A FRACTURE MECHANICS APPROACH TO LIFE PREDICTION OF TURBINE BLADES

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ABSTRACT

Emerging blade technologies are finding it increasingly essential to correlate blade vibrations to blade fatigue in order to assess the residual life of existing blading and for development of new designs. In this paper an analytical code for Dynamic Stress Analysis and Fatigue Life Prediction of blades is presented. The life prediction algorithm is based on a combination method, which combines the local strain approach to predict the initiation life and fracture mechanics approach to predict the propagation life, to estimate the total fatigue life. The conventional stress based approach involving von Mises theory along with S-N-Mean stress diagram suffer from the drawback that they do not make allowance for the possibility of development of plastic strain zones, especially in cases of low cycle fatigue. In the present paper, strain life concepts are employed to analyse the crack initiation phenomenon. Dynamic and static stresses incurred by the blade form inputs to the life estimation algorithm. The modeling is done for a general tapered, twisted and asymmetric cross section blade mounted on a rotating disc at a stagger angle. Blade damping is nonlinear in nature and a numerical technique is employed for estimation of blade stresses under typical nozzle excitation. Critical cases of resonant conditions of blade operation are considered. Neuber's rule is applied to the dynamic stresses to obtain the elasto-plastic strains and then the material hysteresis curve is used to iteratively solve for the plastic stress. Static stress effects are accounted for and crack initiation life is estimated by solving the strain life equation. Crack growth formulations are then applied to the initiated crack to analyse the propagation of crack leading to failure. The engineering approximations involved are stated and the algorithm is numerically demonstrated for typical conditions of blade operations.

NOMENCLATURE

\( a, b \) Initial and final crack lengths
\( b \) Fatigue strength exponent
\( c \) Fatigue ductility exponent
\( C \) Torsional stiffness
\( D \) Damping matrix
\( E \) Depth of defect
\( e \) Modulus of Elasticity
\( F \) Engineering nominal strain
\( F_x, F_y \) Correction factor for stress intensity factor
forcing functions
\( f \) Shape function for bending deflections
\( F \) Shape function for angular deflections
\( h \) \( m \)th harmonic response in the kth mode
\( A \) \( A_0, A_1, \ldots, A_n \) Shape function for bending and twisting moments
\( l, 1, 2 \) Second moment of area about \( X_1, X_1, Y_1 \) axis
\( x, y \) Second moment of area about \( Y_1, Y_1 \) axis
\( I, J \) Product moment of area about \( X_1, Y_1 \) axis
\( I_{xx}, I_{yy}, I_{x'y'} \) Moment of inertia per unit length
\( K, K' \) Reissner's functional
\( K_c, K_f \) Strength coefficient under cyclic loading, fatigue toughness
\( K_{max}, K_{min} \) Fatigue stress concentration factor, Maximum stress intensity factor
\( K_t, K_{Ic} \) Minimum stress intensity factor, Theoretical stress concentration factor
\( \Delta K \) Plain strain fracture toughness
\( \delta \) Stress intensity range = \( K_{max} - K_{min} \)
\( L \) Reissner's dynamic functional
\( l \) Blade length

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INTRODUCTION

Turbine blade fatigue is a multi-discipline problem. Blade life is mainly influenced by the static and dynamic stress fields on the blade, fatigue properties of the blade material, loading history and the environment of operation. Fatigue crack usually initiates in a region of high stress at some metallurgical or structural discontinuity and if critical conditions of operation sustain, the crack may grow to failure. The various factors may affect the blade behavior in several different ways to make the failure problem very specific. It also makes it increasingly essential to correlate blade vibrations research to the problem of blade fatigue.

The steps towards fatigue analysis can be, broadly, summarised as, (Rao and Vyas, 1987)
- Generating the stress loading biography.
- Definition of blade fatigue parameters under operating environment
- Life estimation using fatigue theories.

Various aspects of the problem have been subjects of extensive research. Reference can be made to articles by Rao (1987), Srinivasan (1984) and Rierger (1985) for an overall review of the problem. Attempts have been made by several researchers to obtain mathematical models for accurate determination of blade response under steady as well as transient excitations. Stress response of single rotating blades was obtained by Rao et al. (1986). Transient vibration analysis has been undertaken by Irriiter (1986) and Vyas et al. (1987). Models for multiple blade systems have also been developed. Some recent works include Jones and Muszynska (1983), MacBain and Whaley (1984) and Sinha and Griffin (1984).

Nonlinearity of damping in blades makes the problem, of stress response determination, more complex. Energy dissipation through Coulomb, hysteretic, and viscous mechanisms of damping is a blade specific problem. Rao, Gupta and Vyas (1986) developed an experimental technique for determination of damping properties under the overall influences of the above mechanisms for rotating blades and defined damping ratios in various vibratory modes as nonlinear functions of rotor speed and the vibratory strain amplitude. A numerical procedure has been developed by Rao and Vyas (1989, 1990) to account for the nonlinearity in damping for determination of resonant stresses during steady and transient rotor operations. The influence of transient resonant stresses, experienced by the blade with nonlinear damping, during step-up/down operations and its influence on fatigue life is investigated by Vyas and Rao (1992). S-N and Mean Stress Diagrams are coupled to define the fatigue failure surface for the blade for specific conditions of operating environment.

The conventional stress based approach involving von Mises theory along with S-N-Mean stress diagram suffer from the drawback that they do not make allowance for the possibility of development of plastic strain zones, especially in cases of low cycle fatigue and attempts need to be made for development of a blade fatigue model based on more fundamental fracture mechanics concepts (Rierger, 1980; Irriiter, 1991). In the present study an attempt is made to develop a life prediction algorithm through a combination approach (Dowling, 1979; Socie, 1984) involving strain life concepts to determine the crack initiation life and then employing linear elastic fracture mechanics principles to estimate the crack propagation life to obtain the total life as the summation of the initiation and propagation lives. Dynamic and static stresses incurred by the blade form inputs to the life estimation algorithm. A typical rotating turbine with nonlinear damping and under representative nozzle excitation is considered. Critical cases of resonant conditions of blade operation are taken to provide stress inputs to the life estimation routine. Neuber’s rule is applied to the dynamic stresses to obtain the elasto-plastic strains and then the material hysteresis curve is used to iteratively solve for the plastic stress. Crack initiation life is estimated using by solving the strain life equation. Crack growth formulations are then applied to the initiated crack to analyze the propagation of crack leading to failure.
BLADE VIBRATION MODEL

A typical turbine blade with taper, pretwist and asymmetric aerodynamic section mounted on a rotating disk at a stagger angle is shown in Figs. 1 and 2. The centroid of the cross-section is located at G, while O is the center of flexure.

FIG. 1 BLADE CROSS-SECTION

Reissner's functional is employed to obtain good stress and displacement fields simultaneously.

The dynamic Reissner's functional for the blade is obtained as (Rao et al., 1986)

\[
L = T - V + \int_0^l \left( \frac{1}{2} \rho I_o \left( \dot{x}^2 + \dot{y}^2 \right) + \frac{1}{2} \rho I_1 \left( \dot{\theta}_x^2 + \dot{\theta}_y^2 \right) + \frac{1}{2} \rho \omega^2 \left( I_1 \dot{\theta}_x \dot{\theta}_y - \frac{1}{2} \rho \omega^2 \right)^2 + \frac{1}{2} \rho \omega^2 \left( I_1 \dot{\theta}_x \dot{\theta}_y - \frac{1}{2} \rho \omega^2 \right)^2 \right) \, \mathrm{d}z
\]

\[
+ \int_0^l \left( \frac{1}{2} \rho I_o \left( \dot{x}^2 + \dot{y}^2 \right) + \frac{1}{2} \rho I_1 \left( \dot{\theta}_x^2 + \dot{\theta}_y^2 \right) \right) \, \mathrm{d}z
\]

\[
- \int_0^l \left( -M_{xy} \ddot{x} + M_{xy} \ddot{y} + T_{\theta} \ddot{\theta}_x + \frac{T_{\theta}^2}{2C} \right) \, \mathrm{d}z
\]

\[
+ \int_0^l \left( -M_{xy}^{(1)} \ddot{x} + M_{xy}^{(2)} \ddot{y} + T_{\theta} \ddot{\theta}_x + \frac{T_{\theta}^2}{2C} \right) \, \mathrm{d}z
\]

Using shape functions

\[
x = \Sigma a_i(t) f_i(z) \quad M_x = \Sigma D_i(t) h_i(z) \]

\[
y = \Sigma b_i(t) f_i(z) \quad M_y = \Sigma E_i(t) h_i(z) \]

\[
\theta = \Sigma C_i(t) f_i(z) \quad T_{\theta} = \Sigma F_i(t) h_i(z) \]

where

\[
f_i(z) = \frac{(i+2)(i+3)}{6} \frac{z^{i+1}}{3} - \frac{(i+3)}{3} \frac{z^i}{6} + \frac{(i+1)}{6} \frac{z^i}{3}
\]

\[
h_i(z) = \frac{(i-2)}{i+1}
\]

which satisfy the boundary conditions of a cantilever, Ritz process

\[
\frac{\partial L}{\partial a_i} = 0 \text{ etc.}
\]

is applied to the dynamic Reissner functional above to obtain the equations of motion

\[
p^2 \|M\| \dot{q} - \|K\| q = 0
\]

where \([M]\) and \([K]\) are mass and stiffness matrices and the vector \([q]\) contains

\[
[q]^T = (A_1, ..., D_i, ...)
\]

For the blade under consideration the geometric properties as functions of blade length are described in Fig. 3. The eigenproblem (7) is solved to obtain natural frequencies and mode shapes.

FIG. 3 BLADE GEOMETRIC PROPERTIES AS FUNCTIONS OF BLADE LENGTH

NONSTEADY NOZZLE FORCES

For constant speed operations the nozzle forces \(F_x(Z, t), F_y(Z, t), F_m(Z, t)\) and \(M(Z, t)\) on the blade in the \(x, y, \theta\) directions respectively are periodic with the nozzle passing frequency, \(v = n_\theta \omega\) and are expressed in Fourier form as

\[
F_x(Z, t) = a_{ox}(Z) + \sum \frac{a_{mx}(Z)}{m_{mx}} \cos \frac{\mu\pi}{m_{mx}} + \sum b_{my}(Z) \sin \frac{\mu\pi}{m_{my}}
\]

\[
F_y(Z, t) = a_{oy}(Z) + \sum \frac{a_{my}(Z)}{m_{my}} \cos \frac{\mu\pi}{m_{my}} + \sum b_{my}(Z) \sin \frac{\mu\pi}{m_{my}}
\]

\[
F_m(Z, t) = a_{om}(Z) + \sum \frac{a_{mM}(Z)}{m_{mM}} \cos \frac{\mu\pi}{m_{mM}} + \sum b_{mM}(Z) \sin \frac{\mu\pi}{m_{mM}}
\]

Typical nozzle forces simulated by electromagnets for a laboratory spin-rig are shown in Fig. 4.

Resonant frequencies for the blade are found out from the Campbell diagram (Fig. 5), e.g. interaction of the I mode of vibration with the VI, V and IV harmonics of nozzle forces gives rise to blade
FIG. 4  EXCITATION FORCES

FIG. 5  CAMPBELL DIAGRAM

resonant speeds of 619, 743 and 930 rpm respectively. It should be noted here that for the blade under consideration the increase in natural frequencies with rotational speed is small and on the Campbell Diagram the natural frequencies get depicted as straight horizontal lines.

FORCED VIBRATION STRESSES UNDER NONLINEAR DAMPING

Blade damping is modeled as nonlinear function of strain amplitude and speed of operation for each mode of vibration. Initially, damping can be assumed to be proportional and viscous, so that the energy dissipated is

$$ W = \frac{C \cdot \dot{q}}{\rho} $$

(6)

The process of incorporating the nonlinear model will be considered shortly. The equations for forced damped motion are then obtained as

$$ (M(q) + C \ddot{q} + Kq) \ddot{q} = (\dot{q})_T = \{ A_1, ..., D_i, ... \} $$

$$ M_{II} = \rho \int_{0}^{l} \alpha_i \dot{f}_j \, dz $$

$$ C_{II} = \frac{C \cdot M_{II}}{\rho} $$

$$ Q_{II} = \int_{0}^{l} a_{o,k} \dot{f}_j \, dz $$

etc.

(7)

Modal analysis technique is employed to obtain the decoupled equations of forced motion. Hence, using

$$ \dot{q} = [\mathbf{U}] (\mathbf{q}) $$

we get

$$ M_{k} \ddot{q}_k + 2\zeta_k \omega_k \dot{q}_k + \omega_k^2 q_k = Q_0 N_k + \sum_{m} Q_{m} N_k \cos m \omega t $$

$$ + \sum_{m} Q_{m+6} N_k \sin m \omega t $$

(9)

The solution of the above in terms of displacement and moment vectors, due to each harmonic is of the form

$$ M_k \ddot{q}_k = Q_0 N_k $$

$$ H_{m_k} = \frac{Q_0}{P_k} $$

$$ H_{m_k} = \frac{Q_0}{P_k} $$

$$ H_{(m+6)k} = \frac{Q_0}{P_k} $$

$$ H_{(m+6)k} = \frac{Q_0}{P_k} $$

$$ H_{(m+6)k} = \frac{Q_0}{P_k} $$

where

$$ \psi_{m_k} = \tan^{-1} \left[ \frac{2\zeta_k \omega_k (m \omega)}{P_k - (m \omega)^2} \right] $$

$$ \delta_{m_k} = \tan^{-1} \left[ \frac{Q_{m+6} N_k}{Q_{m} N_k} \right] $$

(10)

With the help of equations (10), (8) and (2), the deflections and moments of the blade for the force harmonics can be determined. The stress components are computed using standard formulae (Roark & Young, 1976) and the basic mean stress (zeroth harmonic), \( S_m \), and the basic alternating stress component, \( S_a \), are determined using the von Mises theory.

Damping in turbine blades primarily consists of friction at the mating surfaces and material hysteresis. The overall damping is known to be dependent on the rotational speed (influencing the blade-disk root friction) and strain amplitude (influencing hysteresis). The above dependency was quantitatively assessed for each mode by Rao, Gupta and Vyas (1986a) with tests conducted in vacuum spin rig. From the filtered decay signals of each mode, equivalent modal damping ratios were obtained as function of rotational speed and strain amplitude. Typical equivalent modal damping values for mode 1, obtained from free vibratory response are shown in Fig. 6 as a function of strain amplitude for different rotor speeds. There is a threshold speed in each mode, below which friction damping is predominant. In the first mode, this threshold speed is approximately between 500 to 600 rpm. Beyond the threshold speed the effect of friction decreases due to the centrifugal force causing increased interlocking between the blade root and the
disk and only hysteresis is available for energy dissipation. There is a transition region between the friction threshold speed, and the hysteresis threshold speed, where the energy dissipation occurs due to both friction and material hysteresis simultaneously.

The damping ratios for each mode are expressed as polynomial functions of strain amplitude at various rotor speeds, thus

$$\xi = a_0 + a_1 \varepsilon + a_2 \varepsilon^2 + \ldots + a_5 \varepsilon^5$$  \hspace{1cm} (11)

The constants $a_i$ in equation (11) for mode I are obtained by a curve fitting routine.

The stress calculation procedure starts with an assumed value of strain and corresponding modal damping ratio, point I, Fig. 8 and the principal component of stress and resultant strain is estimated, point 1'. The average of the assumed and estimated values of strain and the corresponding modal damping ratio are taken as the starting values for the next iteration. The process is repeated till two successive values of stress are obtained within a specified limit of accuracy. The stress harmonics thus estimated up to a rotor speed of 1200 rpm are shown in Fig. 9.

**Fig. 6 BLADE DAMPING IN MODE I AS FUNCTION OF STRAIN AMPLITUDE AND ROTOR SPEED**

For the blade data considered in this paper, the fundamental natural frequency is 743 Hz. As the excitation is due to 12 magnets simulating the nozzles in a turbine stage, there are three critical speeds for the blade, viz., 619 rpm, 743 rpm and 930 rpm, corresponding to the sixth, fifth and fourth harmonics respectively, in the speed range for which damping data was experimentally obtained. The damping data for these speeds is obtained by interpolation from the plots in Fig. 6 and are shown in Fig. 7.

**FIG. 7 MODE I DAMPING FOR THREE RESONANT SPEEDS**

**FIG. 8 ITERATION PROCEDURE**

**FIG. 9 STRESS HARMONICS**

**LIFE PREDICTION ALGORITHM**

The life estimation algorithm is based on the combination approach suggested by Dowling (1979) which combines the local strain concepts to predict the crack
initiation life and fracture mechanics principles to predict the propagation life. In the combination approach usage of strain life concepts for prediction of initiation life avoids the inherent difficulty that would have been faced in application of linear elastic fracture mechanics to describe the short crack behaviour at the defect root. Although the turbine blade is designed such that the nominal loads remain elastic, stress concentrations often cause plastic strains to develop in the vicinity of defects within the blade material. Linear elastic fracture mechanics principles are inherently constrained to be applied only after the overall crack dimensions have become large in comparison to the plastic zone size. Combination approach employs the fact that within a distance, \( l_t \), from the defect, the local notch stress field dominates the stress intensity solution. When the crack is smaller than this length \( (1 < l_t) \), crack initiation can be estimated using the strain-life concepts. Once the crack is larger than this length \( (1 > l_t) \), crack growth can be modeled with the fracture mechanics principles.

The procedure adopted for life estimation can be summarised as:

Crack Initiation -
(1) With the nominal stress (from stress analysis) as input, determination of elasto-plastic strain using Neuber's rule (1964).
(2) Coupling with material hysteresis curve and Massing's hypothesis (1926) and solving for plastic stress through iterative technique.
(3) Accounting for mean stress by Morrow's hypothesis (1968) and determination of crack initiation life.

Crack Propagation -

The total life, then, is:

\[ \text{Total life} = \text{Initiation life} (N_i) + \text{Propagation life} (N_p) \]

\[ (1 < l_t) \quad (1 > l_t) \]

Strain life approach Fracture mechanics approach

Cyclic Stress - Strain Behaviour

The response to cyclic inelastic loading is in the form of the well known hysteresis loop. The relation between the total strain range, elastic strain range and the plastic strain range is:

\[ \Delta e = \Delta e_e + \Delta e_p \]

or,

\[ \frac{\Delta e}{2} = \frac{\Delta e_e}{2} + \frac{\Delta e_p}{2} \]

(12)

The log-log plot of the completely reversed stabilised cyclic true stress versus true plastic strain can be approximated by a straight line using a power law function:

\[ \sigma = K' (\varepsilon_p)^{n'} \]

(13)

Doubling the cyclic stress-strain value from the stabilised cyclic stress-strain curve, the equation of the hysteresis loop is (Massing's hypothesis)

\[ \Delta e = \frac{\Delta e}{2} + \left( \frac{\Delta e}{2K'} \right)^{1/n'} \]

(14)

Neuber's Rule

For nominal strains within the elastic range, the relationship between nominal stress and true stress and strain in the vicinity of the defect, using Neuber's rule, is:

\[ \sigma = (K'_t \sigma) / E \]

(15)

In terms of the stress and strain amplitudes, the above is written as:

\[ \Delta \sigma = (K'_t \Delta S) / E \]

(16)

Using equation (14) in (16) one gets

\[ \Delta \sigma \left( \frac{\Delta \sigma}{E} + 2 \frac{\Delta \sigma}{2K'} \right)^{1/n'} = \frac{(K'_t \Delta S)}{E} \]

(17)

Strain Life Equation

The true plastic strain \( \epsilon' \) data can be plotted linearly on a log-log scale (Basquin, 1910), thus:

\[ \frac{\Delta \sigma}{2} = \sigma' \left( \frac{2N_i}{b} \right)^b \]

(18)

Similarly, plastic strain-life data could be linearised on log-log scale (Manson, 1953; Coffin, 1954) as:

\[ \frac{\Delta \varepsilon}{2} = \epsilon' \left( \frac{2N_i}{b} \right)^b \]

(19)

The total strain amplitude is then written as the following strain life equation:

\[ \frac{\Delta \varepsilon}{2} = \sigma' \left( \frac{2N_i}{b} \right)^b + \epsilon' \left( \frac{2N_i}{c} \right)^c \]

(20)

The strain life equation is further modified to include the effects of mean stress as (Morrow, 1968):

\[ \frac{\Delta \varepsilon}{2} = \sigma' \left( \frac{2N_i}{b} \right)^b + \epsilon' \left( \frac{2N_i}{c} \right)^c \]

(21)

Initiation Life Estimation

Starting with the nominal stress range \( \Delta S \) between the maximum and minimum values of the alternating stress obtained from stress analysis, the true stress range \( \Delta \sigma \) can be determined from equation (17) if the material properties \( E, K', n' \) are known. Instead of the fatigue stress concentration factor \( K'_t \), the theoretical stress concentration factor \( K_t \) is used as an approximation (Socie, 1984) in the present calculations. \( K_t \) is a function of the geometry of crack and the mode of its loading. These values are compiled in various texts (Peterson, 1934). From the hysteresis loop equation (14) the true strain range value, \( \Delta e \), is then obtained which is used in the strain life equation (21) along with material parameters \( \sigma'_f, \epsilon'_f, b \) and c to get an estimate of the crack initiation life \( N_i \). Fatigue strength coefficient, \( \sigma'_f \), is the intercept of the true stress amplitude - reversals to failure plot at one reversal. This is determined by curve fit of actual experimental data. In the absence of actual tests a fairly good approximation is \( \sigma'_f = \sigma'_f (\text{true stress at fracture}) \).
For steels with Brinnel hardness upto 500 it may be approximated as

\[ \sigma' = S_u + 345 \text{ (in MPa)} \]

Fatigue strength exponent, \( b \), is the exponent obtained on curve fitting experimental cyclic fatigue data. Its value varies from -0.05 to -0.12 for most metals with an average value of -0.085. Fatigue ductility coefficient, \( \varepsilon_f' \), is approximated by the true fracture ductility ie.

\[ \varepsilon_f' = \ln \left( \frac{1}{1 - \frac{1}{RA}} \right) \]

where \( RA \) is the percentage reduction in area. Fatigue ductility exponent, \( c \), is not as well defined as others and a rule of thumb has to be followed for its estimation. Its value has been found by Coffin to be about -0.5, by Manson to be about -0.6 and by Morrow to be lying between -0.5 and -0.7.

**Propagation Life Estimation**

The crack growth life, from an initial crack size \( a_i \) to a final size \( a_f \), is written in terms of number of propagation cycles, using Paris formulation as

\[ N_p = \int_{a_i}^{a_f} \frac{da}{N_p} = \frac{\pi}{C (\Delta K)^m} \]

(22)

The material constants, \( C \) and \( m \), are determined by curve fit of experimental data. ASTM E647 sets guidelines for determination of these parameters and are available for some steels in literature (eg.Osgood, 1982).

The stress intensity factor \( K \) which defines the magnitude of the local stresses around a crack tip for a given nominal stress, \( S_n \), on a crack of length \( a \), is given as

\[ K = F S \sqrt{2a} \]

(23)

In the above \( F \) is the correction factor depending on crack shape and geometric boundaries. Reference can be made to Murakami (1987) for a wide variety of stress intensity factor solutions. As the stress intensity factor reaches the material fracture toughness value, \( K_c \) (denoted by \( K_{IC} \) in the case of plain strain), unstable fracture occurs. The failure crack length, \( a_f \), is obtained as

\[ a_f = \frac{1}{F} \left[ \frac{K_c}{\varepsilon_f} \right]^2 \]

(24)

The value of the initial crack length is given by

\[ a_i = D_n + l_t \]

(25)

\( D_n \) is the depth of the defect. The estimates for \( l_t \) can be obtained from Dowling (1979). Socie (1984) proposed a simpler procedure, yielding similar results as Dowling, where the initial crack size is taken equal to the defect depth.

**Implementation of The Algorithm**

With reference to Fig. 9 rotor speed of 930 rpm, at which the blade experiences resonant stresses due to the interaction between the first mode and the fourth harmonic of the excitation, is chosen for analysis. The maximum section stresses occur at the root. The total mean stress on the blade comprises of the zeroth harmonic of the stress and the centrifugal stretching of the blade. The centrifugal stresses due to rotation can be determined as

\[ \sigma_{mc} = \frac{(R + z)}{A} \left( \frac{2\pi \text{ RPM}}{60} \right)^2 \left( \int \rho A \frac{dz}{z} \right) \]

(26)

For the purpose of modeling the blade for life estimation an equivalent rectangular section is obtained for the blade root section, where the chord length of the root aerofoil is taken as the width of the section. The thickness of the rectangular cross section is obtained from the equivalent section modulus of the root aerofoil. The blade is assumed to be defect free except for surface / body metallurgical discontinuity incurred during manufacture. The depth of such a defect is to be determined using non-destructive evaluation techniques for inspection of turbine blades and detection of defects if no defect is detected then the least count of the measurement technique can be taken as the initial depth of the defect. For illustration the depth of the defect is taken as 1.00 mm.

For the blade material AISI-4340 steel the properties listed in Table 1 are employed (Osgood, 1982).

**TABLE 1**

**PROPERTIES OF BLADE MATERIAL AISI-4340 STEEL**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity (E)</td>
<td>193 GPa</td>
</tr>
<tr>
<td>Cyclic Yield Strength (S_y)</td>
<td>758 MPa</td>
</tr>
<tr>
<td>Cyclic Strength Coefficient (K')</td>
<td>1730 MPa</td>
</tr>
<tr>
<td>Cyclic Strain Hardening (n')</td>
<td>0.14</td>
</tr>
<tr>
<td>Fatigue Strength Coefficient ((\varepsilon_{f}'))</td>
<td>1655 MPa</td>
</tr>
<tr>
<td>Fatigue Strength Exponent (b)</td>
<td>-0.076</td>
</tr>
<tr>
<td>Fatigue Ductility Coefficient ((\varepsilon_{f}''))</td>
<td>0.73</td>
</tr>
<tr>
<td>Fatigue Ductility Exponent (c)</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

**MATERIAL CONSTANTS**

<table>
<thead>
<tr>
<th>Fracture Toughness ((K_{IC}))</th>
<th>137.375 MPa√m</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6.6 x 10^-9</td>
</tr>
<tr>
<td>m</td>
<td>2.25</td>
</tr>
</tbody>
</table>

**ILLUSTRATION**

Case 1:

An initial defect, modeled as a circular defect of radius 1.0 mm, is considered in the blade root section. This is depicted in Fig. 10a. For 930 rpm, the stresses obtained from the stress analysis are as follows

Alternating stress (\(S_a\)) = 0.056 MPa

Mean stress due to zeroth harmonic (\(S_{mv}\)) = 0.00966 MPa

Mean stress due to centrifugal effects (\(S_{mc}\)) = 1.044 MPa

Total mean stress (\(S_{mv} + S_{mc}\)) = 1.05386 MPa

The width of the plate is determined as 3.8 cm and the thickness as 1.0 cm. The theoretical stress concentration factor, \(K_t\), is determined as 3.0. The expression for correction factor \(F\) used, is

\[ F = \left[ \sec \left( \frac{\pi a}{2 b} \right) \right]^{1/2} \]
The results of the blade life estimate obtained are as follows

Initiation life estimate = \(1.7 \times 10^{52}\) cycles
Propagation life estimate = \(2.2 \times 10^{11}\) cycles
Total life estimate = \(1.7 \times 10^{52}\) cycles

The total life of the blade is very large. This is due to the fact that the nozzle excitation forces simulated in laboratory spin rig by means of electromagnets are small (refer Figure 4) and consequently the dynamic stresses experienced by the blade due to this electromagnetic excitation are small (refer Figure 9). Moreover, the resonance under consideration at 930 RPM is a weak one, being caused by the fourth order harmonic of the electromagnetic excitation (refer Figure 5). Consideration of stronger resonances, which occur at higher rotor speeds, is constrained due to the fact that the experimentation on the rig had to be restricted to a rotor speed of 1200 RPM (Rao et al 1986, 1986a) and damping data necessary for stress computations were experimentally extracted and the analytical stresses further experimentally validated within the above range of rotor speed. In practice, the nozzle forces driving the machine would be many times larger and inflict stresses which would be considerably greater than those obtained, using available data, in the illustrated case. It is therefore assumed, for further illustration, that the forces are such that a thousand times larger stresses are inflicted on the blade. The stresses are now

Alternating stress \(S_a = 56.0\) MPa
Mean stress due to zeroth harmonic \(S_{mv} = 9.86\) MPa
Mean stress due to centrifugal effects \(S_{mc} = 1.044\) MPa

Total mean stress = \(S_{mv} + S_{mc} = 10.904\) MPa
The life estimates obtained are

Initiation life estimate = \(0.54 \times 10^{13}\) cycles
Propagation life estimate = \(0.32 \times 10^5\) cycles
Total life estimate = \(0.54 \times 10^{13}\) cycles

Case 2:
Two initial defects, modeled as two semicircular defects of depth 1.0 mm each, are considered in the blade root section. This is depicted in Fig. 10b. For 930 rpm, the stresses obtained from the stress analysis are the same as for Case 1. The theoretical stress concentration factor, \(K_t\), is determined as 3.0. The expression for correction factor used is

\[ F = 1.12 + 0.203 \left( \frac{a}{b} \right) - 1.197 \left( \frac{a}{b} \right)^2 + 1.930 \left( \frac{a}{b} \right)^3 \]

The results of life estimate obtained are as follows

Initiation life estimate = \(0.54 \times 10^{13}\) cycles
Propagation life estimate = \(0.2 \times 10^5\) cycles
Total life estimate = \(0.54 \times 10^{13}\) cycles

Comparison between Case 1 and Case 2 reveals that the crack initiation lives in the two cases are the same. This is because the phenomenon of crack initiation is critically dependent on the theoretical stress concentration factor, which is the same in the two cases since the ratio of flaw radius to the blade width is the same in the two cases. The propagation lives, though different, can be seen to be of the same order. On the basis of these observations, it can be stated the size of the flaw is more critical parameter than its location on the blade surface. Such an observation would suit a flaw detection technique whereby in case no flaw is detected and the least count of the detection technique is taken as the flaw size with its location being arbitrary.

Case 3:
An initial defect, modeled as a semicircular defect of radius 1.0 mm, is considered in the blade root section. This is depicted in Fig. 10c. The expression for the correction factor used is

\[ F = 1.12 - 0.231 \left( \frac{a}{b} \right) + 10.55 \left( \frac{a}{b} \right)^2 - 21.72 \left( \frac{a}{b} \right)^3 + 30.39 \left( \frac{a}{b} \right)^4 \]

The theoretical stress concentration factor, \(K_t\), is determined to be 2.79. The results of life estimate obtained are as follows

Initiation life estimate = \(1.4 \times 10^{13}\) cycles
Propagation life estimate = \(0.17 \times 10^5\) cycles
Total life estimate = \(1.4 \times 10^{13}\) cycles

Case 4:
An initial elliptical surface defect into the three dimensional blade root section is considered (Fig. 10d). The correction factor, \(F\) for the stress intensity factor expression has been estimated using empirical relations given by Murakami (1987). The stress intensity factor for elliptical defect varies along the periphery and its maximum value has been taken for computation. Two cases, are considered:

1) Semicircular surface defect
The diameter of the semicircular defect is taken as 1.0 mm for illustration. The correction factor, \(F\), is determined to be 0.732. The life estimates are

Initiation life estimate = \(0.76 \times 10^{13}\) cycles
Propagation life estimate = \(0.47 \times 10^5\) cycles
Total life estimate = \(0.76 \times 10^{13}\) cycles

2) Semi-elliptical surface crack
The length of the major and minor diameters of the elliptical defect are taken as 1.0 mm and 0.5 mm respectively. The value of the correction factor obtained is 0.657. The life estimates are

Initiation life estimate = \(0.65 \times 10^{13}\) cycles
Propagation life estimate = \(0.81 \times 10^5\) cycles
Total life estimate = \(0.65 \times 10^{13}\) cycles

Case 5:
An initial elliptical discontinuity is assumed to be embedded inside the three dimensional root section is considered (Fig.10e).

1) Semicircular embedded defect
The diameter of the defect is taken as 1.0 mm. The correction factor, \(F\), is estimated as 0.638. The life estimates are

Initiation life estimate = \(2.0 \times 10^{13}\) cycles
Propagation life estimate = \(5.9 \times 10^5\) cycles
Total life estimate = \(2.03 \times 10^{13}\) cycles
2) Semi-elliptical embedded crack

The major and minor diameter are taken as 1.0 mm and 0.5 mm respectively. The correction factor, $F$, is found to be 0.58. The life estimates are

Initiation life estimate = $0.76 \times 10^{13}$ cycles

Propagation life estimate = $0.78 \times 10^{5}$ cycles

Total life estimate = $0.76 \times 10^{13}$ cycles

As in cases 1 and 2, a comparison between case 4 and case 5 reveals that the order of the life estimates is approximately the same irrespective of the location of the flaw.

As in cases 1 and 2, a comparison between case 4 and case 5 reveals that the order of the life estimates is approximately the same irrespective of the location of the flaw.

**EFFECT OF INITIAL FLAW SIZE**

As seen that for a given loading condition, the flaw size is the crucial parameter in determining the total life of the blade. For the case 3 type of defect and stresses ten times larger than those in case 3, the effect of initial flaw size is illustrated in Table 2.

The initiation life can be seen to increase with increasing flaw size. This is due to the fact that with an increase in flaw radius there is a lowering of the stress concentration factor. Once initiated, a larger flaw can be seen to propagate more rapidly to failure.

**TABLE 2**

<table>
<thead>
<tr>
<th>INITIAL CRACK SIZE (mm)</th>
<th>$K_t$</th>
<th>$N_t$ (cycles)</th>
<th>$N_p$ (cycles)</th>
<th>$N_r$ (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>3.00</td>
<td>356</td>
<td>174</td>
<td>530</td>
</tr>
<tr>
<td>0.50</td>
<td>2.95</td>
<td>378</td>
<td>134</td>
<td>512</td>
</tr>
<tr>
<td>0.75</td>
<td>2.93</td>
<td>388</td>
<td>113</td>
<td>501</td>
</tr>
<tr>
<td>1.00</td>
<td>2.90</td>
<td>402</td>
<td>98</td>
<td>500</td>
</tr>
<tr>
<td>1.25</td>
<td>2.85</td>
<td>429</td>
<td>88</td>
<td>517</td>
</tr>
<tr>
<td>1.50</td>
<td>2.81</td>
<td>452</td>
<td>79</td>
<td>531</td>
</tr>
</tbody>
</table>

**COMPARISON OF THE PRESENT METHOD WITH THE S-N APPROACH**

A comparison is made between the present method and the stress based approach (Rao and Vyas (1986)), involving Bagci's line (1981), (which is an update on the Goodman Diagram) in conjunction with the S-N Diagram to define the fatigue failure surface.

**Bagci's Fatigue Failure Surface Line**

Bagci's fatigue surface line is of the fourth order and is given by

$$S_{af} = S_e \left( 1 - \left( \frac{S_{mf}}{S_y} \right)^4 \right)$$  \hspace{1cm} (27)

The S-N plane reads

$$\log S^* = -A \log N + B$$

for $10^3 < N < 10^6$

$$S_e^* = S_e = k S_u$$

for $N > 10^6$

with

$$A = \frac{1}{3} \log \left( 0.9 \frac{S_u}{S_e} \right) \hspace{1cm} B = \log \left( R_f \left( 0.9 \frac{S_u}{S_e} \right)^2 \right)$$  \hspace{1cm} (28)

$k = 0.5$ for a standard rotating beam fatigue specimen made of steel. $R_f$, the endurance limit modifying factor, is given by

$$R_f = K_a K_b K_c K_d K_e K_f$$

where

$K_a$ : surface factor

$K_b$ : size factor

$K_c$ : reliability factor

$K_d$ : temperature factor

$K_e$ : stress concentration factor

$K_f$ : miscellaneous effect factor

The value of $S_y$ for AISI 4340 steel is 1241 MPa and $S_u$ is 1172 MPa. It is assumed that a flaw of the type of Case 3 exists in the blade root section. The theoretical stress concentration factor for this geometry is found to be 2.79 (from Peterson's curves ). The alternating stress and the mean stress values used for the comparison are 85 MPa and 16 MPa respectively.

The value of $R_f$ computed is 0.113, based on suitable values of the modifying factors. The life estimate using the Bagci's fatigue surface line is found to be $0.96 \times 10^5$ cycles.
By the strain-life - fracture mechanics combination approach the life estimate obtained is:

Initiation life estimate = $5.6 \times 10^{10}$ cycles
Propagation life estimate = $6632.8$ cycles
Total life estimate = $5.6 \times 10^{10}$ cycles

It can be seen that the S-N approach gives a life lower than that obtained from the present combination approach, since it does not specifically account for the development of the plastic zone.

CONCLUSION

The stress analysis code along with the life estimation algorithm forms an overall turbine blade analysis package. The present study is constrained and engineering approximations regarding numerical data have been made, as stated, wherever essential, due to lack of better information on blade performance and fatigue data and the present fatigue algorithm is to be viewed, not as an exact solution of the problem but as an improvement on the conventional stress based approach for life estimation.

REFERENCES


