from Dow Corning. A copper wire is held straight using a screw mechanism as shown in Fig. 1. A well-like arrangement is created by using double sided tapes. The well is partitioned into two parts; a cross-linker and an elastomeric base. Two parts of the well are prepared; one of which is the development section and the other is called the test section[21]. Initially, 10% cross-linker is mixed with 90% elastomeric base, degassed, and then gently poured on one side of the well. Double sided tapes are used to separate the development and the test section. The well along with the PDMS is kept at $80°$C for 3 hours for curing the PDMS mixture. After curing, the tapes separating the two sections are removed. Varying amounts of cross-linkers in the range $2.5 - 2.75\%$ is now added to the test section. The PDMS well along with the template is now cured at $80°$C for 12 hours. Owing to the excellent swelling properties of toluene[44], the cured PDMS block is now kept in toluene for 2 to 3 hours for swelling. The copper wire is carefully removed from the swelled PDMS block. The swelled PDMS block is de-swelled at $5°$C to avoid crack-formation. By using this technique for fabricating deformable micro-tubes, tubes of diameter in the range $300 - 400\mu$m are prepared with a rigid development section and the deformable test section. Shear modulus of the test section is varied by varying the cross-linker percentage in PDMS. Higher cross-linker concentration increases the shear modulus of the deformable tube.

B. Characterization of PDMS elastomer

Freshly mixed cross-linker and elastomer base is simultaneously poured in a 5cm×5cm square well made by sticking microscopic slides on glass plates (Fig. 2). The resulting well, filled with PDMS mixture is cured at $80°$C for 12 hours. Thus, rectangular PDMS blocks are created for characterizing shear modulus using small amplitude oscillatory strain experiments with the help of a Thermal Analysis (TA) Discovery Hybrid Rheometer (DHR-3). A parallel plate geometry with diameter 40mm is used to measure shear modulus by performing small amplitude oscillatory

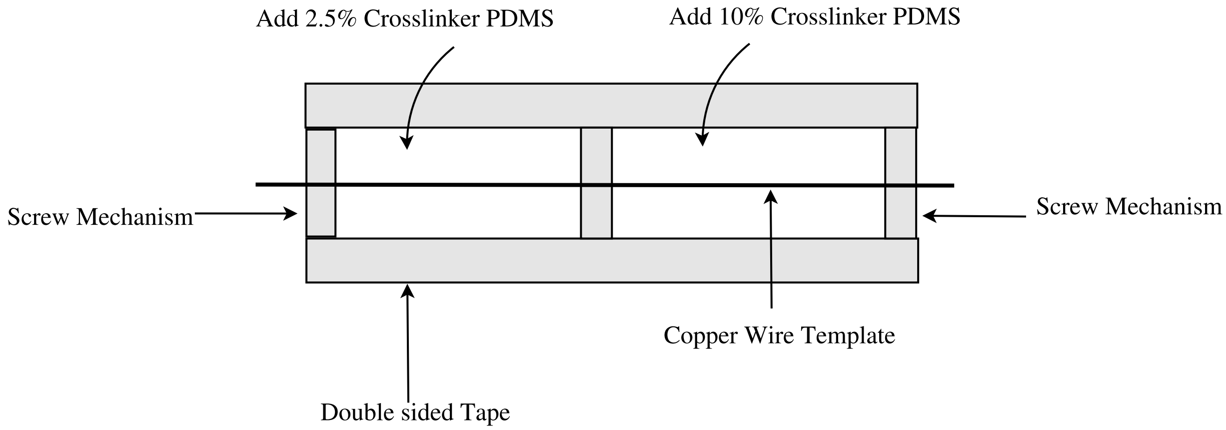


FIG. 1. Fabrication process for preparing deformable micro-tubes.

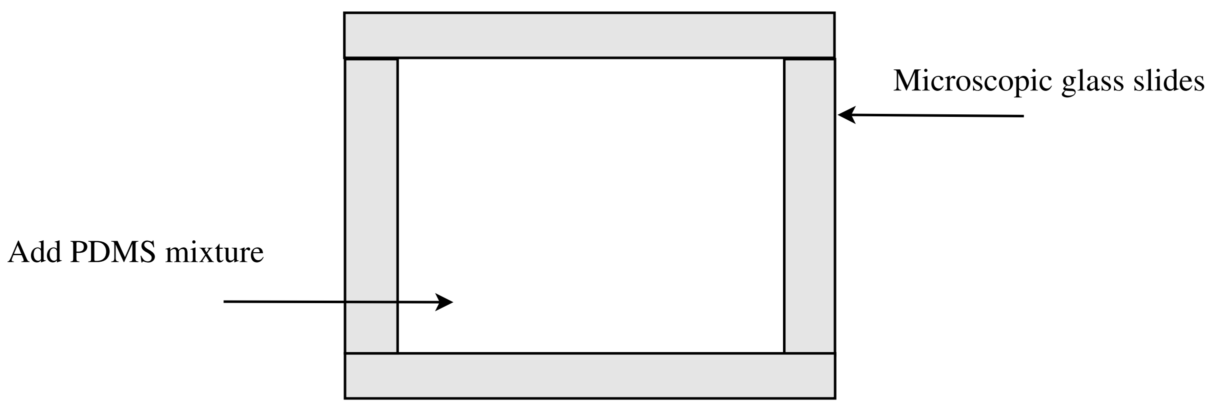
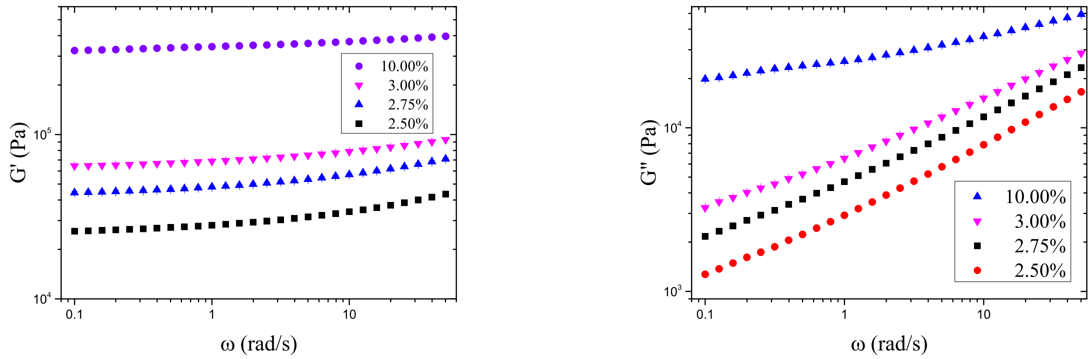


FIG. 2. Preparation of square PDMS block for characterizing the soft PDMS tube.

experiments. The PDMS block is kept on the bottom surface and the top plate is lowered until it experiences a normal force form the gel surface. A normal force of 0.5 N is used to perform oscillatory tests to measure shear modulus. A near-plateau region for shear modulus is obtained in the frequency range 0.1 Hz to 50 Hz. The shear modulus is calculated by taking the average of the shear modulus in the frequency range of $0.1 - 50$ Hz (Fig. 3). The shear and loss moduli increases with increase in the cross-linker percentage in PDMS.

C. Characterization of the polymer solutions

The polymer solution is prepared by slowly mixing polyacrylamide (MW=5×10^6) in deionised water by using a magnetic stirrer for 12 hours at a room temperature of $25°$C. Viscosity of polymer solutions is measured in the shear-rate controlled mode of the TA Discovery DHR-3 rheometer. Figure 4 shows that viscosity variation with shear rate is negligible in the measured shear rate range. However, the shear rates prevalent in our experiments are much higher because of the microtubes tubes used in our experiments, for which viscosity cannot be measured using our rheometer. The longest relaxation time for the polymer solutions is obtained by performing small amplitude oscillatory strain experiments and fitting the oscillatory data using the single-mode Maxwell model. Viscosity of the polymer solution is increased by adding glycerine to



(a) (b)

FIG. 3. Rheological characterization of the PDMS elastomer used to fabricate the deformable wall. (a) Shear, and (b) loss modulus as a function of angular frequency. Data obtained by performing small amplitude oscillatory strain experiments using a TA Discovery DHR-3 rheometer.

C_p (ppm)	Viscosity (mPa-s)	λ (ms)
50	1.00	1.41
100	1.07	1.41
300	1.25	1.76
600	1.78	2.60
900	2.05	3.44
1500	3.20	5.12
2000	4.30	6.52
2500	5.30	7.92

TABLE I. A summary of concentration (C_p) dependence of viscosity and relaxation time (λ) as inferred from rheological measurements.

provide sufficient stress signals for oscillatory experiments. Table. I summarises concentration (C_p) dependent viscosity and relaxation time (λ) as inferred from rheological measurements. The protocol used is similar to the one used in Ref. [29].

III. MICRO-PIV ANALYSIS FOR FLOW OF NEWTONIAN SOLVENT THROUGH DEFORMABLE MICRO-TUBES

Micro-PIV analysis is carried out for pressure-driven flow of Newtonian solvent (water) through deformable micro-tubes sized $\sim 400\mu$m. The micro-PIV setup is very similar to the one used in Ref. [29]. A syringe pump (Chemyx, Nexus 6000) is used for pumping the fluid through the deformable microtubes. All measurements are carried out at a distance of 3.5cm from the end of the development section. Processing of micro-PIV images is carried out using two different techniques. The first one involves averaging of multiple images to obtain the averaged velocity data. Velocity at the centerline is taken and then divided by the cross-sectional average velocity of the fluid flow. The laminar Newtonian value for center line peak to average velocity

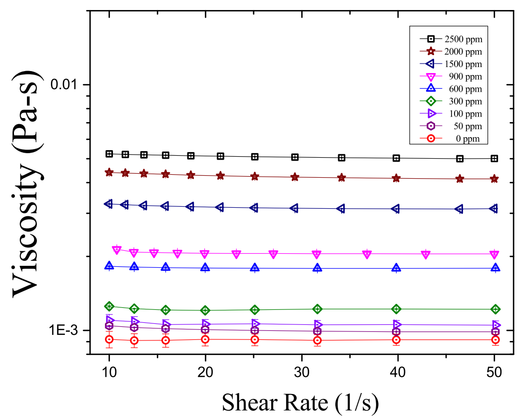
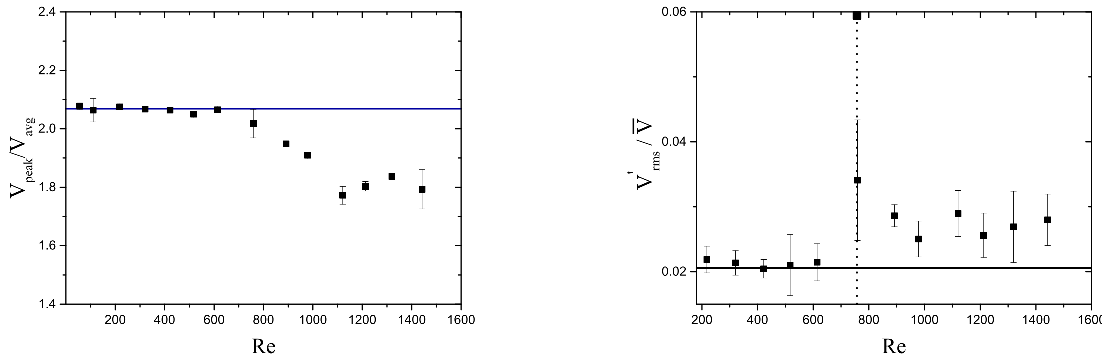


FIG. 4. Viscosity variation with shear rate for various concentrations of polyacrylamide solutions used in our experiments measured using a TA DHR-3 rheometer. This shows that shear thinning is negligible for all the solutions considered in our study.

is 2 for a rigid tube. However, for a deformable tube, this may slightly vary depending on the converging or diverging section. For a fixed location, for a fixed diameter, and for a fixed shear modulus of the tube, the center-line peak to average velocity remains constant. Second type of image processing involves calculating velocity at each time instant and then calculating standard deviation in velocity for that particular point. The standard deviation thus obtained is normalized by the time-averaged center-line velocity. A jump in the normalized standard deviation in the center-line velocity is considered as a signature for onset of transition. Further, a flattening of the velocity profiles also indicates the onset of transition. For all our micro-PIV data plots, V_{peak} is the center-line velocity obtained from micro-PIV measurements, and V_{avg} is the cross-sectional average velocity obtained from flow rate measurements. The root mean squre center-line velocity and the time-averaged center-line velocity are obtained from micro-PIV experiments. Figure 5 shows the onset of transition for the flow of pure water in a deformable microtube of shear modulus 53 kPa. A deviation in the peak to cross-sectional average velocity from its laminar value is considered as a signature for transition. Further, a jump in the normalized velocity fluctuations is also observed at the same Re as observed in deviation of the peak to cross-sectional average velocity. Transition for flow in deformable tube is observed at $Re \sim 800$ for a tube of shear modulus 53 kPa, which is much lower than that for Newtonian rigid tube transition which typically occurs at $Re \sim 2000$. Further, the magnitude of velocity fluctuations in the post-transition regime is much lower as compared to the rigid-wall transition. Experiments are performed for different shear moduli, different tube diameters, and different fluid viscosities to achieve varying wall elasticity parameter regimes.

The processing of images obtained by using micro-PIV is carried out as follows. For each experiment, hundred image-pairs are taken at the center plane of the tube, and each image pair is processed to obtain the velocity at each time instant. The center-line peak velocity is the average of the velocities obtained from each image pair. Similarly, two more sets of experiments are performed by obtaining hundred image pairs in each case. The center-line peak velocity is normalized by the cross-sectional average velocity. The error-bar in the plots of the ratio of peak to cross-sectional average velocity represents the standard deviation in the data from three sets of experiments, over three hundred velocity data. For obtaining the r.m.s velocity, the standard



(a) (b)

FIG. 5. (a) The variation of peak to cross-sectional average velocity with Re for a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa at a distance 4 cm from the development section for the flow of pure water. Horizontal straight line indicates laminar regime. (b) Normalized velocity fluctuations for the same deformable tube. Here V_{peak} is the center-line velocity obtained from micro-PIV measurements and V_{avg} is the cross-sectional average velocity obtained from flow rate measurements. The ratio of peak to cross-sectional average velocity is found to be slightly more than 2. This could be because the measurements are taken in the converging section of the deformed tube.

deviation in velocity is obtained from each experiment comprising of hundred image pairs. The data point representing the velocity fluctuation is the average of the standard deviations across the three experiments over three hundred velocity data points. Error bars on each data point represents the standard deviation of the r.m.s velocity obtained from the three experiments, which is less than 5% in our experiments. For obtaining the velocity profile, for each experiment, hundred pairs of images are processed by using the ensemble averaging algorithm. The averaging scheme calculates the average of velocity for the entire field of view across hundred images. The velocity represented in the velocity profile is the average of the velocity obtained on a line across the the three experiment set comprising of three runs.

The experiments are repeated three times to mark the onset of transition. The reported critical Reynolds number is determined by taking the average of the Re at which there is a jump in the normalized velocity fluctuations for three different experiments. The Reynolds number for each flow rate is calculated by accounting for the deformation in the tube in the radial direction caused by the applied pressure gradient. The deformation in the tube causes a change in the tube diameter which must be accounted for in calculating the actual Re. The deformation is larger for tubes with smaller shear modulus.

One of the methods to detect transition used by Srinivas et al[42] in deformable channels is to check for a flattening of velocity profile. A flattening of velocity profile is observed in the post-transition regime which is similar to the post transition regime for rigid wall transition for a Newtonian flow. Figure 6 represents the change in velocity profile with increasing Re in the post-transition regime for the flow of pure water through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. Figure 7 shows the onset of transition for the flow of 35% glycerine through a deformable micro-tube of diameter $\sim 400\mu$m and shear moduli 53 kPa. The transition Re is decreased as compared to the flow of pure water through the same deformable tube. Adding glycerine increases the fluid viscosity, which decreases the wall elasticity parameter Σ of the system. This decreases the transition Re, because the transition Re decreases with decrease in Σ.

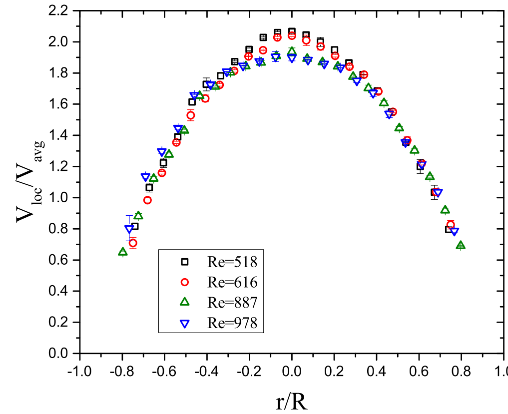
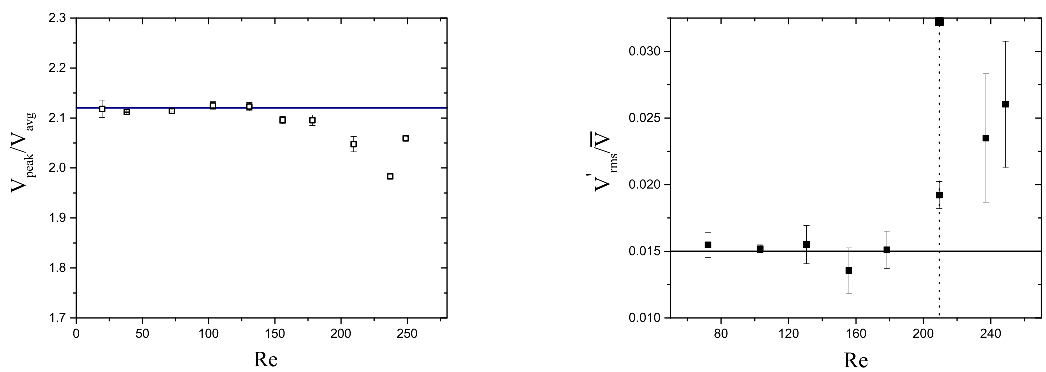


FIG. 6. Flattening of velocity profile in the post-transition regime for the flow of pure water through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. Here V_{loc} is the local velocity and V_{avg} is the cross-sectional average velocity.



(a) (b)

FIG. 7. (a) The variation of peak to cross-sectional average velocity with Re for a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa at a distance 4 cm from the development section for flow of 35% glycerine in water. Horizontal straight line indicate laminar regime. (b) Normalized velocity fluctuations for the same deformable tube. Here V_{peak} is the centerline velocity obtained from micro-PIV measurements and V_{avg} is the cross-sectional average velocity obtained from flow rate measurements.

Addition of glycerine causes no change in fluid elasticity and only increases the viscosity of the fluid.

Figure 8 compares the Re_t with varying wall elasticity numbers for Newtonian fluid flow in deformable tubes. The wall elasticity number Σ in the present study is lower (due to use of tubes of much smaller diameters) as compared to the earlier studies[21,22]. The transition Re_t obtained in our experiments agree reasonably well with earlier works[21,22]. The data from the three different experiments seem to reasonably follow the scaling relation of $Re_t \sim \Sigma^{0.65}$. It is instructive to discuss the possible reasons behind the slight disagreement between various experimental observations. Firstly, the method to characterize the onset is different among various studies, as Verma

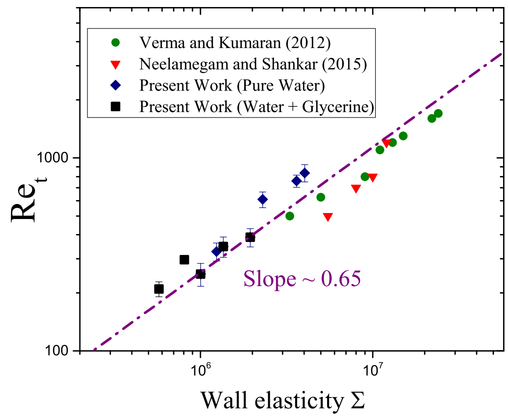


FIG. 8. Comparison of Re_t vs. Σ for Newtonian flow in deformable tubes: data obtained in the present work is shown along with the results of Verma and Kumaran (2012) and Neelamegam and Shankar (2015).

and Kumaran[21] and Neelamegam and Shankar[22] used the pressure difference between a point upstream of the test section and the end of the tube (assumed to be at atmospheric pressure), while the present study uses micro-PIV measurements within the test section. This difference in the procedure for detecting the transition could result in some difference in the transition Reynolds number. Further, it should be noted that transition experiments are often difficult to reproduce, even in standard cases such as flow in a rigid tube, a fact well-known since the pioneering experiments of Osborne Reynolds[1] for (rigid) pipe flows of Newtonian fluids[2]. This scenario for Newtonian pipe flows is a consequence of the finite-amplitude, subcritical nature of the transition, which makes the transition Reynolds number extremely sensitive to experimental conditions. As a result, the transition Reynolds number for Newtonian (rigid) pipe flows can range all the way from 2000 to 10^5. Considering this state of affairs in Newtonian pipe flows, it is indeed noteworthy that there is reasonable agreement (Fig. 8) for the dependence of transition Reynolds number with Σ for (Newtonian) flow through deformable tubes between the present experiments and previous results. For flow through deformable tubes, it is now well understood that the instability is triggered by infinitesimal perturbations[36], but the nature of the bifurcation is not yet fully understood. If the bifurcation is of a subcritical kind, then the transition Reynolds numbers from different experiments would be sensitive to experimental conditions, leading to some variations in results from different experiments. The only cases where the onset of transition can be reproduced well in experiments is when the bifurcation is supercritical, such as in Taylor-Couette flow in the gap between two concentric rotating cylinders[3].

The tube diameters used our study are very small ($\sim$400 μm) as compared to the tube diameters used in earlier works ($\sim$1200 μm). While smaller tubes require much larger pressure gradients for the flow, it has been well-documented[39] that due to the imposed pressure difference, there is some radial deformation in the tube, and the tube diameter has a weak variation ($\sim 1\%$) with the flow direction. This will sensitively depend on the thickness of the deformable wall, its elastic modulus etc. Such effects are not being factored by the single dimensionless group Σ (in fig. 8) that characterizes wall elasticity. More importantly, Verma and Kumaran[21] have carried out experiments in a *rigid* tube with diameter variation that mimics the actual variation in a deformable tube and showed that the transition Reynolds number is not lowered for this case when the wall is rigid. Thus, it is the fluctuations in the wall displacement that is the primary reason for the instability.

This is also borne out by theoretical studies[36], where the key mechanism that drives the instability in flow through deformable tubes is the coupling that occurs between the laminar flow and perturbations via the velocity and stress continuity conditions at the interface, and this is a "local" phenomenon at a given axial location, largely independent of the (weak) tube diameter variation. Thus, the weak diameter variation in the flow direction is not *esential* for the instability, and is a secondary effect that is difficult to be subsumed in a *single* dimensionless group. We nonetheless observe a reasonable data collapse (in the Re_t vs Σ plot) among different experiments in Fig. 8. It is also important to point out that for flow of Newtonian fluids through rigid tubes, there is good agreement for Re_t with Ref. 4. It is further well established with the use of linear stability analysis that the wall mode instability follows the scaling relationship of $Re_t \sim \Sigma^{0.75}$. The scaling relationship obtained by combining the data for Re_t from different experiments, viz., $Re_t \sim \Sigma^{0.65}$ is in close agreement with the scaling relationship obtained using linear stability analysis.

IV. MICRO-PIV ANALYSIS FOR FLOW OF POLYMER SOLUTION THROUGH DEFORMABLE MICRO-TUBES

A similar procedure procedure is carried out to detect the onset of transition in the flow of polymer solutions. Transition is deemed to occur when there is a decrease the in the ratio of peak to cross sectional average velocity, and when a jump is detected in the normalized velocity fluctuation data. Figure 9 shows the onset of transition for the flow of 50ppm and 100ppm polyacrylamide solutions through a deformable micro-tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. Addition of 50ppm of polyacrylamide (abbreviated PAAm henceforth) destabilizes the flow at $Re_t \sim 550$ which is lower than the transition $Re_t \sim 700$ obtained for the flow a Newtonian fluid through the same deformable tube. For the case of flow of 50ppm PAAm solution through rigid tubes, there was a slight delay in the transition Reynolds number compared to the Newtonian value. For flow of the same polymer solution in a deformable tube, the drastic reduction in Re_t to 550 indicates the destabilizing role played by wall deformability. Fluid elasticity is also found to destabilize the flow as the addition of 50ppm PAAm destabilizes the flow at $Re_t \sim 550$ as compared to as compared to the Newtonian value of $Re_t \sim 700$ through the same deformable tube. Hence, both wall elasticity and fluid elasticity play destabilzing roles. Flow of 100ppm PAAm solution through a deformable tube of shear modulus 53 kPa becomes unstable at a similar $Re_t \sim 700$ as compared to the flow of Newtonian fluid through the same deformable tube. Flow of 100ppm PAAm through a rigid tube delays the transition Re_t when compared to the flow of Newtonian fluid through a rigid tube. Hence wall elasticity has a destabilizing effect on the onset of transition for the flow of 100ppm PAAm solution.

Figure 10 shows the onset of transition for the flow of 300ppm, 600ppm and 900 ppm polyacrylamide solutions through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. The onset of transition for the flow of 300ppm PAAm solution through a rigid tube occurs at $Re \sim 1700$. With the presence of a deformable wall, the transition Re_t falls to $Re_t \sim 450$. Hence, the soft wall clearly has a destabilizing effect on the onset of transition. The transition Re_t is lower than the transition Re_t obtained for the flow of Newtonian fluid through the same deformable tube. Hence fluid elasticity also has a destabilizing effect on the onset of transition. Figure 11 shows the onset of transition for the flow of 1500 ppm, 2000 ppm and 2500 ppm polyacrylamide solutions through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. With increasing polymer concentration, we observe that increase in fluid elasticity destabilizes the flow at lower Re_t. For the highest polymer concentration used, i.e., 2500ppm, the transition Re_t reaches as low as 75. Figure 12 represents the evolution of velocity profile observed in the post-transition regime

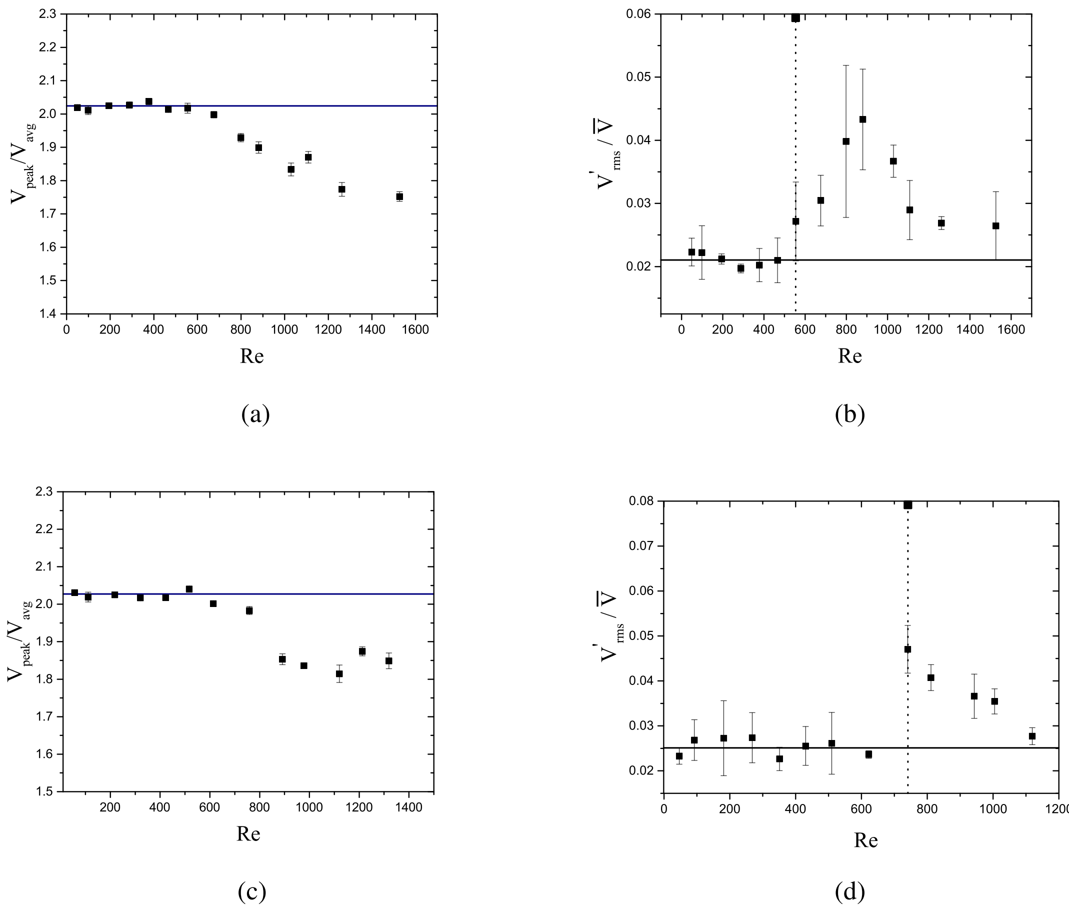
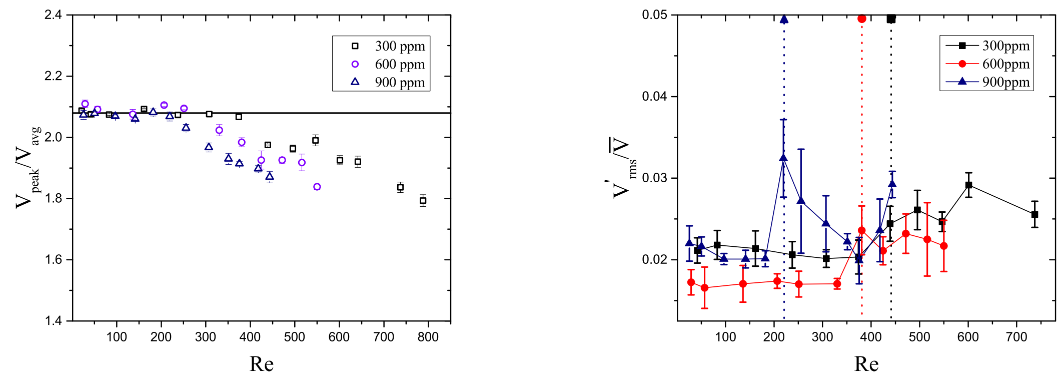


FIG. 9. The variation of peak to cross-sectional average velocity with Re for a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa at a distance 4 cm from the development section for flow of (a) 50ppm Paa solution and (b) 100ppm PAAm solution. Horizontal straight line indicates laminar regime. Normalized velocity fluctuations for the same deformable tube for (c) 50ppm PAAm and (d) 100ppm PAAm (d). Here V_{peak} is the center-line velocity obtained from micro-PIV measurements and V_{avg} is the cross-sectional average velocity obtained from flow rate measurements.

for the flow of 2500ppm polyacrylamide through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa. A flattening of velocity profile is observed in the post-transition regime.

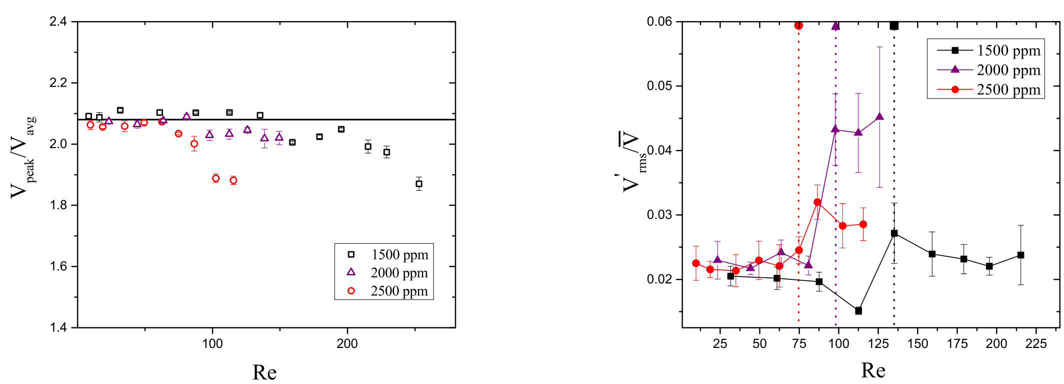
V. DISCUSSION

Micro-PIV analysis is carried out to investigate transition in flow of Newtonian and viscoelastic solutions through deformable micro-tubes. Soft-wall transition occurs at Re_t much lower than Re at which transition occurs in rigid-walled tubes for the flow of both Newtonian and polymer solutions. For 50ppm and 100ppm polyacrylamide solutions, transition to turbulence occurs at a lower Re_t as compared to the flow of Newtonian solution through deformable tube of similar shear modulus. When data for the Re_t vs wall elasticity parameter (Fig. 13) is plotted for polymer fluid flow of different concentrations, diameters and shear moduli is plotted, it obeys the relation $Re \sim \Sigma^{0.7}$ which is suggestive of a wall mode instability[21,41]. Further, when the data of different polymer



(a) (b)

FIG. 10. (a) The variation of peak to cross-sectional average velocity with Re for a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa at a distance 4 cm from the development section for flow of 300-900ppm polymer solution. Horizontal straight line indicate laminar regime. (b) Normalized velocity fluctuations for the same deformable tube and for the same 300-900ppm polymer solution. Vertical dotted line indicates onset of transition. Here V_{peak} is the centerline velocity obtained from micro-PIV measurements and V_{avg} is the cross-sectional average velocity obtained from flow rate measurements.



(a) (b)

FIG. 11. (a) The variation of peak to cross-sectional average velocity with Re for a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa at a distance 4 cm from the development section for flow of 1500-2500ppm polymer solution. (b) Normalized velocity fluctuations for the same deformable tube and for the same system. Vertical dotted line indicates onset of transition.

concentrations is plotted along with the data for a Newtonian fluid, it seems to obey a similar scaling relation. This indicates that both fluid elasticity and wall elasticity have a destabilizing role in the flow of polymer solutions through deformable tubes. Neelamegam et al[22] observed a power law scaling of $3/2$ for the flow of Newtonian fluid through deformabe tubes, which might be due to the fact that the transition was investigated in the diverging section of the tube. Further, the number of data points to obtain the scaling relationship was as low as 4, which might not be enough to get a reliable scaling law.

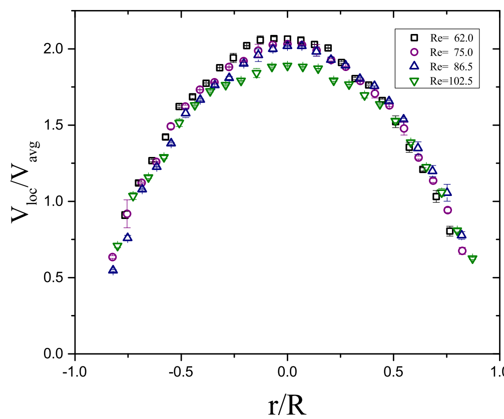


FIG. 12. Flattening of velocity profile in the post-transition regime for the flow of 2500ppm polyacrylamide solution through a deformable tube of diameter $\sim 400\mu$m and shear modulus 53 kPa.

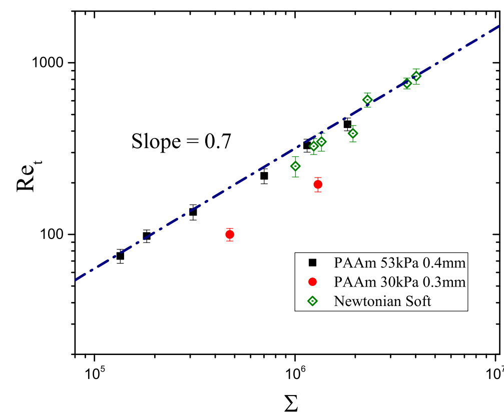


FIG. 13. Variation of Re_t with wall elasticity parameter Σ (for different shear modulus and different tube diameters) for the flow of Newtonian fluids and PAAm solutions through deformable tubes with varying shear moduli, fluid viscosities and tube diameters. Table II contains details of the shear modulus and tube diameter corresponding to Re_t for Newtonian fluid and Table III contains details of the shear modulus and tube diameter corresponding to Re_t for PAAm solutions.

Figure 14 shows a plot of Re_t vs $E(1-\beta)$ to compare the transition in the flow of polymer solution through rigid and deformable tubes. The dimensionless group $E(1-\beta)$ is used because the use of the factor $1-\beta$ shows the effect of the amount of solvent present in the solution. If the value of β is 1, then the fluid is purely Newtonian and as the value of β decreases, the solvent contribution decreases. The transition Re decreases with the introduction of a soft wall for the same values of elasticity number. Hence the introduction of a soft wall has a destabilizing effect on the 'early transition' observed in flow of polymer solutions through rigid tubes. It is not possible to isolate the physical effect underlying the destabilization as being that of fluid elasticity or wall elasticity. In all likelihood, our results suggest that it is a combined consequence of both fluid. Srinivas et al[25] studied the onset of transition for the flow of polymer solution through a

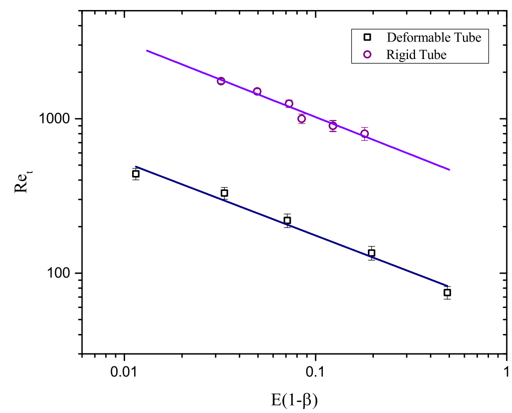


FIG. 14. Variation of Re_t with the dimensionless group $E(1-\beta)$ for onset of transition in rigid and deformable tubes. Dependence of Re_t with $E(1-\beta)$ follows a scaling relation as $Re_t \sim E(1-\beta)^{-0.5}$ for flow through rigid and deformable tubes.

deformable channel with one wall made of soft PDMS. Onset of transition is observed at $Re = 30$ for a deformable channel with bottom wall soft with shear modulus 17 kPa for 50ppm polymer solution. We do not observe transition at such low Re for a 50ppm polymer solution for our system. This may be attributed to the higher shear modulus of our tubes (53 kPa) used for our polymer flow experiments. Only one of the channels walls is soft for Srinivas et al[25], which may account for a different transition mechanism as compared to a symmetric system.

In order to provide a quantitative basis to the experimental scaling of the Reynolds number, it is instructive to derive the relation between Re and Σ using a scaling argument, under the assumption that the unstable modes belong to a class of wall modes[36] in Newtonian flow through a deformable tube. For these modes, at high Re, the velocity disturbances in the fluid are confined in a thin region ('wall layer') near the fluid-deformable solid boundary. The thickness of this wall layer is found to scale as $Re^{-1/3}R$, where R is the tube radius. Thus, the viscous stresses in the fluid at the interface would scale as $V\eta/(Re^{-1/3}R)$. There is no wall layer in the solid, and consequently no sharp regions of rapid variations, and hence the elastic stresses in the solid simply scales as G. By balancing the viscous stress in the wall layer to the elastic stress in the solid, we obtain $Re \sim \Sigma^{3/4}$.

The scaling relation between Re_t and E can be inferred as follows[30] for unstable center modes. For this class of modes, the disturbances are confined in a region of thickness $\delta \sim O(Re^{-1/3}R)$. However, in order for the elastic stresses and viscous relaxation to be of the same order, it is necessary to have $\delta \sim E^{1/2}$. Combining these two relations, we obtain $Re \sim E^{-3/2}$. For a polymer solution, the appropriate quantity that characterizes elasticity is $E(1-\beta)$, which vanishes in the limit ($\beta \to 1$) of a pure solvent. Thus, we obtain $Re \sim (E(1-\beta))^{-3/2}$. This is obtained in the absence of shear thinning effects. Inclusion of shear thinning will further reduce the power law exponent, and the theoretical prediction will be more consistent with the experimental exponent of -0.5.

Table II provides a list of the values of shear modulus of the deformable tube G (kPa), wall elasticity parameter (Σ) and transition Re_t for the flow of pure water through deformable tubes. Increase in shear modulus increases Re_t which shows that introduction of a deformable wall has a destabilising effect on the Newtonian transition for rigid tubes. Table III shows the values of concentration (C_p), shear modulus of the deformable tube G (kPa), wall elasticity parameter (Σ),

G (kPa)	Glycerine (%)	Diameter (mm)	$\Sigma \times 10^5$	Re_t
30	0	0.3	12.4	327
53	0	0.4	36.1	759
74	0	0.4	40.3	836
30	0	0.4	23.0	327
30	10	0.4	10.0	250
53	10	0.4	19.5	388
30	20	0.4	13.6	347
53	20	0.4	8.07	297
53	35	0.4	5.75	209

TABLE II. A summary of our experimental results for the onset of transition of Newtonian fluid (water) flow in a deformable tube.

C_p (ppm)	G (kPa)	$\Sigma \times 10^5$	$E(1-\beta)$	Re_t
50	53	27.9	0.002	554
100	53	26.9	0.004	740
300	53	18.3	0.011	439
600	53	11.8	0.033	330
900	53	7.02	0.071	219
1500	53	3.11	0.196	135
2000	53	1.82	0.349	98
2500	53	1.35	0.196	75
300	20	13.0	0.0091	196
600	20	4.72	0.046	100

TABLE III. A summary of our experimental results for tubes of different shear moduli, and for solutions of different polymer concentrations. Also shown are the relevant dimensionless groups Σ, $E(1-\beta)$ and Re_t.

elasticity number ($E(1-\beta)$) and transition Re_t for the flow of PAAm solution through deformable tubes. Addition of polymer to pure water is seen to have a destabilizing effect on the soft wall instability for the flow of Newtonian fluid.

VI. CONCLUSION

Micro-PIV measurements are carried out to investigate the onset of laminar-turbulent transition in the flow of Newtonian and viscoelastic fluids through deformable tubes. In general, we find that the addition of polymer to a Newtonian fluid destabilizes the flow in a deformable tube, leading to instabilities at lower Re for polymer solutions compared to Newtonian flow in deformable tubes. Similarly, the effect of wall deformability is also destabilizing for the flow of polymer solutions through rigid tubes. The post-transition behaviour for Newtonian and polymer fluid flow through deformable tubes is very different as compared to the classical Newtonian rigid wall transition. A plot between transition Re and the wall elasticity parameter Σ shows a scaling relation of $Re \sim \Sigma^{0.7}$, which is an indicator of the presence of wall modes[21,23,36]. The data for Re_t vs Σ seems to collapse for Newtonian fluid flow through deformable tubes, and for the flow of polymer solutions through rigid and deformable tubes. Our results suggest that we may view the transition in flow of polymer solutions through deformable tubes tubes via two different viewpoints: (i) Modification of the elastoinertial instability[27,29] for the flow of polymer solutions through rigid tubes, by making the wall elastic, and (ii) modification of the soft-wall instability[21,22] by making the fluid elastic via addition of polymers. Based on the present experiments, it is not possible for us to unambiguously ascribe the mechanism that causes the instability to either fluid elasticity or wall elasticity.

The scaling relation obtained in our study for the plot of transition Re vs wall elasticity ($Re_t \propto \Sigma^{0.7}$) is strongly suggestive of the presence of wall mode instability in the flow[21,23,36]. Further, the plot of Re_t with elasticity number E shows a scaling relation similar to the one reported in [29] for elasto-inertial instability in rigid tubes. Hence, it seems plausible that the observed instability is an outcome of the combination of soft-wall transition which is present in the flow Newtonian solvent through deformable tubes and elasto-inertial instability present in the flow of polymer solutions through rigid tubes. The minimum transition Re_t obtained in the present work, for a combination of wall elasticity and fluid elasticity, is found to be as low as 75. This can be potentially exploited in microfluidic mixing applications, where mixing at low Re is desirable.

The authors thank the Department of Science and Technology, Government of India, for financial support (Grant EMR/2016/001218). The authors would also like to thank Mr Ramkarn Patne, Mr Parag Joshi and Mr Abhishek Ratnam for providing insights and useful suggestions while conducting the experiments.

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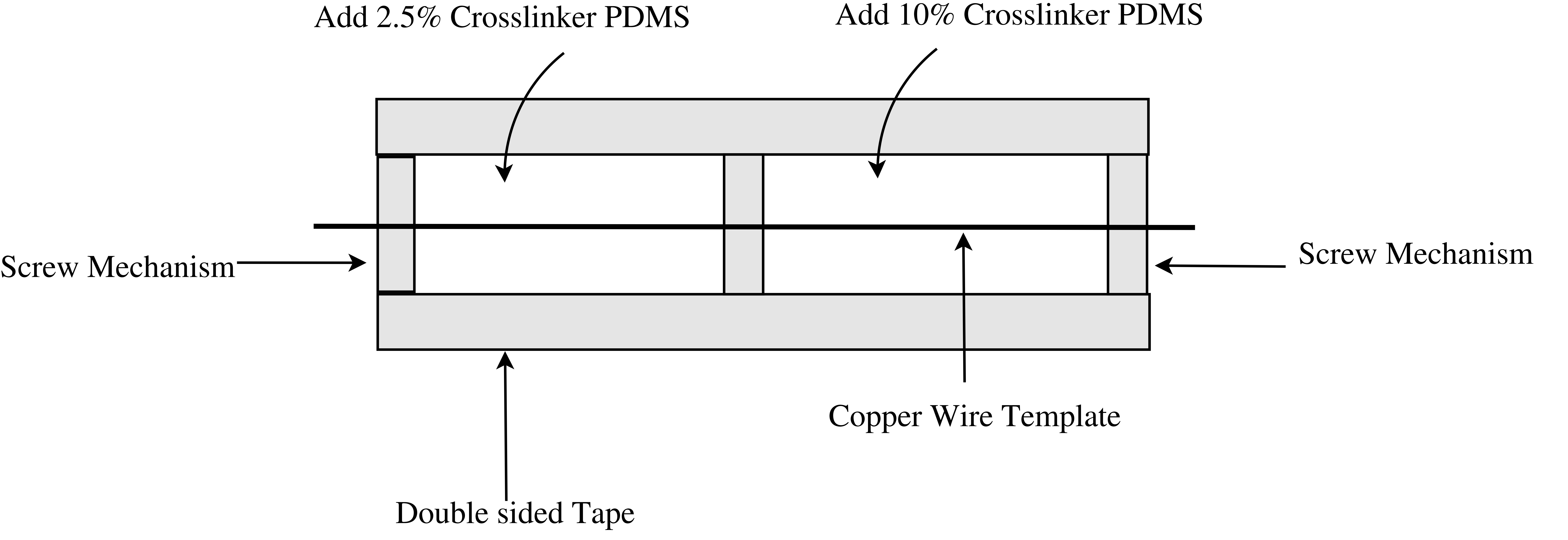
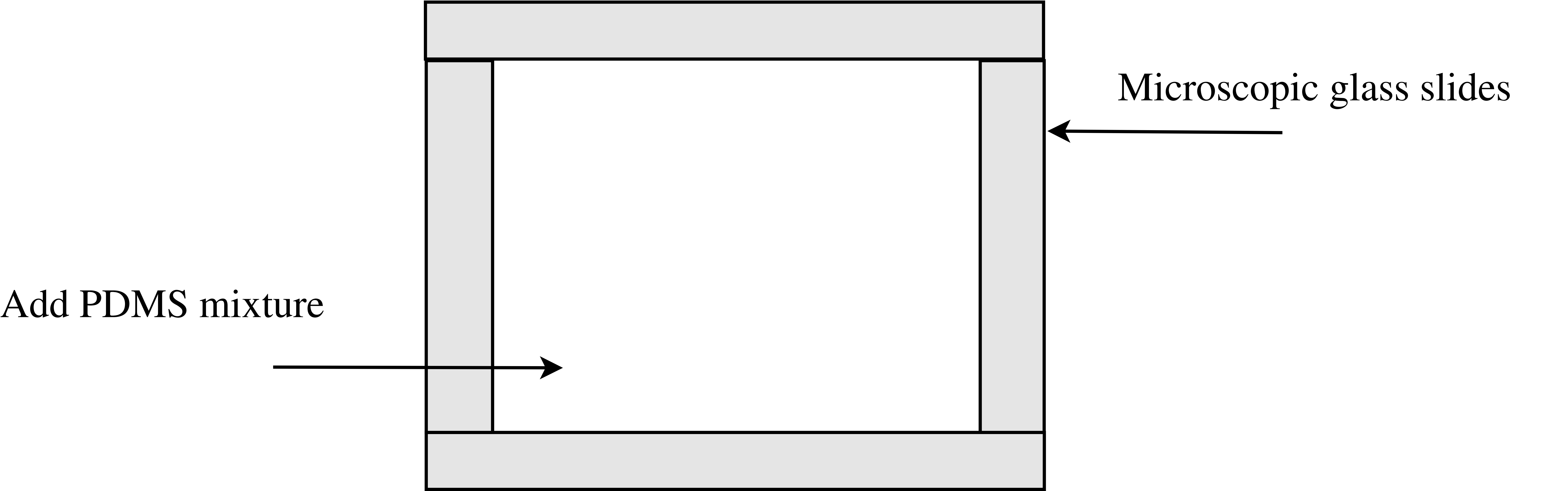
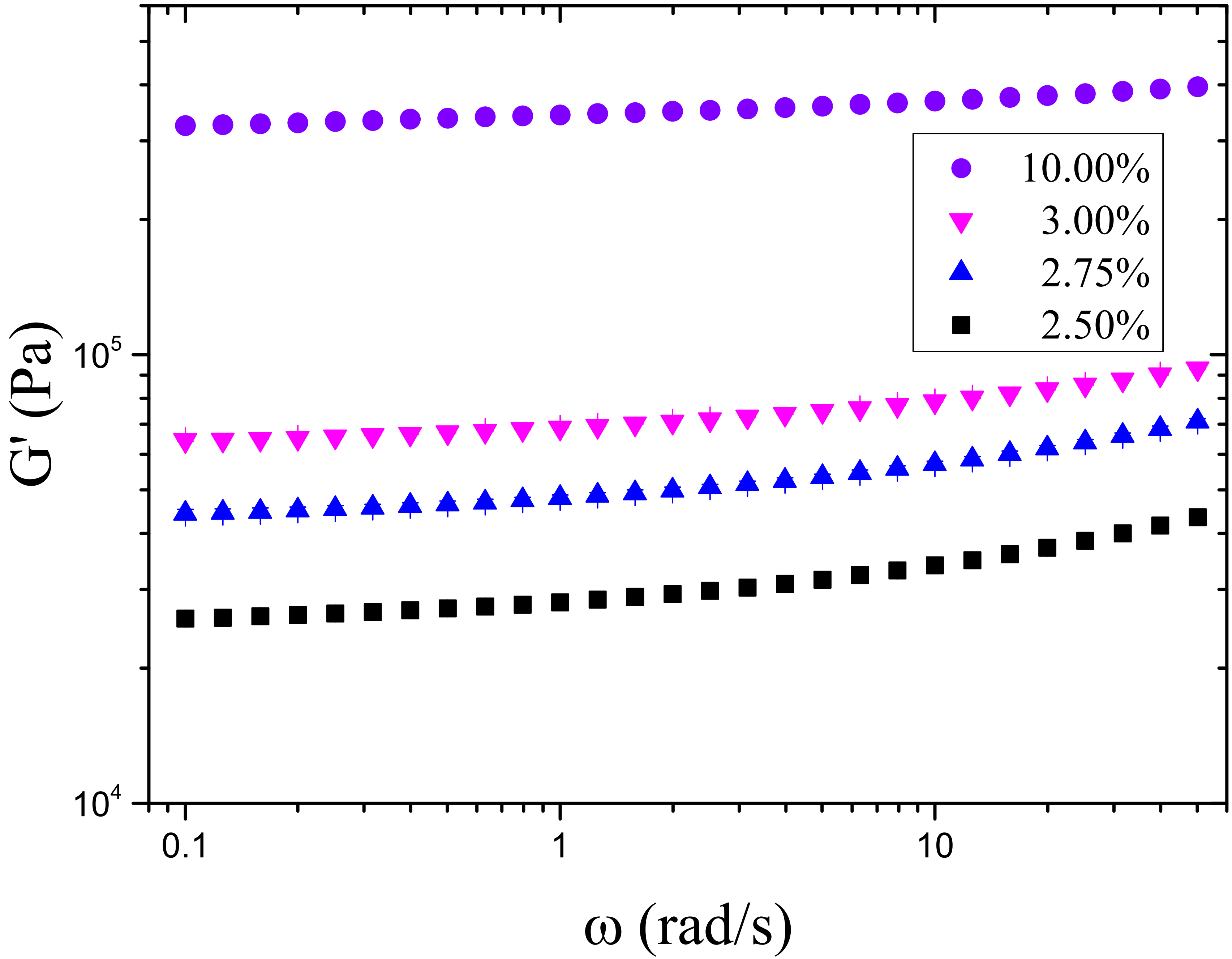
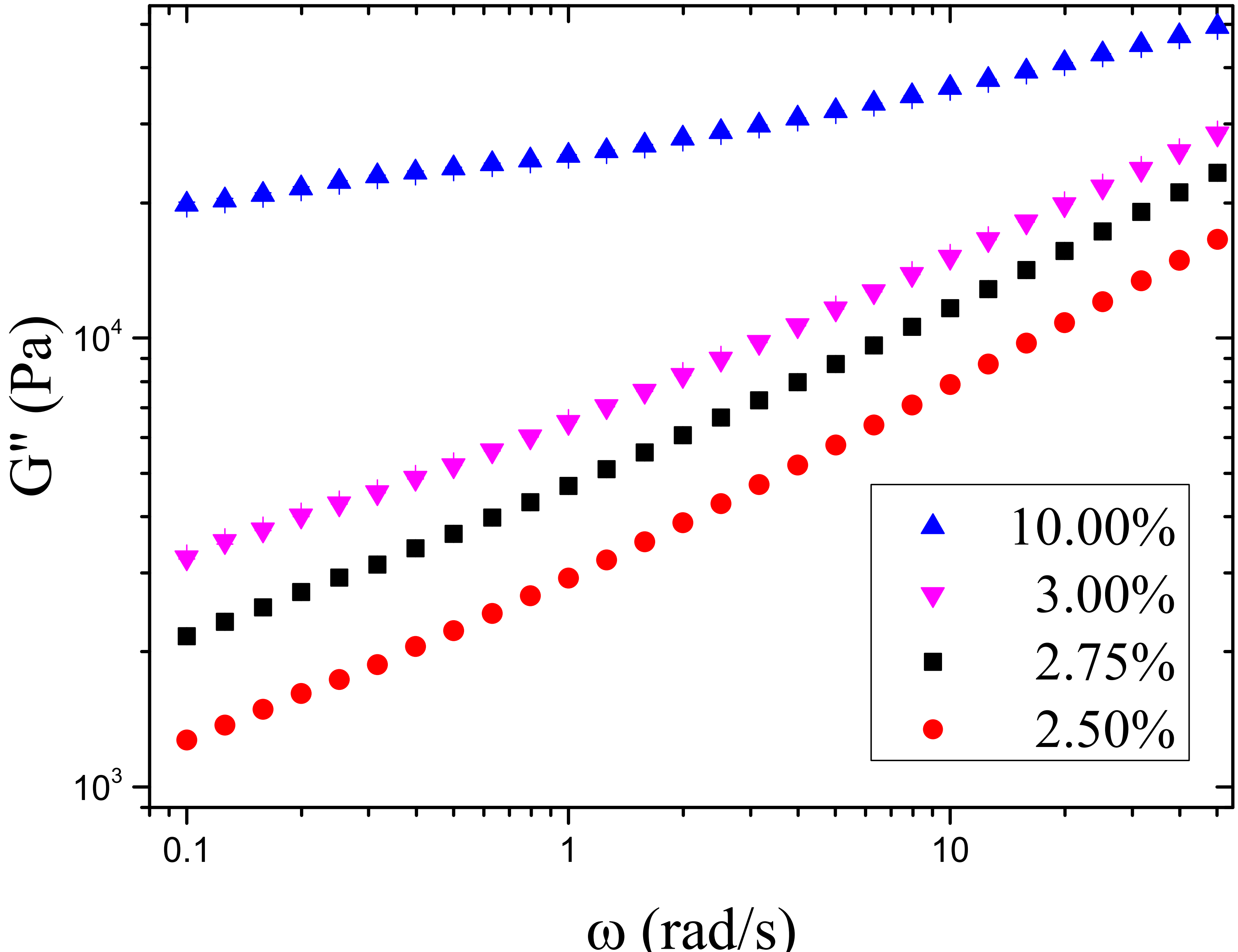
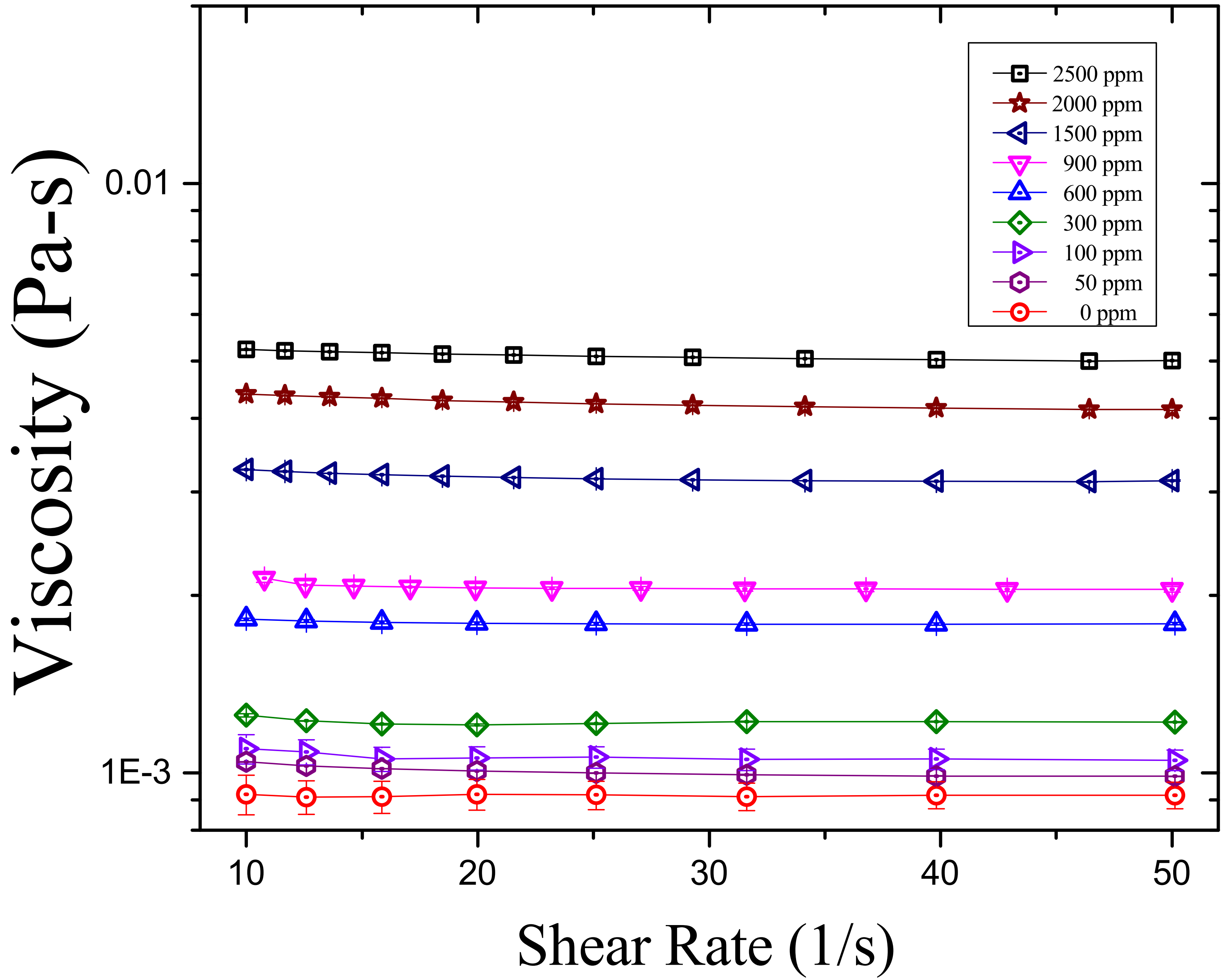

Add 2.5% Crosslinker PDMS
Add 10% Crosslinker PDMS
Screw Mechanism
Screw Mechanism
Copper Wire Template
Double sided Tape


Microscopic glass slides
Add PDMS mixture

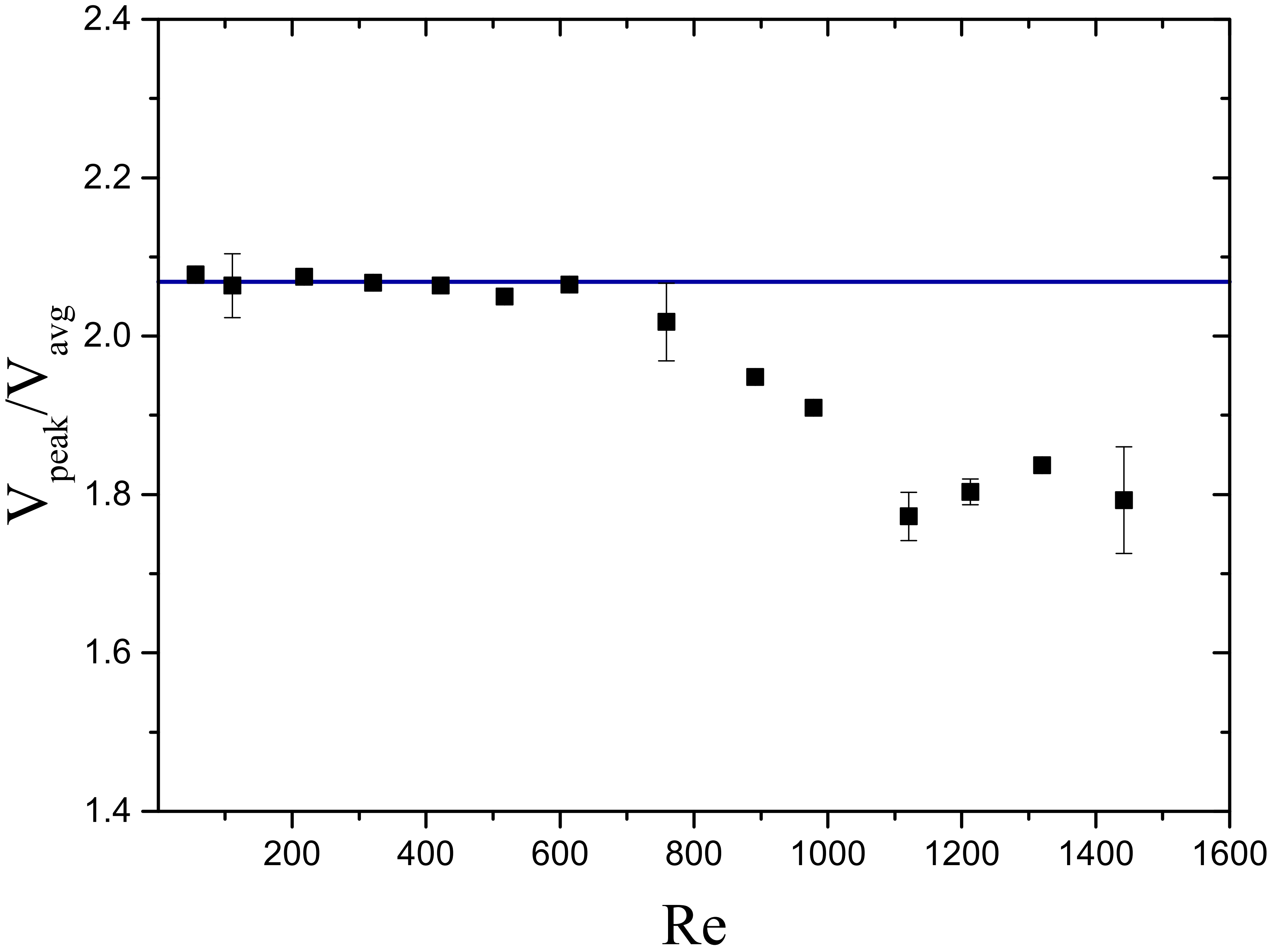

10.00%
3.00%
2.75%
2.50%
G' (Pa)
10⁵
10⁴
0.1
1
10
ω (rad/s)



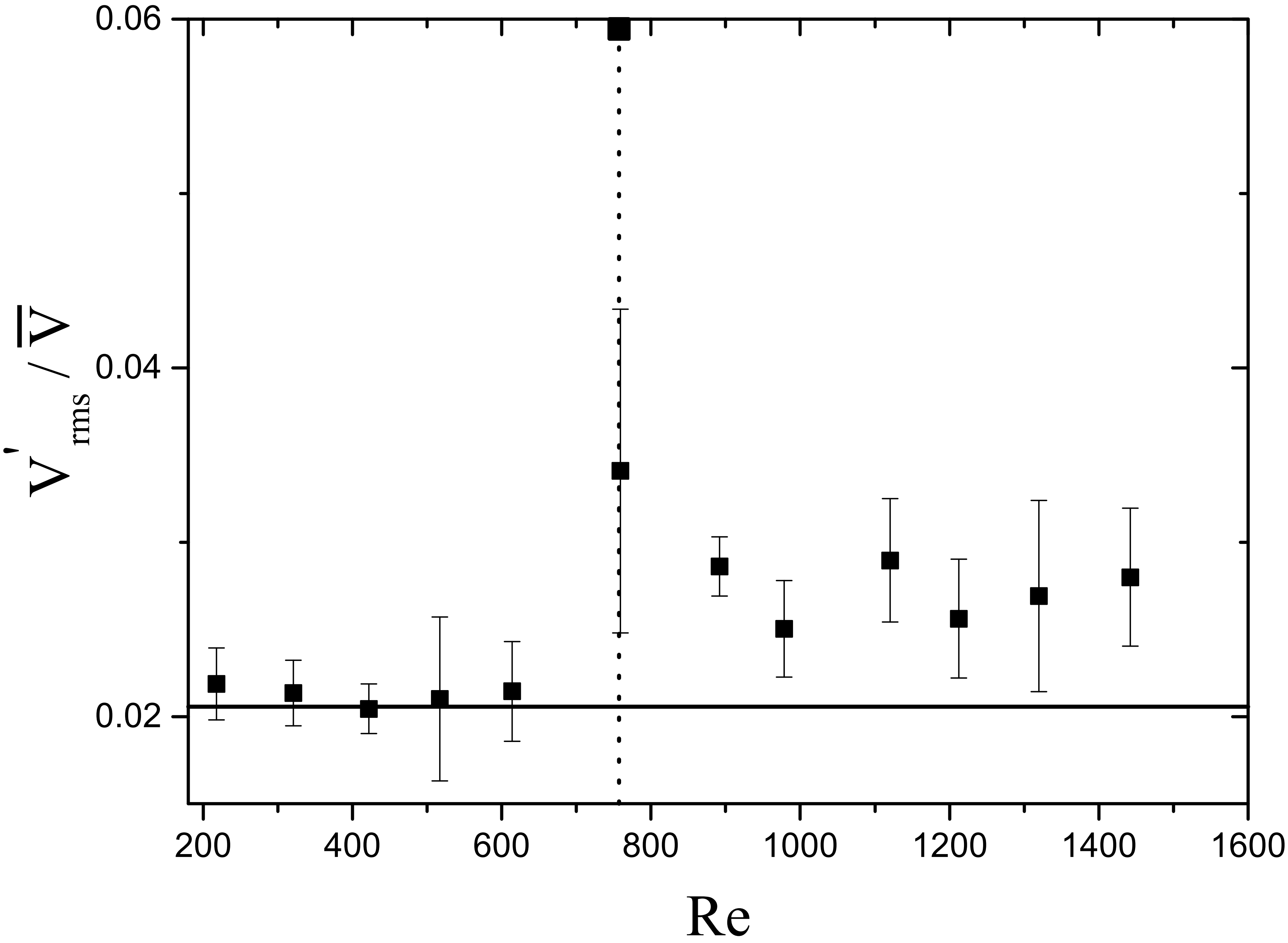
G'' (Pa)
10^4
10^3
0.1
1
10
ω (rad/s)
10.00%
3.00%
2.75%
2.50%

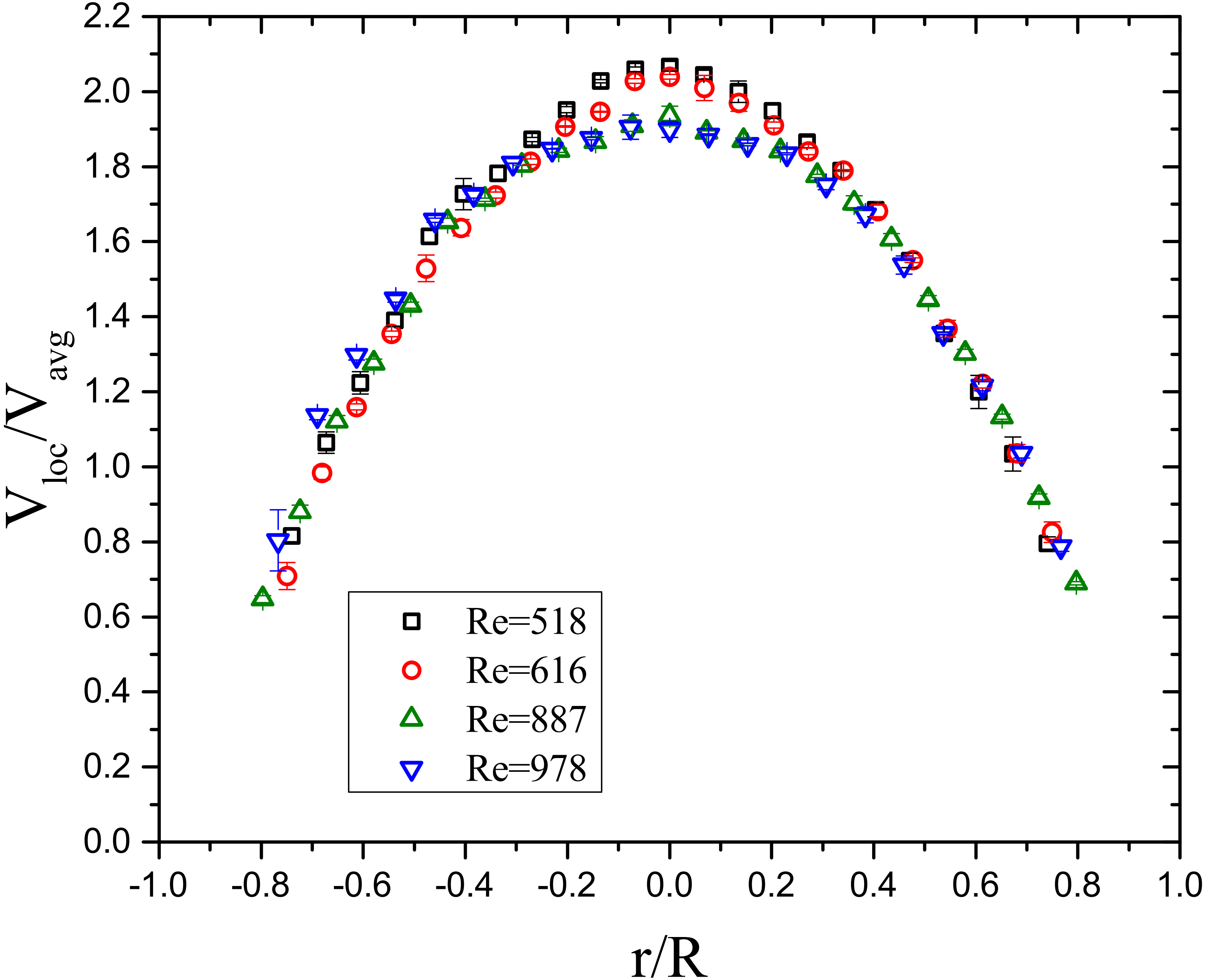


Viscosity (Pa-s)
0.01
1E-3
10
20
30
40
50
Shear Rate (1/s)
2500 ppm
2000 ppm
1500 ppm
900 ppm
600 ppm
300 ppm
100 ppm
50 ppm
0 ppm

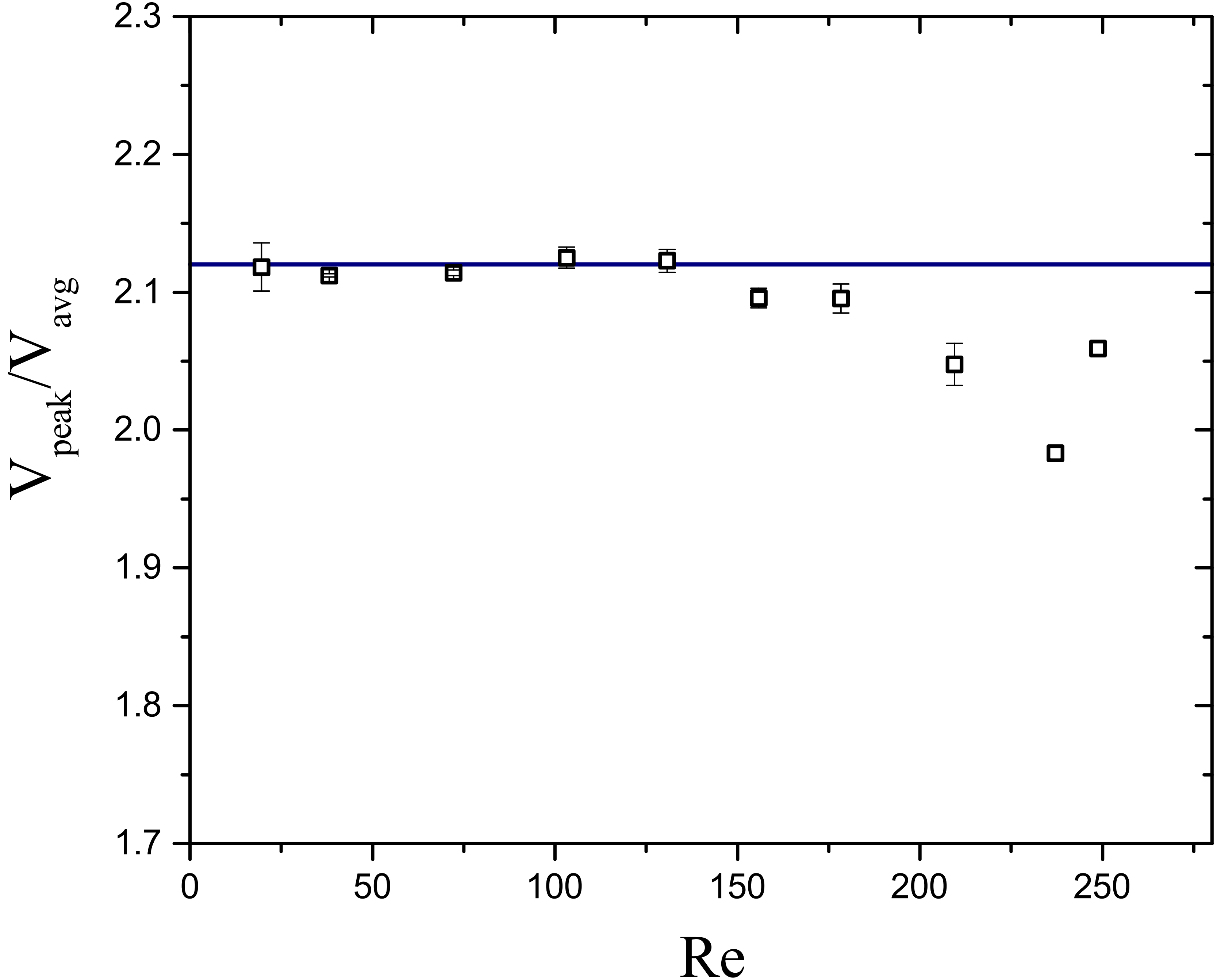


2.4
2.2
2.0
1.8
1.6
1.4
200
400
600
800
1000
1200
1400
1600
V_{peak}/V_{avg}
Re

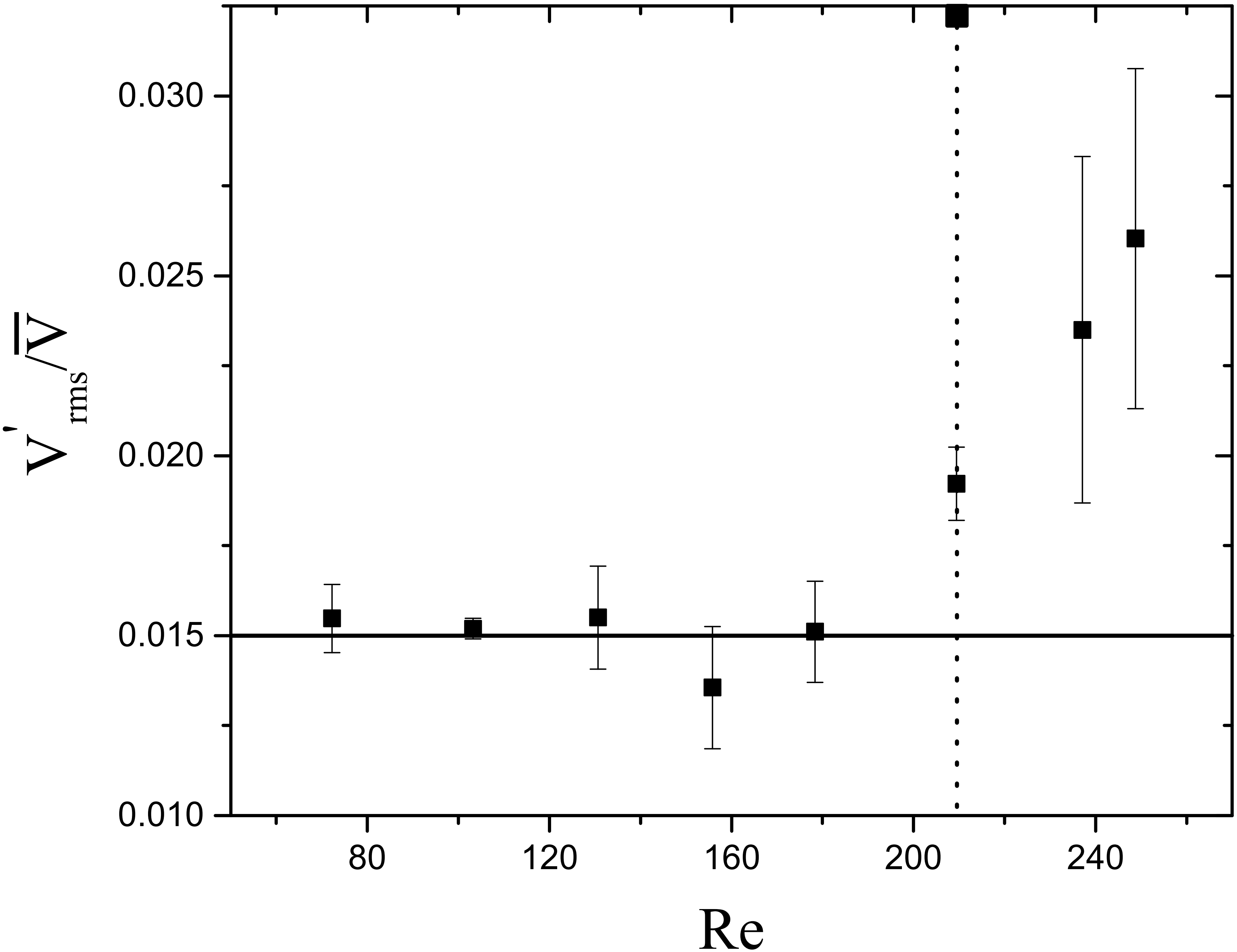

0.06
0.04
0.02
200
400
600
800
1000
1200
1400
1600
Re
V'rms / V̄



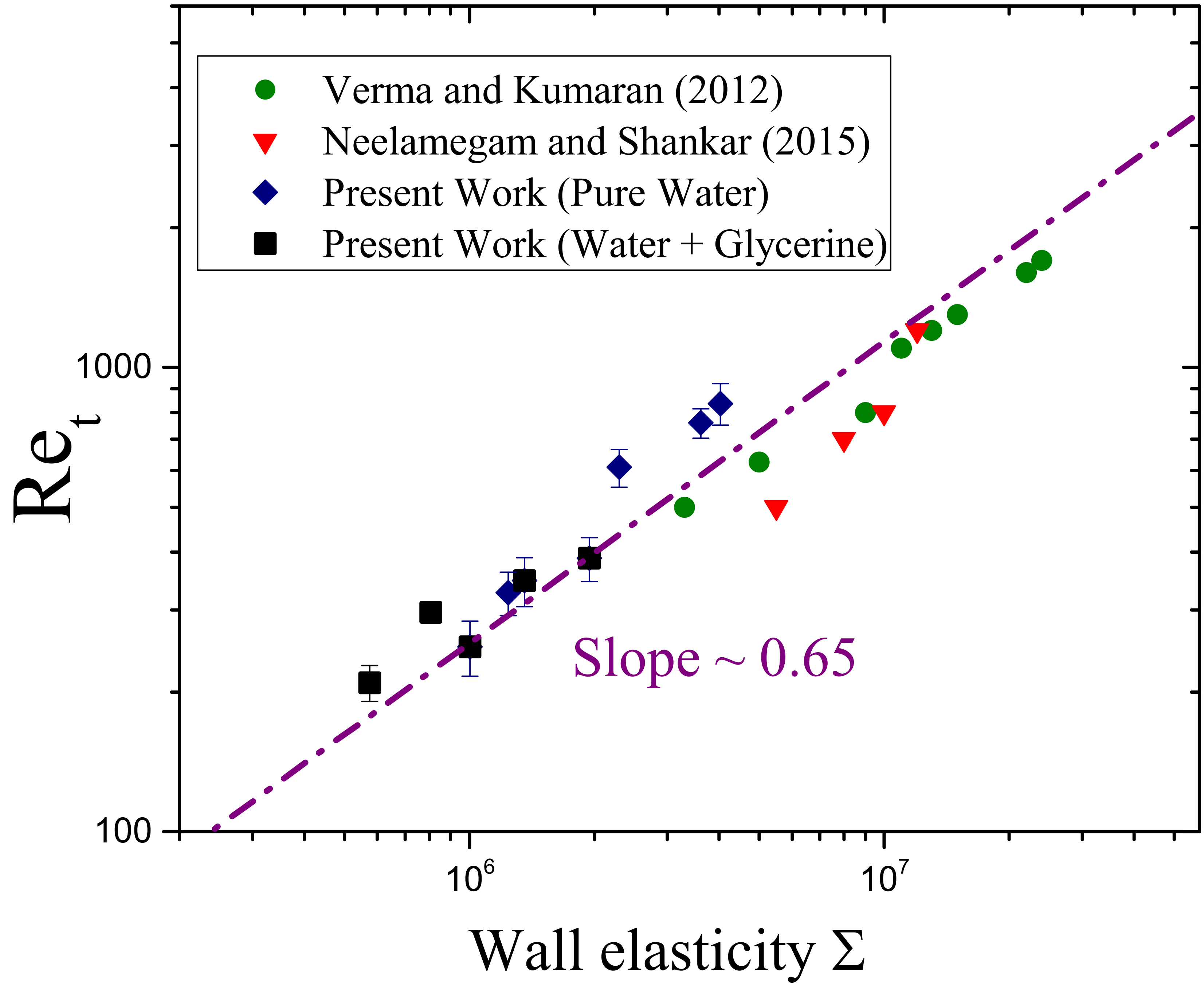
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
V_{loc}/V_{avg}
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
r/R
Re=518
Re=616
Re=887
Re=978



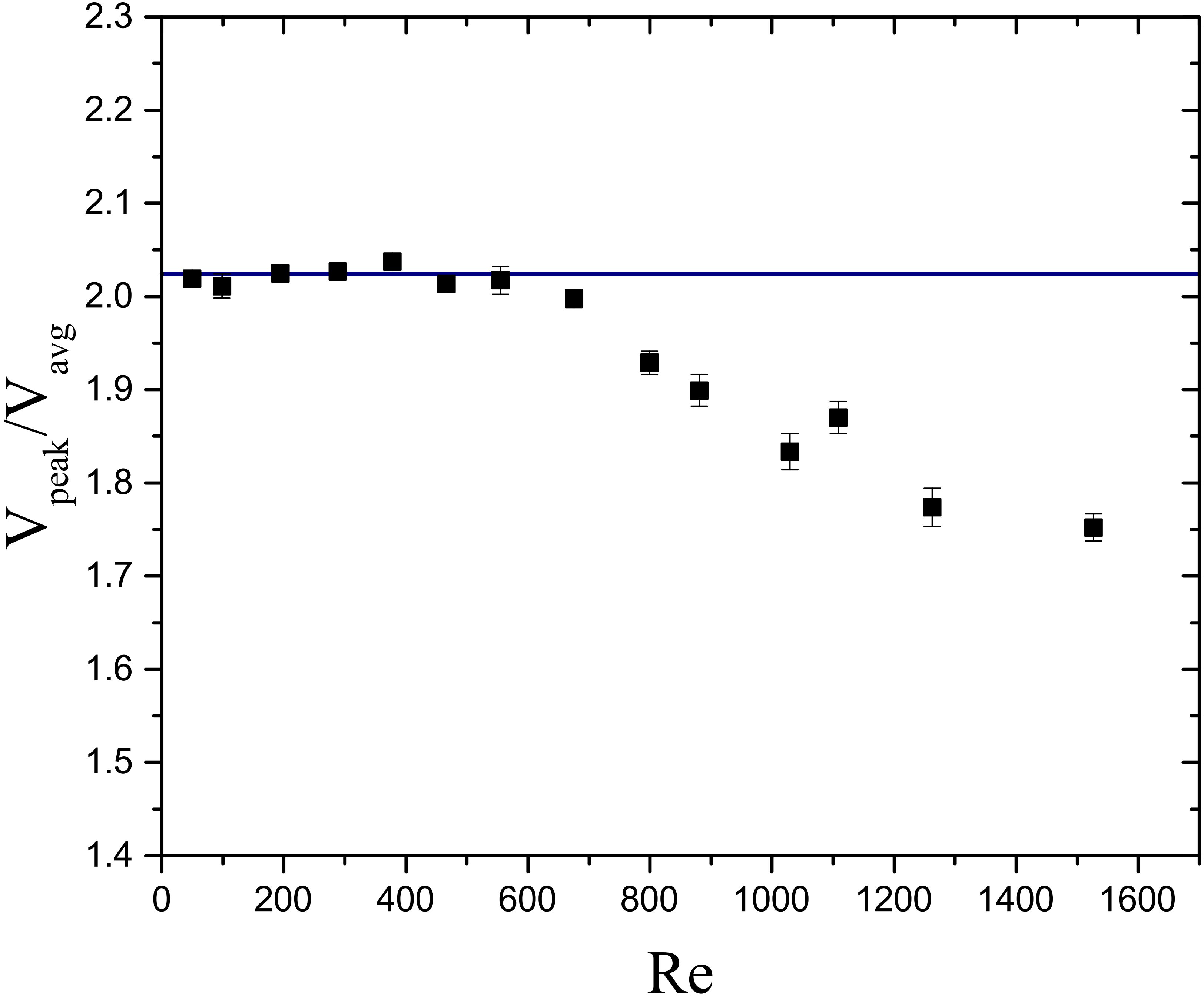
2.3
2.2
2.1
2.0
1.9
1.8
1.7
0
50
100
150
200
250
V_{peak}/V_{avg}
Re



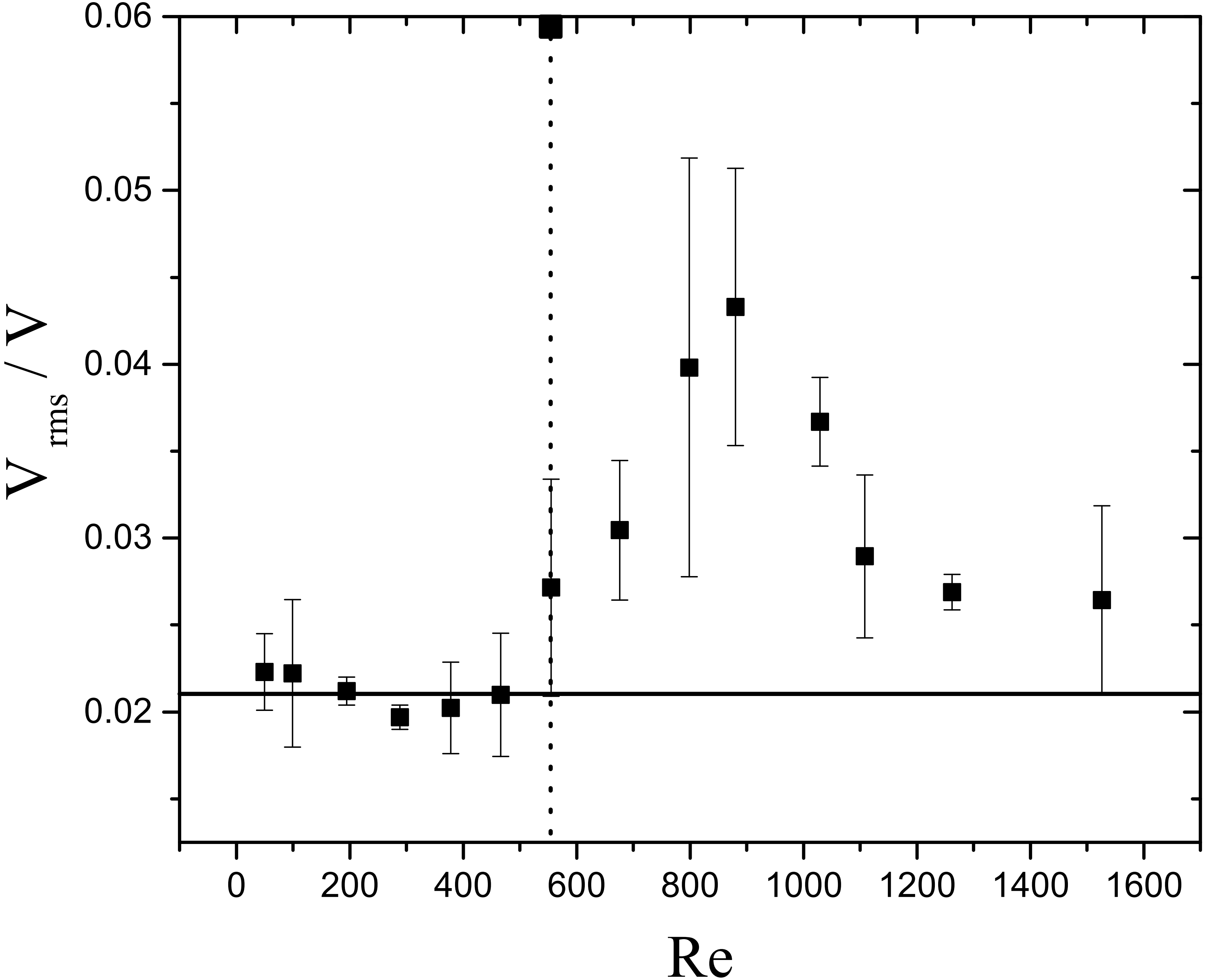
0.030
0.025
0.020
0.015
0.010
80
120
160
200
240
Re
$V'_{rms}/\overline{V}$



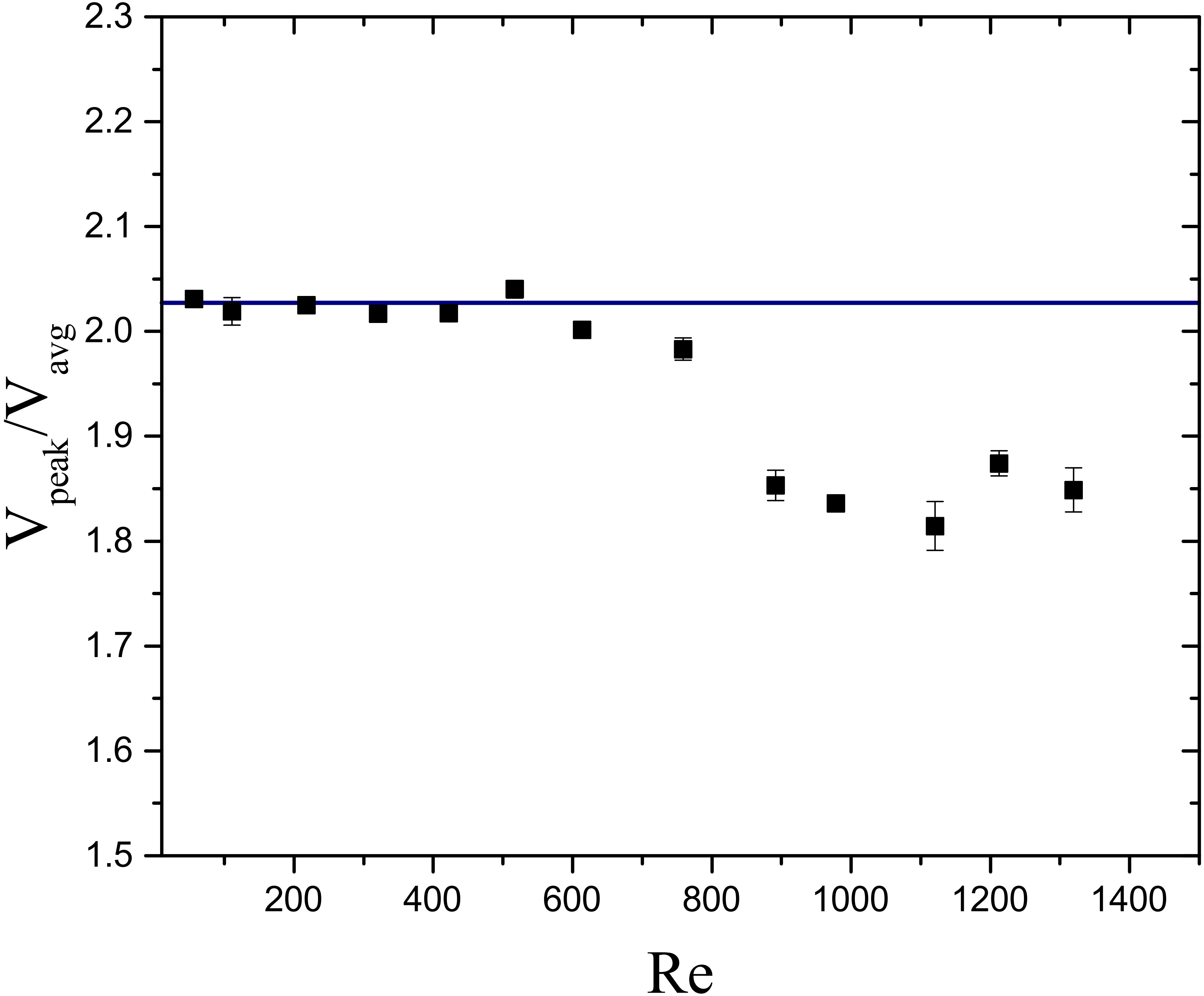
Verma and Kumaran (2012)
Neelamegam and Shankar (2015)
Present Work (Pure Water)
Present Work (Water + Glycerine)
Slope ~ 0.65
1000
100
Re_t
10^6
10^7
Wall elasticity Σ



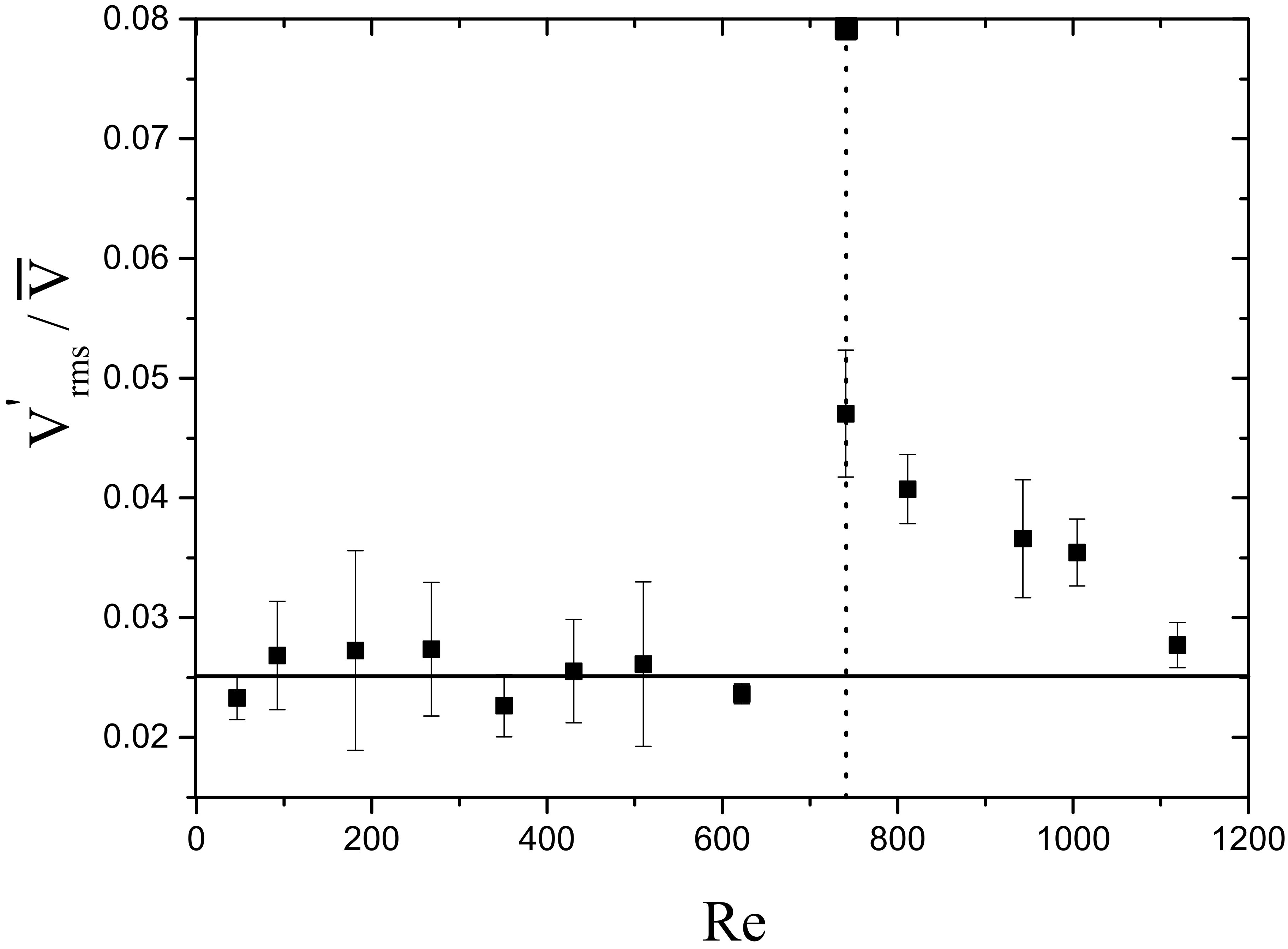
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
0
200
400
600
800
1000
1200
1400
1600
V_{peak}/V_{avg}
Re



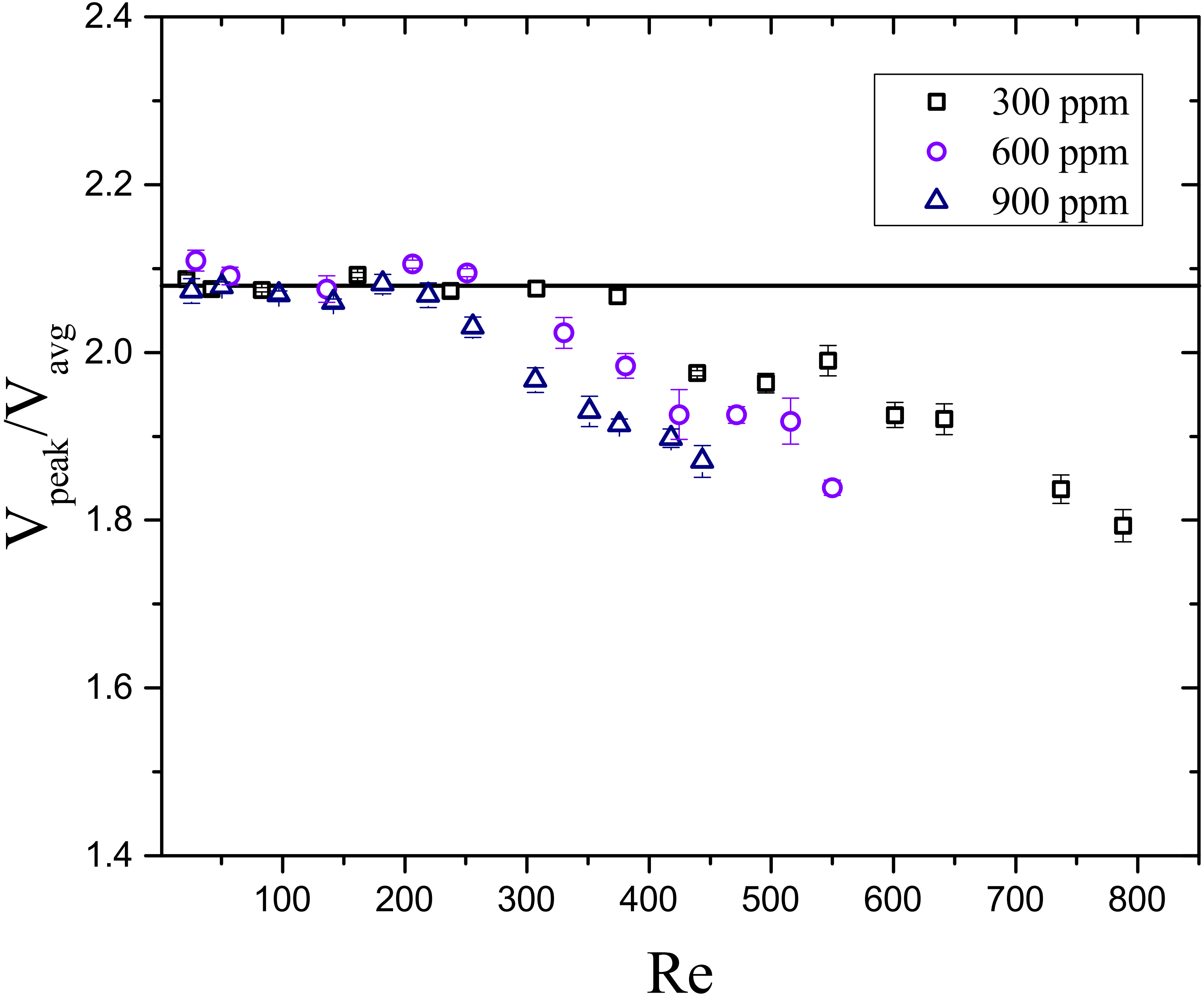
0.06
0.05
0.04
0.03
0.02
V'rms / V̄
0
200
400
600
800
1000
1200
1400
1600
Re



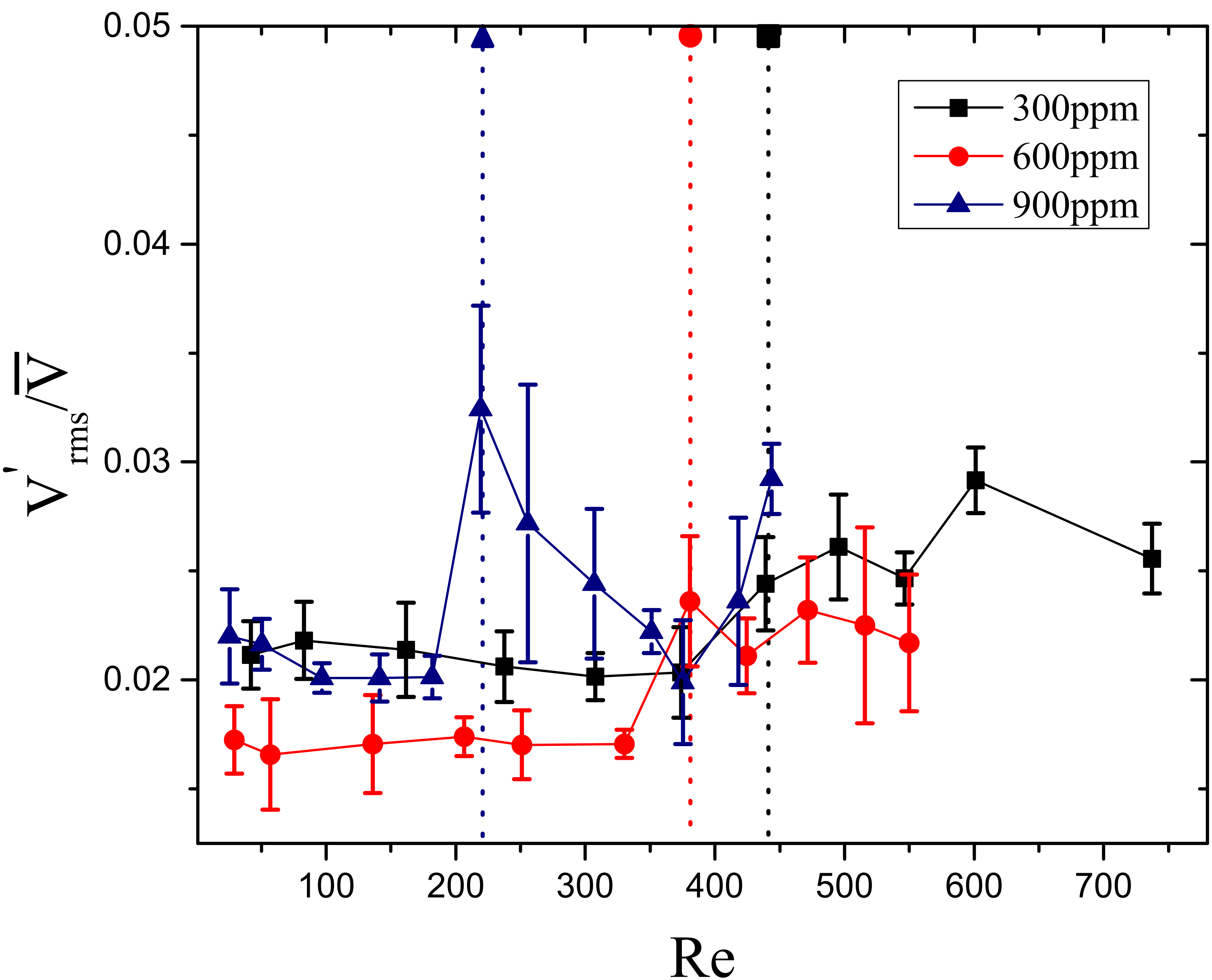
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
200
400
600
800
1000
1200
1400
Re
V_{peak}/V_{avg}



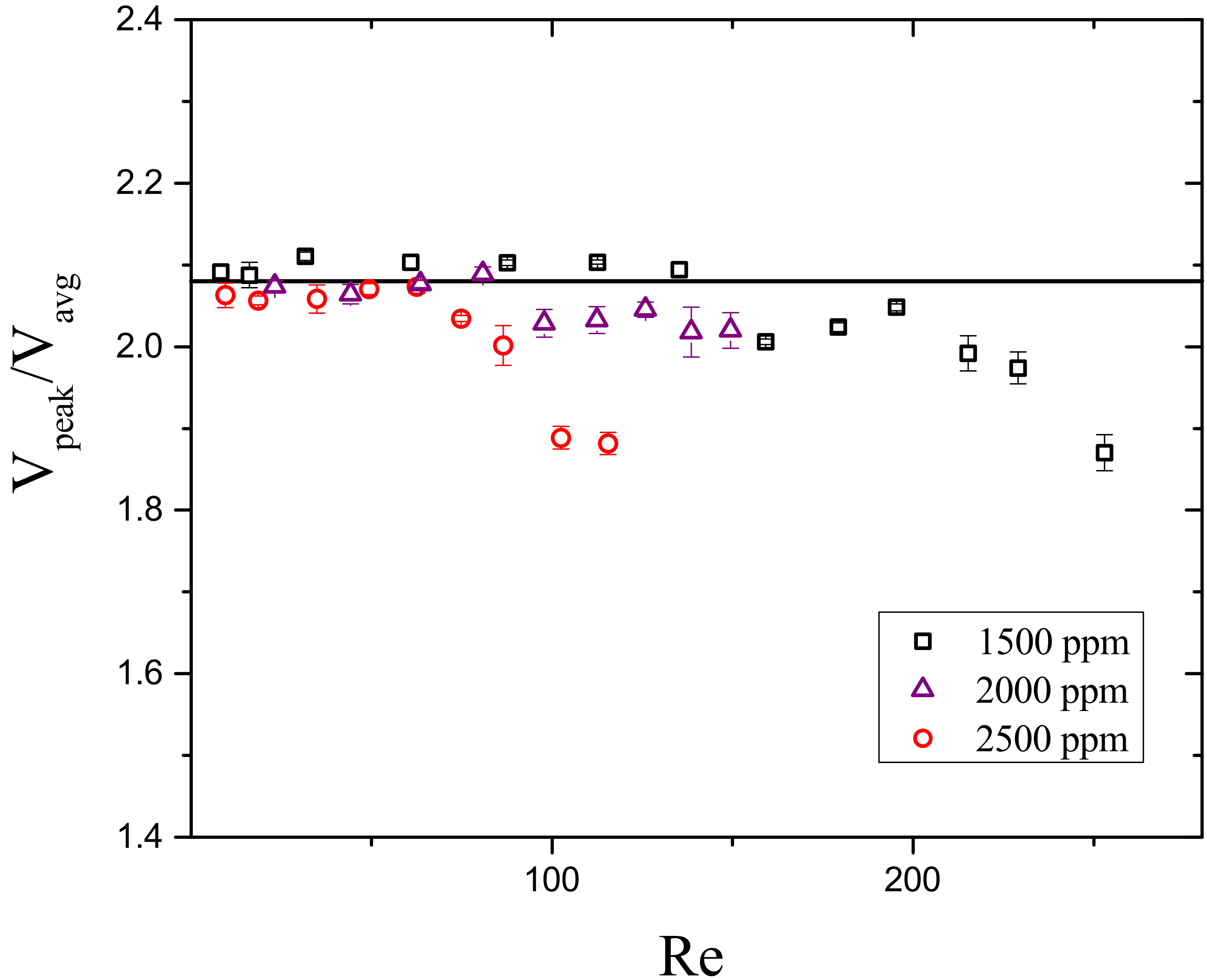
0.08
0.07
0.06
0.05
0.04
0.03
0.02
$V'_{rms}/\overline{V}$
0
200
400
600
800
1000
1200
Re



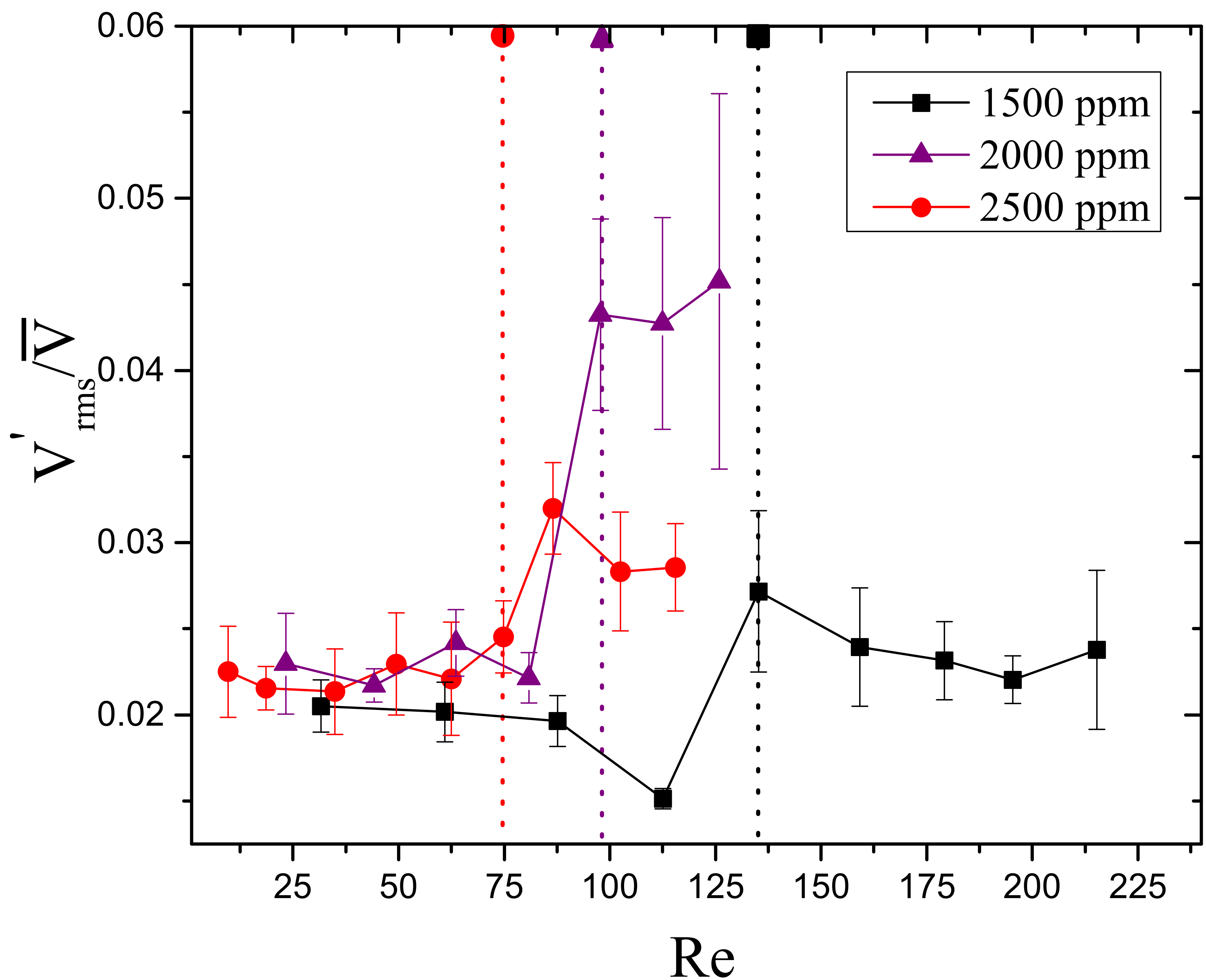
2.4
2.2
2.0
1.8
1.6
1.4
100
200
300
400
500
600
700
800
300 ppm
600 ppm
900 ppm
V_{peak}/V_{avg}
Re



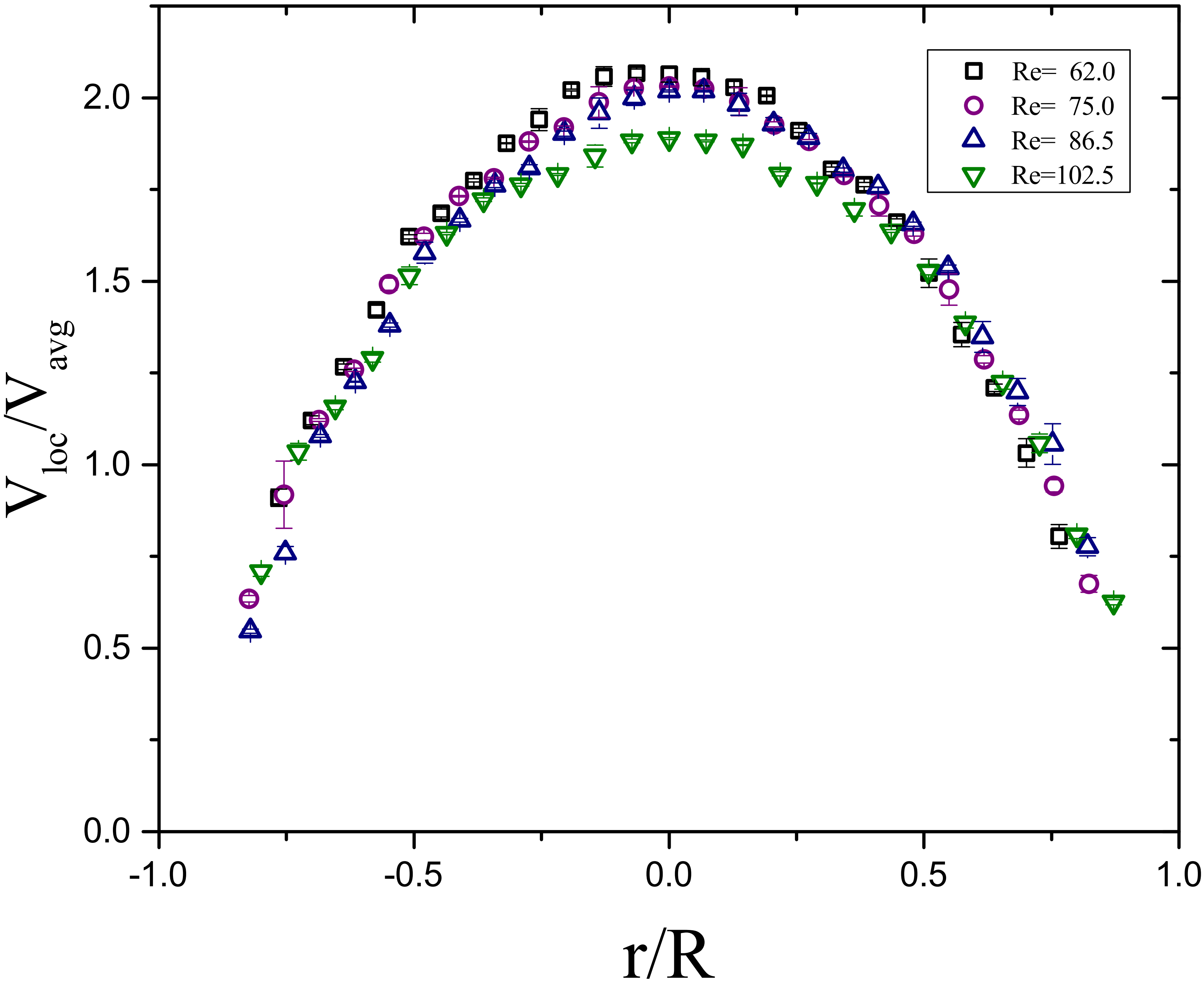
0.05
0.04
0.03
0.02
100
200
300
400
500
600
700
Re
$V'_{rms}/\overline{V}$
300ppm
600ppm
900ppm



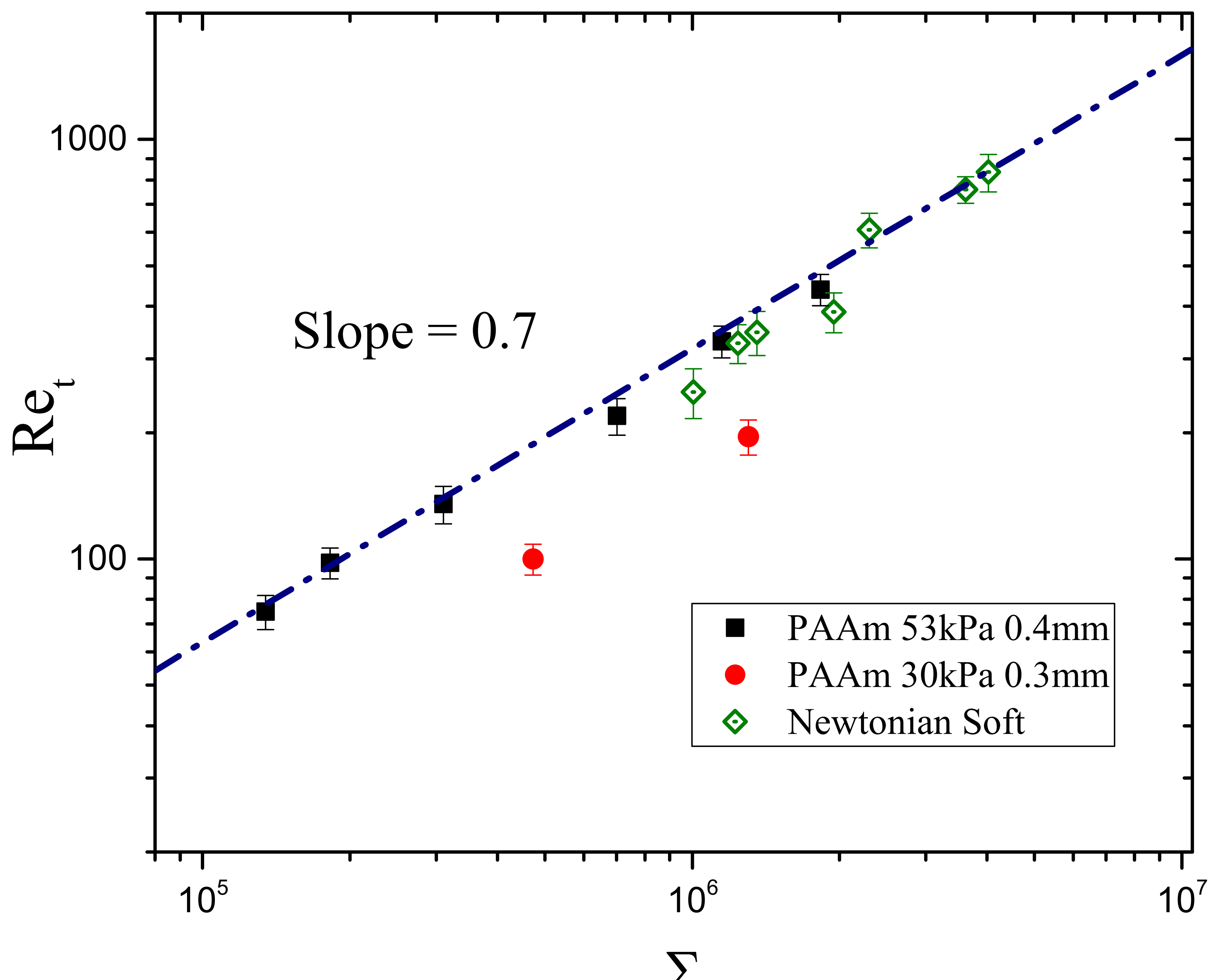
2.4
2.2
2.0
1.8
1.6
1.4
100
200
V_{peak}/V_{avg}
Re
1500 ppm
2000 ppm
2500 ppm



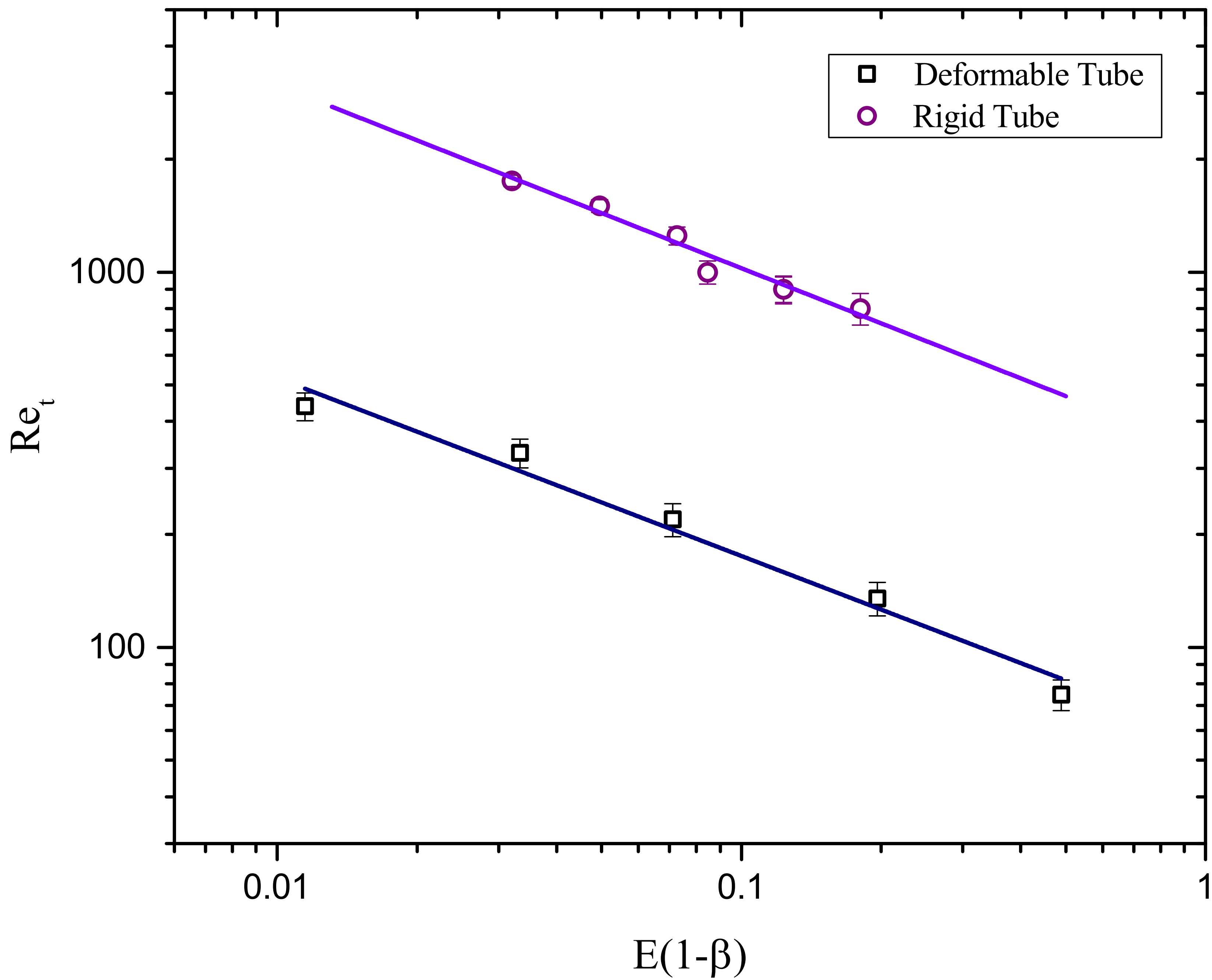
1500 ppm
2000 ppm
2500 ppm
0.06
0.05
0.04
0.03
0.02
25
50
75
100
125
150
175
200
225
Re
V'rms/V̄



Re= 62.0
Re= 75.0
Re= 86.5
Re=102.5
2.0
1.5
1.0
0.5
0.0
-1.0
-0.5
0.0
0.5
1.0
V_{loc}/V_{avg}
r/R



Slope = 0.7
Re_t
1000
100
10^5
10^6
10^7
Σ
PAAm 53kPa 0.4mm
PAAm 30kPa 0.3mm
Newtonian Soft



Deformable Tube
Rigid Tube
1000
100
0.01
0.1
1
Re_t
$E(1-\beta)$