PLACEMENT OF OPTICAL AMPLIFIERS IN OPTICAL NETWORKS

A Thesis Submitted
in Partial Fulfillment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY
by
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to the
CENTRE FOR LASER TECHNOLOGY
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
APRIL, 1999

CERTIFICATE

This is to certify that the work contained in this M.Tech thesis titled **PLACEMENT OF OPTICAL AMPLIFIERS IN OPTICAL NETWORKS** *has been carried out by* **ARVIND KUMAR** *under my supervision and has not been submitted elsewhere for any degree or diploma.*

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Dedicated to
My Parents, Sisters and Brothers-in-law.

Acknowledgement

I would take this opportunity to express my sincere gratitude towards Dr. **Y. N. SINGH**, my thesis supervisor for his invaluable guidance and for giving me complete freedom in choosing a problem for the thesis work. He has always been very patient and encouraging regarding the doubts and discussions during the thesis work. I express my sincere respects to all my teachers who introduced me to different interesting subjects in the field of optical communications and laser technology.

I would like to thank my friends Khem Chandra, Punya Sheel, Maj. Trandeep, Maj. Narad and Prasad who were helping me during the thesis work. I would also like to thank Vivek, Pawan, Pratul, Chander and Jitendra their support during my IIT stay. I am deeply indebted to my parents and my sisters for their love and encouragement.

My stay at IITK has been marked with many memorable moments. During this period, I was fortunate enough to listen to some of the eminent personalities from different fields. From this environment, apart from academics, I got an exposure to different ideas regarding the various aspects of life, science and technology.

Finally, I thank to CSIR for giving me the financial add to pursue this M.Tech Thesis.

Abstract

Optical networking has revolutionized long distance transport and has resulted in capacity expansion, cost reduction and simplified operation. Optical amplifiers play a key role in the construction of optical networks to overcome the impact of attenuation. Positions of amplifiers in the optical path determine the bit error rate of the digital optical links. In this thesis the impact of placement of amplifiers in optical networks is analyzed. Effectiveness of OAs in a subscriber access network and multi-star topology in terms of increase in number of users has been investigated. In the analysis OA with i) unsaturated gain, ii)average gain saturation, and iii) average gain saturation with gain fluctuations have been considered. Effect of extinction ratio has also been taken in care of. Results show that the number of users in any network can not increase indefinitely by placing more and more amplifiers because after a certain limit, system performance starts degrading with increase in number of amplifiers due to the effect of ASE and gain saturation.

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Chapter 1

INTRODUCTION

1.1 ALL OPTICAL NETWORK

All-Optical Networks (AONs) form a class of networks where optical technology (rather than electronics) plays the major role in network functionality. Before defining the various types of AONs, it is instructive to recall that today's "traditional" networks consist, for the most part, of a collection of electronic switches interconnected by point-to-point optical fiber links, and spanning local to national (or even global) distances. To accommodate continuously increasing demand for bandwidth and flexibility, such networks are being enhanced by adding more fibers and switches, by increasing the bit rate per fiber (e.g., to 2.5- or 10-Gb/s SONET rates), and by upgrading the switches size and throughput. It is anticipated that such enhancements will eventually lead to very large and complex networks that are difficult and expensive to construct, operate and maintain. Recent advances and envisioned future capabilities of optical technology promise novel all-optical networks capable of providing improved economy, flexibility and robustness, while still capable of making use of the embedded fiber plant. There are three general types of AONs, which, from a functionality point of view, can be classified as follows

1. Passive Optical Networks (PONs)

2. Transparent AONs

3. Ultra- High-Speed AONs

1.1.1 Passive Optical Networks (PONs)

PONs, traditionally, refer to all-optical networks utilizing only passive optical components (e.g., fibers, directional couplers, star couplers, wavelength routers and multiplexers, and filters). The emphasis in such networks is on low cost and low (or no) maintenance, and the intended applications are fiber in the loop and fiber to the home. Thus, economy and the "bury it and forget it" philosophy are prime objectives, even with a sacrifice in performance. Occasionally, optical amplifiers, are employed to overcome propagation and distribution losses in PONs, even though they are clearly not passive. The optical signaling formats in PONs can employ wavelength division multiplexing (WDM), sub-carrier multiplexing, time division multiplexing (TDM) or any combination of these.

1.1.2 Transparent All Optical Networks

Transparent AONs resemble PONs in many ways, but, rather than economy, they stress on flexibility, high performance and local to global coverage. The idea is to provide optical transparency enabling each of a large number of optical WDM channels to propagate from source to destination without optical to electronic data conversion within the network. The signaling format of each of the WDM channels can be virtually arbitrary, providing tremendous potential for flexibility and economic upgradability. Besides the aforementioned passive optical components used in PONs, transparent WDM AONs can employ devices that control the flow of the optical signals, such as optical switches and configurable wavelength routers and wavelength converters. The control of these devices can be done electronically, as long as the path of the signal itself remains optical. Such control can be slow if it is used mainly to reconfigure the network to accommodate various connection requests and varying traffic patterns. The terminal equipment in transparent AONs would employ tunable laser transmitters and/or tunable filter (or heterodyne) receivers. Two users can establish a communications path through the network by picking the appropriate transmitter and receiver wavelengths, perhaps after making a prior request from a network controller. After providing the connection, the network should be transparent to the modulation format employed. More interestingly, any group of users can pick the right set of wavelengths and reconfigure the AON so as to create their own transparent virtual network. Any user should be able to take part in multiple simultaneous sessions.

1.1.3 ULTRA-HIGH-SPEED ALL OPTICAL NETWORKS

Ultra high speed AONs take advantage of the tremendous speed of various optical phenomena to realize peak channel data rates that are much higher than what can be generated, controlled or switched electronically. In such networks, communication is effected by on-off modulated, ultra high rate (e.g., 100 Gb/s) time division multiplexed (TDM) streams of short duration optical pulses, One can also use high power optical pulses in which dispersion is compensated by the non-linearity in fiber, known as solitons. Virtually all network controls and signal switching functions are accomplished optically i.e., these are truly all optical networks. However, they are not optically transparent.

1.2 THE IMPORTANCE OF OPTICAL NETWORKS

The great importance of optical networks arises from their potential to offer low cost telecommunication capacity because of huge accessible bandwidth of optical fiber. This bandwidth can be accessed using WDM, TDM or CDM techniques. These techniques have reduced the cost per unit bandwidth per user. One more factor which attract us to optical networks is openness, system is easy to use, easy to change and can serve a wide variety of resources that are in use concurrently. These networks are also protocol transparent, and many different bit rates and formats can be used at the same time.

1.3 TOPOLOGIES

Every network is a combination of nodes that can be geometrically arranged in many different ways to get many desirable requirements e.g. scalability (ability to expand the network), modularity (ability to add just one more) and irregularity (any stylized pattern should not be forced that may not meet user requirement). This geometrical arrangement called topology, is highly important since many factor like cost of network, bandwidth distribution and delay etc. depend upon it

There are different type of topological aspects to keep the track of a network e.g. what is going on physically and logically also called physical and logical topology respectively. Physical topology tells us how the traffic flows in the network and logical topology shows how the nodes are connected to each other. These topologies is implemented on a certain geometrical topology (physical-path topology) which are mainly of three types, bus, ring and star.

1.3.1 Bus Topology

In a bus topology (fig.1.1), all the network nodes are connected to a common transmission medium. Each network node has unique address which is used when the information is transmitted. When a message is sent it propagates through out the medium and is received by all the users. To receive the message, each user continuously monitors the medium and copies those messages that are addressed to it as the message passes by.

Bus topology is very common in the copper cable network since it requires less copper cable. When this topology is implemented on optical fiber, it can only support limited number of users. This is because of nonuniform distribution of power among the users, and consequently requirement of receivers with large dynamic range.

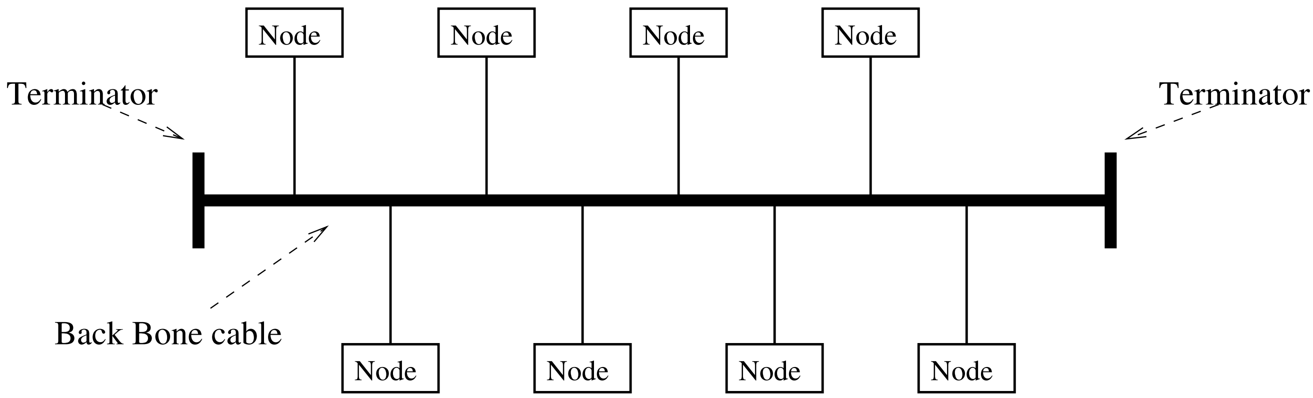

Figure 1.1: Bus Topology

1.3.2 Ring Topology

In ring topology (Fig 1.2) consecutive nodes are connected by point-to-point links which are arranged to form a single closed loop. One can also use passive optical rings since these can provide high reliability because failure of nodes will not break the ring

In this topology number of user is limited by the power budget. Whenever a node is added, links have to be placed between this node and two nearby nodes. This addition or deletion of node disturbs operation of network. While in bus the network operation remain undisturbed when a node is added or removed.

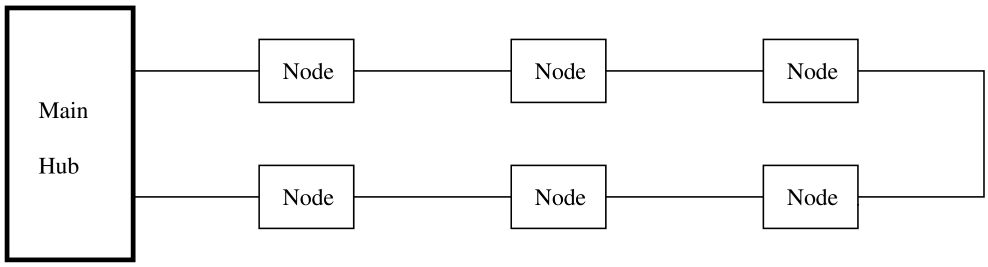

Figure 1.2: Ring Topology

1.3.3 Star Topology

In star topology (fig.1.3) transmitted optical power is equally distributed to all output nodes. Therefore this topology can support maximum number of users as compared to passive ring and bus. Present technological constraints limit the number of users in this topology. Every node have separate connectivity to main hub so amount of fiber required is considerably more but every user have large band width and sectionalization of faults is easy.

1.4 Optical Networks

1.4.1 Subscriber Access Network

Subscriber access network is a network which connects customers to a central office (CO). Two subscriber can communicate via exchange and trunk network. The traffic flow control and management (administration) is done by the exchange along with the network management function of trunk network. The subscriber loop consists of three components, i) feeder ii) distribution and iii) service segments (Fig. 1.4). These segments may be totally fiber based; however near term economic consideration may call for deploying fiber only up to the SAP and using Copper (wire pair or coax) for service segment. The enhanced bandwidth offered by optical fiber will allow the use of subscriber access networks to provide wide-band service (video-phone, Video on demand, Digital audio, facsimile, data etc.) to the subscriber together with narrow band

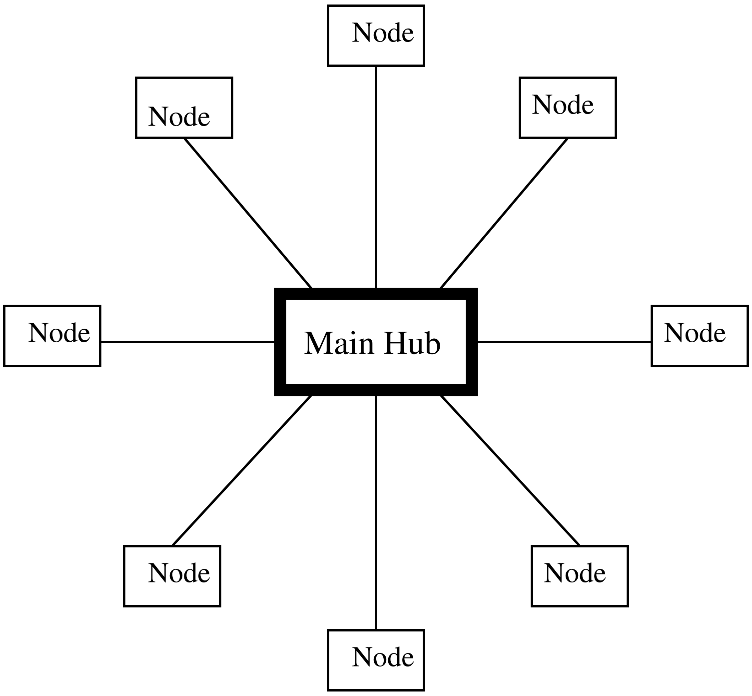

Figure 1.3: Star Topology

services. Each segment of the subscriber access network (feeder, distribution, service) may have different physical topology, which refers to certain physical attributes of that segment.

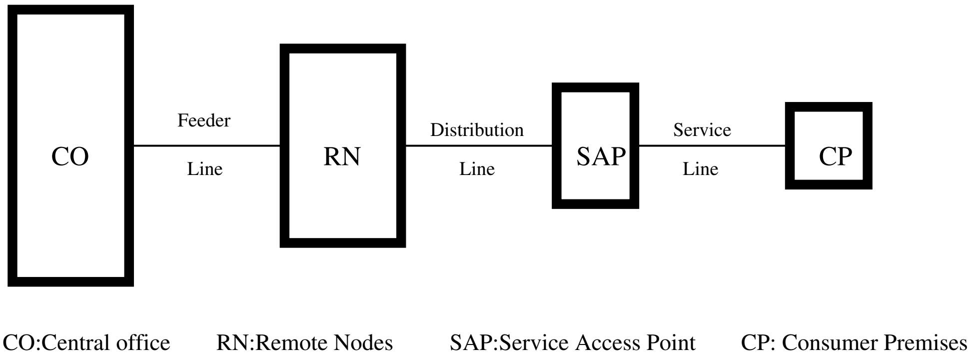

Figure 1.4: Subscriber Access Network

1.4.1.1 Totally Transparent Optical Subscriber System (TTOSS)

The totally transparent optical subscriber system is shown in fig 1.5. It is a single star topology used between CO and CP Thus, dedicated fiber required between CO and each customer. Two fiber can be provide to each customer. One for each direction of transmission or both direction of transmission can be carried over a single fiber using an optical duplexer to separate the upstream and downstream wavelengths [11]. WDM is used to provide multiple optical channels to each customer in the TTOSS system.

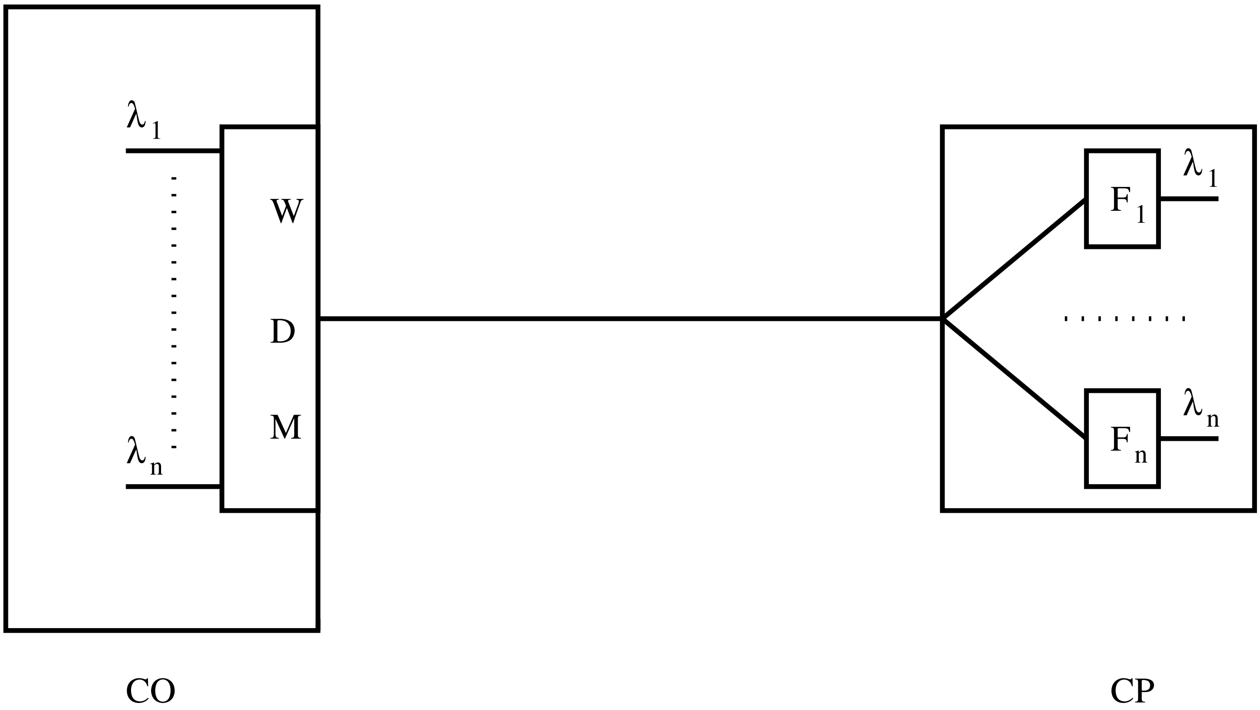

Figure 1.5: Totally Transparent Optical Subscriber System

1.4.1.2 TELEPHONY PASSIVE OPTICAL NETWORKS (TPON)

The telephony passive network is shown in figure 1.6. It uses physical star and logical bus topology used feeder,distribution and service segments of local subscriber access network. Broadcast mechanism is used for communication. In this network an optical filter that passes only the TPON system wavelengths is built into Optical network unit This enables wavelengths to be added at a later date to provide new services without disturbing existing services. In TPON the subscriber security and privacy are needed, encryption and time slot hopping can also be used.

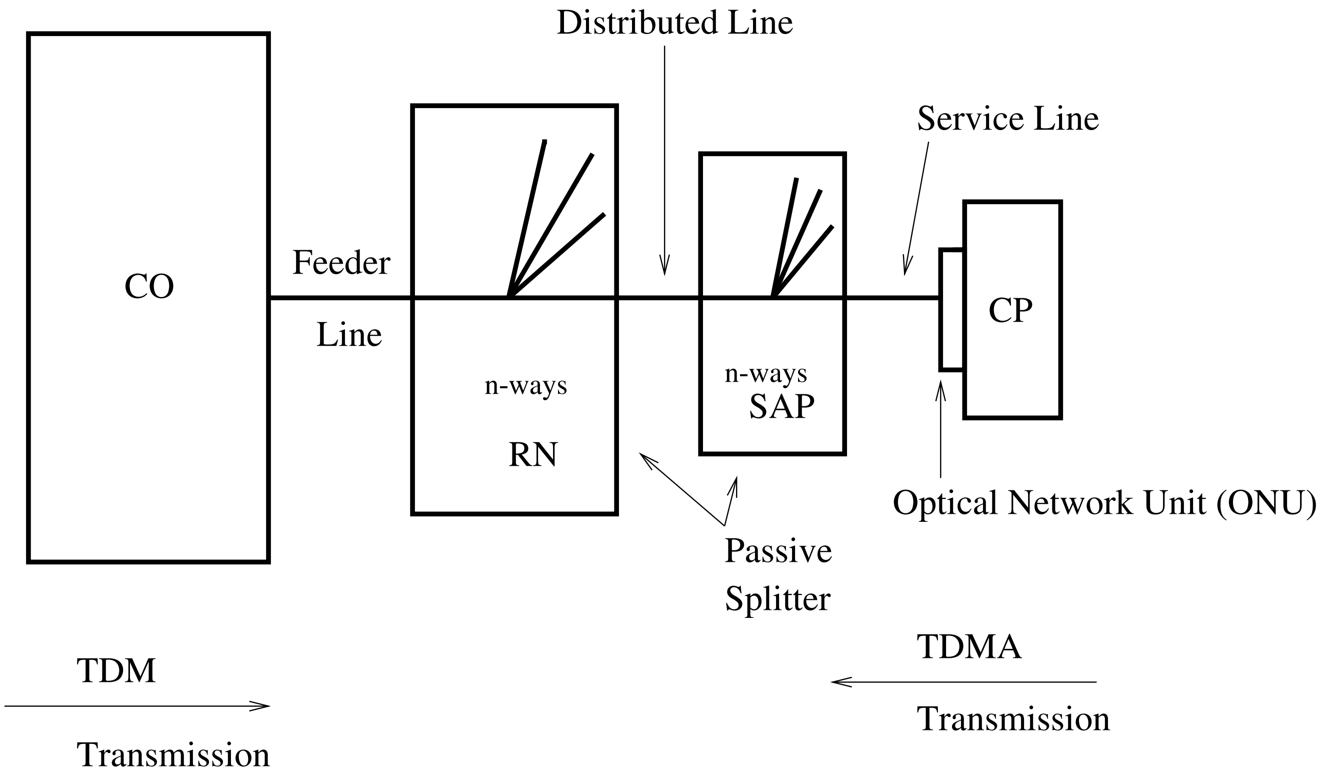

Figure 1.6: Telephony Passive Optical Network

1.4.1.3 PASSIVE PHOTONIC LOOP

Passive Photonic Loop system is shown in fig 1.7 . It is double star network and uses dense WDM techniques to provide each subscriber with a dedicated down stream wavelength and a dedicated upstream wavelength. PPL has physical star and logical star topology in both the feeder and service segments of the subscriber access network.

The PPL system allows sharing of the fiber by several customers. Dense WDM is used between the CO and RN or SAP to allow sharing of single feeder fiber. Dedicated optical fiber links connect the RN or SAP to CP.

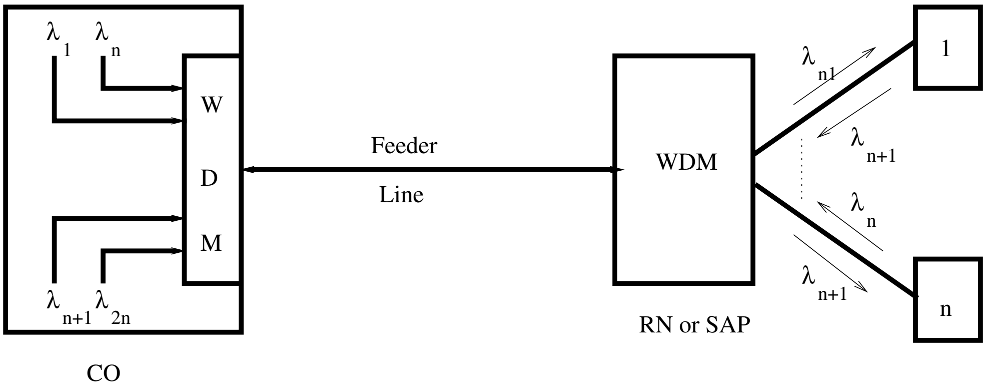

Figure 1.7: Passive Photonic Loop

1.4.1.4 NEW SUBSCRIBER ACCESS NETWORK

This network is proposed by sakhtivel [12] and shown in fig.1.8 . It is a combination of bus and star topologies. The architecture has two buses, one for down stream transmission and other for upstream transmission. There are OADMs (Optical Add-Drop Multiplexers) on each bus. As shown in fig 1.8. The drop signal is fed to a star coupler. The split signal is distributed to the subscribers connected to star coupler through dedicated fibers.

A specific wavelength is preassigned to each group and the same wavelength is dropped from a set of wavelength in the bus. The signal on this wavelength is equally distributed to all users through a star coupler (SC).

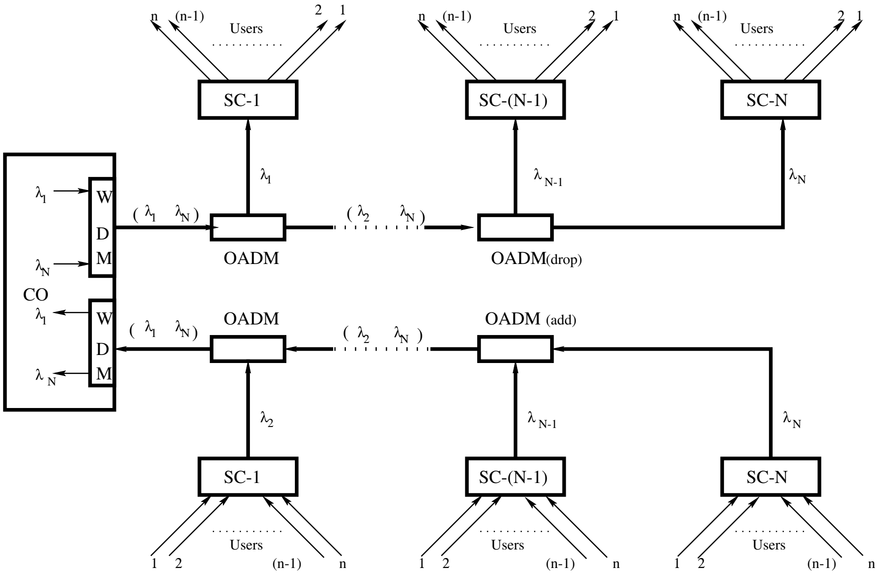

Figure 1.8: New Subscriber Access Network

1.4.2 LANs and MANs

LANs and MANs are characterized in terms of their geographical span. LANs span distance of only few kilometers and MAN span distance of around 50-100 km, MANs serve as back bone networks interconnecting LANs and other high speed information sources and sink over a regional domain. Many different type of topology are applied to LANs and MANs but the common ones are bus, tree and star. Many combinations of these topologies are also used.

1.5 Objective of thesis

The limit on number of users in any optical network is set by the losses and noise due to many optical component and fiber used. These number of users can be increase by inserting the optical amplifiers at appropriate places. Increase in users depend upon gain, noise and flatness of gain of optical amplifiers. The objective of this thesis is to study the placement of amplifiers in topologies for subscribers access network, LANs and MANs to maximize the number of users. In chapter 2 we described the role of optical amplifiers with a model used for our analysis. Noise model is also discussed at the end. The chapter 3 describe the placement of optical amplifiers (OAs) in a subscriber access network topology mentioned in section 1.4.1.4. Comparative study of placement is also done for ideal amplifier (without saturation), amplifier with saturation, amplifier with gain fluctuation. Similar studies for Tree net and Star topologies using only one optical amplifier in a optical path are already available in literature [2] [3]. We present our study to place more than one amplifiers in a optical path in multi-star topology in chapter 4. The conclusions and possibilities for future work are given in last chapter.

Chapter 2

AMPLIFIER MODEL

2.1 OPTICAL AMPLIFIER

Optical amplifiers allow the direct amplification of light, without the need for optical to electrical conversion. They provide the amplification necessary to overcome the attenuation associated with fiber and other optical components. The importance of optical amplifier is that it provides high gain (up to 50dB) over extremely wide bandwidths (approximately 4 THz), and this performance is unmatched by any electronic component. The wide optical bandwidth of the amplifier matches reasonably well to the bandwidth of fiber.

Optical amplifiers are now being considered for a wide range of applications and are becoming an essential element in many future system and network designs. In point-to-point systems they are used as power booster, repeater or receiver preamplifiers. The exact requirements in optical amplifier specification for each applications are different. In optical network optical amplifiers can be used as functional device to perform wavelength conversion in wavelength division multiplexed system (i.e. changing the carrier wavelength of a specific information stream) or as gates in optical switch matrices. In any real application of optical amplifier a number of parameters are required to establish performance, all of which must be considered. Major ones are

- **Net gain:** the gain from input to output fiber
- **Bandwidth:** the bandwidth associated with the gain process
- **Noise:** the amount of noise generated by the amplification process

- **Polarisation sensitivity:** the sensitivity of device gain to variations in polarization of the incident signal
- **Saturation power:** the signal power at which the gain has fallen to 3 dB of its unsaturated value.

2.2 TYPE OF OPTICAL AMPLIFIER

There are two types of optical amplifiers most suited for telecommunication applications

1. Semiconductor optical amplifiers (SOAs)
2. Doped fiber amplifier (DFAs)

2.2.1 SEMICONDUCTOR OPTICAL AMPLIFIER (SOAS)

The SOAs are basically semiconductor lasers which operate below lasing threshold. Electrical energy is used to achieve population inversion in these. The basic difference between a laser and SOA is facets reflectivity. Ideally SOA should have zero facets reflectivity and a laser should have one. SOA which having very low facet reflectivity is called traveling wave amplifier (TWA) since input signal passes through the device gain region only once. If facet reflectivity is reasonable but still quite small then signal passes back and forth through the cavity gain region many times and the device called Fabry-Perot amplifier (FPA).

- **ADVANTAGES OF SOAS**

1. SOAs can be easily integrated in integrated optic transmitter/receiver
2. SOAs are available over a wide wavelength range $(0.8\mu m - 1.55\mu m)$

2.2.2 DOPED FIBER AMPLIFIER (DFAS)

This type of amplifier is made by doping the core of fiber by rare earth elements e.g. erbium, praseodymium, ytterbium etc. The basic principle of DFAs is also based on stimulated emission like SOAs. But optical pumping is used in DFAs in place of electrical pumping used in SOAs. Pumping power can be injected in three ways. It may be injected so that it may propagate in the same direction as the signal. Alternatively it can be injected in the opposite direction of signal. The fiber may be pumped by injecting pumping power in forward as well as reverse direction for high output power amplifier. The signal amplification depends upon the population inversion profile along the length of fiber. For higher power more population inversion is desired over the larger length. So for optimum design we have to trade of between length and pump power. For a given population inversion profile, there is an optimum length of doped fiber which maximizes the overall gain.

2.3 AMPLIFIER MODEL

A traveling wave amplifier has anti-reflection coatings applied to its facets to reduce their reflectivities. In the limiting case, an amplifier with zero reflectivity facets would be a traveling wave amplifier, with the factor $G\sqrt{R_1R_2}$ equal to zero, where G is the single pass gain through the device and R_1 and R_2 are the input and output facet reflectivities of the amplifier respectively. In practice, however, even with the best anti-reflection coatings there is some residual facet reflectivity and the amplifier lies between traveling wave and Fabry-Perot. So we shall define a amplifier to be TWA if $G\sqrt{R_1R_2} \leq 0.17$. For a TWA gain is given by [7]

$$G = G_0 \exp\left(-\left[G-1\right]\frac{P_{in}}{P_{sat}}\right) \tag{2.1}$$

where G_0 is unsaturated gain, P_{in} and P_{sat} the input and saturation power levels, respectively. Parameter G_0 and P_{sat} depend on amplifier and biasing conditions. This equation is derived from steady-state analysis and ignores dynamic effects and possible four-wave mixing (FWM) which may be important at high bit rate.

Gain model of a amplifier presented here does include some simplifying assumptions. We have assumed that all wavelengths equally to the gain saturation of the amplifier. Hence we can aggregate the available gain on the individual wavelength to determine the available from fig.2.1. While this assumption is acceptable for the wavelengths which are close together in the spectrum. It is an approximation for the wavelengths which are wide apart [10].

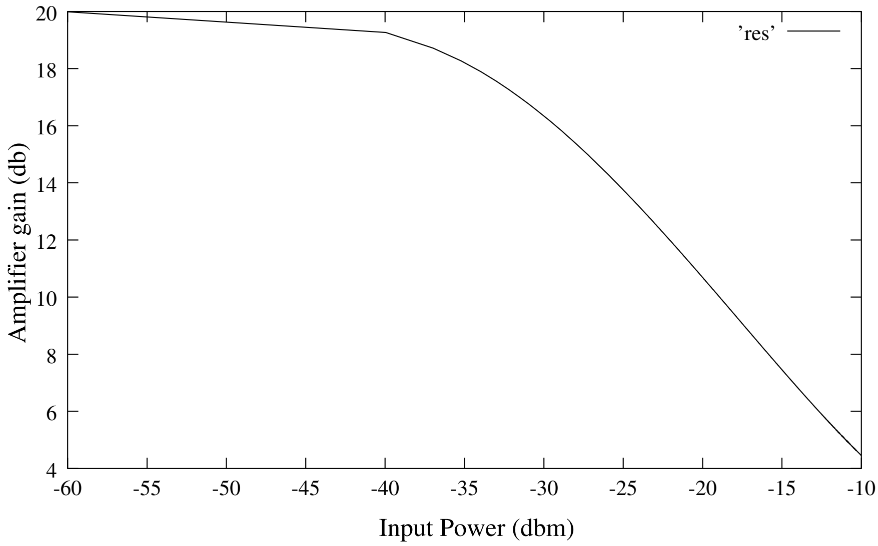

Figure 2.1: Amplifier Gain

2.4 AMPLIFIER NOISE MODEL

In Semiconductor Optical amplifier when input signal is amplified, amplified spontaneous emission noise is also produced due to finite life time of excited atom. This noise can be modeled as a white Gaussian noise with power spectral density

$$S_{sp} = n_{sp}\left(G-1\right)hv \qquad W/Hz \tag{2.2}$$

where n_{sp} is the spontaneous emission factor, G is amplifier gain, h is plank constant and v is the optical frequency of input signal.

When this ASE noise falls on photo detector, due to the square law it beats with itself, input signal and shot noise. This produces ASE-ASE, ASE-signal and ASE-shot noises. The variances of different beat noise current components are[7]

ASE-ASE beat noise

$$\sigma^2_{ASE-ASE} = R_0^2 S_{sp}\left(2B_eB_o - B_e^2\right) \tag{2.3}$$

ASE-Signal noise

$$\sigma^2_{ASE-Sig} = 4R_0^2 P_R S_{sp} B_e \tag{2.4}$$

ASE-Shot noise

$$\sigma^2_{ASE-Shot} = 2eR_0 \left(S_{sp} B_e\right) B_o \tag{2.5}$$

where R_0 represents responsitivity of the photo detector, B_e is the bandwidth of the electrical filter (assumed to be ideal) in receiver, B_o, the band with of optical filter prior to photo detector and P_R ,the received power level.

Chapter 3

AMPLIFIERS IN NEW SUBSCRIBER ACCESS NETWORK

The topology of this network is described in sec 1.4.1.4 of chapter 1. The number of users are limited by the losses due to many optical component like star coupler (SC), optical add-drop multiplexer (OADM) etc, instead of dispersion in fiber. So the number of user can be increased by placing the optical amplifiers at appropriate places. In this chapter amplifiers with a model already discussed in previous chapter are placed in the network topology. Interchannel cross talk in the presence of optical filter, SC, and OADM is neglected. In the calculation, number of wavelengths used is equal to the number of branches, therefore number of branches can not be increased indefinitely, as the loss varies with wavelength in fiber. Bandwidth constraint put by the OADM device used in network limits the number of users per branch that can be supported for maximum desirable bandwidth per user.

3.1 PLACEMENT OF OPTICAL AMPLIFIERS

In this network, amplifier can be placed in two ways either between OADM and SC or on the bus which carries many wavelengths. The second placement option is better. In this, the amplifier can amplify all the wavelength simultaneously. Since all the WDM channels share the optical amplifier, it is expected that small number of OAs would be needed. Henceforth this placement option is investigated further.

Since the number of users depends on the position of amplifiers on the bus, therefore the optimal placement of optical amplifiers will support maximum number of users. In the given topology, one amplifier is placed at a time. In order to place the first optical amplifier, the bit error rate is evaluated for the users in all branches after placing the optical amplifier between CO and OADM. The branch of the topology means the group of users connected to SC which is connected to OADM. The number of users in a branch are fixed and calculated in the example. By the computed value of BER in all branches, it is possible to find the maximum number branches which can be supported. (It is assumed that the branch is supported if BER for its users is less than 10^{-9}.)

If the network supports N number of branches without any amplifier then the first OA can be placed at $N+1$ positions as shown by encircled number in fig 3.1. The worst of the BER in all branches is computed for all these $N+1$ OA positions. This gives the number of branches which can be supported by the each OA position. Now the optimum position to place the OA is decided . Once the first amplifier is placed, let the N_1 branches are supported by the network. Thereby making available N_1+1 positions for placing second amplifier. If second amplifier is also placed adjacent to first one but before the same branch, a drastic decrease in number of users is noted, because the ASE of both the OAs degrades the BER at the receivers leading to reduction in number of users. This will be evident from the calculation done for the example in the coming sections. Due to the above reason, it has been assumed that between two consecutive branches only one OA can be placed. The placement of second OA is optimized for all the remaining N_1 positions. More and more amplifier are placed in the same manner until there is no further increase in number of branches.

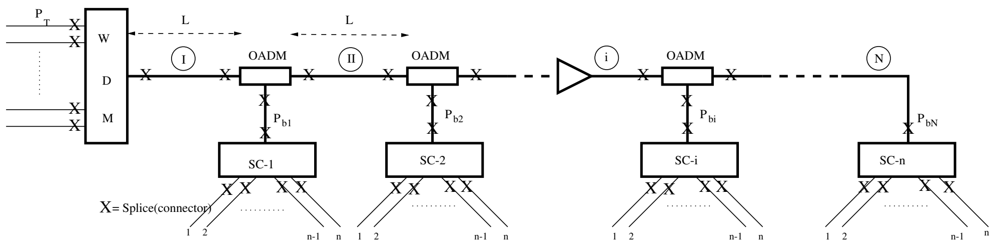

Figure 3.1: System model for new subscriber access network

3.2 EVALUATION OF MAXIMUM NUMBER OF USER

Once the relationship between BER and transmitted power for a given number of users is known. Number of users for a given transmitted power can be easily calculated. The relationship between BER and transmitted power is evaluated with the assumption that responsitivity of photodetector is independent of wavelength and network uses ideal optical filters. In the calculations with OAs, three cases for OAs are considered. These are i) ideal Amplifier, ii) amplifier with average gain saturation and iii) amplifier with average gain saturation and gain fluctuation.

3.2.1 SYSTEM LOSS MODEL

The system model for power budget analysis is shown in Fig 3.1. To determine the number of users supported by the network, it is assumed that all the users are at the equidistant from the access point i.e., SC in the Fig. 3.1. The symbols which are used is given in Table 3.1.

Transmitter optical power per channel	$P_T\ dBm$
Optical power received at branched	$P_{bk}\ dbm$ where k=1,2,3,.....,N
Signal splice loss	$L_{sp}\,dB$
Fiber attenuation	$\alpha\,dB/Km$
Fiber loss	$L_f\ dB$
Losses between transmitter and OA placed before i^{th} branch	L_{TA_i}
Losses between OAs placed before i^{th} and j^{th} branches	$L_{A_iA_j}$
Losses between OA placed before i^{th} branch and receiver	L_{A_iR}
Input power in a^{th} OA	P_{in_a}
Insertion loss of WDM	$L_w\ dB$
Insertion loss for dropping signal in OADM	$L_{12}\ dB$
Insertion loss for passing signal in OADM	$L_m\,dB$
Insertion loss of signal $3\,dB\,2\times 2$ coupler	$L_i\,dB$
Distance between branches	$L\,Km$
Total number of branches	N
number of user per branch	n

Table 3.1: Table for used symbol for new subscriber network

In order to determine the received power level, loss in the path from CO to receiver is needed. When OA is not used the loss between transmitter and receiver of i^{th} branch is given by

$$L_{TR} = L_w + L_{sp}(2i+5) + i\alpha L + (i-1)L_m + L_{12} + L_i \log_2 n + 10\log_{10}(n)\,. \tag{3.1}$$

When the OAs are used then the losses between transmitter and amplifier nearest to it, between two consecutive amplifiers, and between receiver and nearest amplifier are required. If the amplifier nearest to CO is placed before i^{th} OADM (Fig. 3.2) then the loss between transmitter and nearest amplifier will be

$$L_{TA_i}(dB) = L_w + 2iL_{sp} + i\alpha L + (i-1)L_{m.} \tag{3.2}$$

If the two consecutive amplifiers are placed before i^{th} and j^{th} OADM then the loss between two amplifiers will be

$$L_{A_iA_j}(dB) = 2(j-i)L_{sp} + (j-i)L_m + (j-i)\alpha L, \tag{3.3}$$

and the loss between receiver of l^{th} branch and nearest amplifier placed before t^{th}OADM is given by

$$\begin{aligned} L_{A_tR}(dB) &= [2(l-t)+4]L_{sp} + (l-t)L_m + L_{12} + (l-t)\alpha L + L_i \log_2 n \\ &\quad +10\log_{10}(n) \qquad (when\ t < N) \\ &= [2(l-t)+2]L_{sp} + (l-t)L_m + (l-t)\alpha L + L_i \log_2 n \\ &\quad +10\log_{10}(n) \qquad (when\ t = N) \end{aligned} \tag{3.4}$$

Here, N is the total branches in the network. If r amplifiers are placed in the network between transmitter and receiver, then total loss will be

$$L_{TR}(dB) = L_{TA_1} + L_{A_1A_2} + L_{A_2A_3}...... + L_{A_iA_j} + ... + L_{A_rR.} \tag{3.5}$$

3.2.2 Network with Ideal Amplifier

Supportable number of users can be increased by using optical amplifiers. For a given number of amplifiers network may support more users with proper placement of amplifier at the bus. At the branch N, power received is minimum since losses between the transmitter and receiver is maximum. Therefore, BER at this receiver is considered minimum and used in optimizing the OA placement.The SC in each branch divides the optical power equally among n receivers.

The receiver sensitivity for a given bit rate is defined as the minimum received power level needed to maintain bit error rate (BER) less than a particular value. In our calculations, this particular value of BER is taken as 10^{-9} .

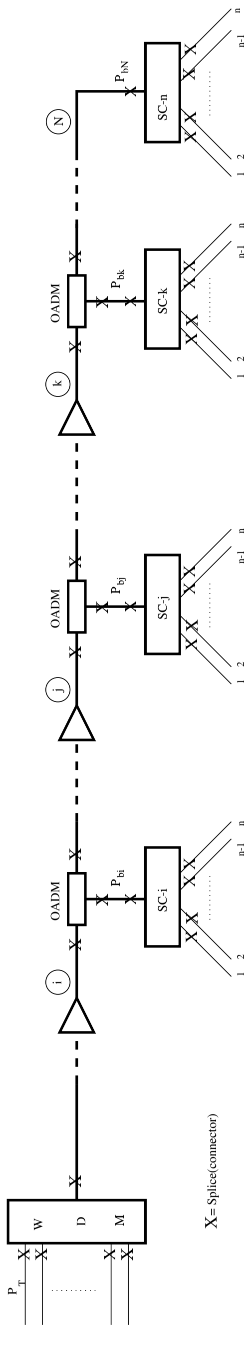

Figure 3.2: Loss model for new subscriber access network

The best optical receivers require a flux of $300 - 100$ photons per bit for bit rates between $10\ Mb/s$ to $2.5Gb/s$ and over the wavelength range of $0.8 - 1.6\mu m$. This corresponds to a sensitivity of

$$P_R = 2\ \times\ 10^{-13} B\, mW, \tag{3.6}$$

where B is the bit rate in bits /sec per channel.

If total a amplifiers are placed between transmitter and receiver and 1^{st} amplifier is placed before i^{th} branch, 2^{nd} amplifier before j^{th} branch and so on. Before the receiver,$(a-1)^{st}$ amplifier before s^{th} branch and a^{th} amplifier before t^{th} branch are placed. The total ASE noise psd produced at the amplifiers' output will be

$$\begin{aligned}
S_{sp_1} &= n_{sp}(G_0 - 1)h\nu \\
S_{sp_2} &= n_{sp}(G_0 - 1)h\nu + S_{sp_1} L_{A_i A_j} G_0 \\
S_{sp_3} &= n_{sp}(G_0 - 1)h\nu + S_{sp_2} L_{A_j A_k} G_0 \\
&\vdots \\
S_{sp_a} &= n_{sp}(G_0 - 1)h\nu + S_{sp_{a-1}} L_{A_s A_t} G_0,
\end{aligned} \tag{3.7}$$

The ASE noise psd at the receiver

$$S_{spr} = S_{sp_a} L_{A_t R}$$

where t^{th} amplifier precedes the receiver. The input power to the amplifiers corresponding to bit b will be

$$\begin{aligned}
P_{in_1}(b) &= P_s(b) L_{TA_i} \\
P_{in_2}(b) &= P_{in_1}(b) G_0 L_{A_i A_j} + S_{sp_1} L_{A_i A_j} \\
P_{in_3}(b) &= P_{in_2}(b) G_0 L_{A_j A_k} + n_{sp}(G_0 - 1)h\nu L_{A_j A_k} \\
&\vdots \\
P_{in_a}(b) &= P_{in_{a-1}}(b) G_0 L_{A_s A_t} + n_{sp}(G_0 - 1)h\nu L_{A_s A_t}
\end{aligned} \tag{3.8}$$

where

$$P_s(1) = \frac{2P_T}{1+\epsilon}$$

$$P_s(0) = \frac{2P_T\epsilon}{1+\epsilon}.$$

In the above equations, the amplifier gain is always taken as G_0 since ideal amplifier implies that there is no gain saturation. But the amplifiers do produce ASE noise corresponding the unsaturated gain G_0. The $P_R(b)$ at the receiver is given by

$$P_R(b) = P_{in_a}(b)G_0L_{A_tR}. \tag{3.9}$$

The signal current and noise variance for bit b are given by

$$I_s(b) = R_0P_R(b). \tag{3.10}$$

and

$$\sigma^2(b) = 2eR_0\left[P_R(b) + S_{spr}B_0\right]B_e + 4R_0^2P_R(b)S_{spr}B_e + R_0^2S_{spr}^2\left[2B_0B_e - B_e^2\right] + \frac{4KTB_e}{R_L} \tag{3.11}$$

respectively. Here $P_R(b)$ is the power received by the photodetector, ϵ is the extinction ratio (the ratio of optical energy transmitted for bit '0' to the one transmitted for bit '1'). The L_{TR} represents the losses between transmitter and receiver.

The average probability of error considering the threshold level which equalizes the BERs for bit '1' and '0' is

$$P_e = \frac{1}{2}erfc\left[\frac{I_s(1) - I_s(0)}{\sqrt{2}\left[\sigma(0) + \sigma(1)\right]}\right], \tag{3.12}$$

where $I_s(b)$ is the signal current and $\sigma(b)$ is noise variance corresponding to bit 'b'.

The average probability of error with threshold level which equalizes the BER for the bit '1' and '0' is determined by using equations (3.10),(3.11) and (3.12).

3.2.3 Network with Average Gain Saturated Amplifier

There will be multiple channels passing through any amplifier which will be placed in the network. Since the amplifier gain is saturated by the input optical power leading to reduction in gain, there will be interchannel gain crosstalk. In order to explain this, consider that two channels (ch #1 and #2) are passing through the OA. Assume that ch #1 is modulated and ch #2, unmodulated. The total sum of powers of two channels will be modulated and hence gain will be modulated. Consequently at the output, one can find that power for ch #2 will be modulated. This phenomenon is called gain crosstalk. In case of multiple channels, a similar gain crosstalk will be there. If the relaxation time of excited atoms, molecules or ions in the amplifying medium is much larger than the bit period, then instead of the modulation of ch #2, one will find that gain seen by this channel will be reduced [7].

In order to analyze the gain crosstalk in multiple channel, let us consider N_t channels passing through an optical amplifier. Out of N_t channels, let N_1 channels are having bit 1. The probability that N_1 channels are having bit '1' is given by [10]

$$P_{N_1} = \begin{pmatrix} N_t \\ N_1 \end{pmatrix} \left[\frac{1}{2}\right]^{N_t} \tag{3.13}$$

Here, The probability distribution of number of channels having bit '1' is preassumed to be binomial. The input power to the 1^{st} amplifier corresponding to bit b is

$$P_{in_1}(b) = P_s(b) L_{TA_i}. \tag{3.14}$$

the total input power to the amplifier is

$$P_{in(amp)} = N_1 P_{in_1}(1) + (N_t - N_1) P_{in_1}(0). \tag{3.15}$$

The corresponding saturation gain $G(N_1)$ of the amplifier is computed using equation 2.1 with the above input power. The average gain for 1^{st} amplifier is given by

$$G_{av_1} = \sum_{N_1=0}^{N_t} P_{N_1} G(N_1). \tag{3.16}$$

One can use G_{av_1} in place of G_0 in equation 3.8 to fine the input power to the 2^{nd} amplifier. Repeating the above procedure, we can find G_{av_2}. Similarly average gain for other amplifiers can be determined. It should be mentioned here that the number of channel passing through an amplifier depends upon its placement. Correspondingly the expression of P_{N_1} and G_{av_1} are modified. Now, the received signal power corresponding to bit b at the receiver is given by

$$P_R(b) = P_{in_a}(b) G_{av_a} L_{A_r R} \tag{3.17}$$

As G_{av_a} is independent of the signal bit, the ASE noise psd at the output of a^{th} for both bit '1' and '0' will be same and easily calculated by equations 3.7. It is given by

$$S_{sp_a} = n_{sp}(G_{av_a} - 1)h\nu + S_{sp_{a-1}} L_{A_t A_r} G_{av_a}. \tag{3.18}$$

Therefore, the ASE noise psd at the receiver is

$$S_{spr} = S_{sp_a} L_{A_t R}. \tag{3.19}$$

The received power level at receiver is

$$P_R(b) = P_{in_a}(b) G_{av_a} L_{A_r R}. \tag{3.20}$$

The signal current and noise variance is given by

$$I_s = R_0 P_R(b) \tag{3.21}$$

and

$$\sigma^2(b) = 2eR_0\left[P_R(b) + S_{spr}B_0\right]B_e + 4R_0^2 P_R(b) S_{spr} B_e + R_0^2 S_{spr}^2\left[2B_0B_e - B_e^2\right] + \frac{4KTB_e}{R_L} \tag{3.22}$$

the average P_e is calculated by using equations (3.19),(3.20) and (3.12).

3.2.4 Network With Amplifier Having Average Gain Saturation and Gain Fluctuation

In this case, the gain statistics of the OAs is not averaged as done in section 3.2.3. Here the effect on a gain of a particular channel due the presence of other channels is calculated. The network is having N channels then there will be N branches. Each channel is dedicated to the users on a particular branch and having bits b_1, b_2, b_3, ..., b_N corresponding to each channel. These bits being transmitted in different channels are considered to be in synchronism. Each bit can have values '1' and '0'. Amplifiers are placed as described in section 3.2.2. If N^{th} is the desired channel then number of interfering channels in 1^{st},2^{nd},3^{rd} , ..., a^{th} amplifier will be $N-i-1$, $N-j-1$, $N-k-1$, ..., $N-t-1$ respectively. Since 1^{st} amplifier have maximum number of interfering channels ($N-i-1$), so there are 2^{N-i-1} possible patterns for the bits in $N-i-1$ channels. Each of these patterns pass through the amplifier with the probability $\frac{1}{2^{N-i-1}}$. Noise variance and signal current corresponding to bit '1' and '0' in desired channel is calculated for all 2^{N-i-1} possible interfering patterns. The threshold level which equalizes the BER corresponding to lowest level for bit '1' and highest level for bit '0' in channels of interest is computed. This threshold is used for computing the BER for all the signal levels, which are then averaged to get average BER. The 1^{st}, 2^{nd}, 3^{rd}, ..., a^{th} amplifiers input power levels corresponding to bits b_1, b_2, b_3, ..., b_N in the N channels are given by

$$\begin{aligned}
P_{in_1(amp)}(b_i,\, b_{i+1},\, ...,\, b_N) &= P_{in_1}(b_N) + P_{in_1}(b_i) + P_{in_1}(b_{i+1}) + ... + P_{in_1}(b_{N-1}) \\
P_{in_2(amp)}(b_i,\, b_{i+1},\, ...,\, b_N) &= P_{in_2}(b_N) + P_{in_2}(b_j) + P_{in_2}(b_{j+1}) + ... + P_{in_2}(b_{N-1}) \\
P_{in_3(amp)}(b_i,\, b_{i+1},\, ...,\, b_N) &= P_{in_3}(b_N) + P_{in_3}(b_k) + P_{in_3}(b_{k+1}) + ... + P_{in_3}(b_{N-1}) \\
&\vdots \\
P_{in_a(amp)}(b_i,\, b_{i+1},\, ...,\, b_N) &= P_{in_a}(b_N) + P_{in_a}(b_t) + P_{in_a}(b_{t+1}) + ... + P_{in_a}(b_{N-1})
\end{aligned} \tag{3.23}$$

The signal power and ASE noise psd at the photodetector is given by

$$P_R(b_i,\, b_{i+1},\, ...,\, b_N) = P_{in_a(amp)}(b_i,\, b_{i+1},\, ...,\, b_N)G_a L_{A_tR,} \tag{3.24}$$

and

$$S_{sp_a}(b_i,\, b_{i+1},\, ...,\, b_N) = S_{sp_{a-1}}(b_i,\, b_{i+1},\, ...,\, b_N)G_a L_{A_sA_t} + n_{sp}\left[G_a - 1\right]h\nu. \tag{3.25}$$

The ASE noise psd at the receiver

$$S_{spr}(b_i,\, b_{i+1},\, ...,\, b_N) = S_{sp_a}(b_i,\, b_{i+1},\, ...,\, b_N)L_{A_tR} \tag{3.26}$$

The signal current and noise variance corresponding to bit b are

$$I_{sig}(b_i,\, b_{i+1},\, ...,\, b_N) = R_0 P_R(b_i,\, b_{i+1},\, ...,\, b_N) \tag{3.27}$$

and

$$\begin{aligned}
\sigma^2(b_i,\, b_{i+1},\, ...,\, b_N) &= 2eR_0\left[P_R(b_i,\, b_{i+1},\, ...,\, b_N) + S_{spr}(b_i,\, b_{i+1},\, ...,\, b_N)B_0\right]B_e \\
&\quad + 4R_0^2 P_R(b_i,\, b_{i+1},\, ...,\, b_N)S_{spr}(b_i,\, b_{i+1},\, ...,\, b_N)B_e \\
&\quad + R_0^2\left[S_{spr}(b_i,\, b_{i+1},\, ...,\, b_N)\right]^2\left[2B_0B_e - B_e^2\right] + \frac{4kTB_e}{R_L}
\end{aligned} \tag{3.28}$$

The threshold corresponding to highest level $I_{sig}(b_i,\, b_{i+1},\, ...,\, b_{N-1}, 0)$ and lowest level $I_{sig}(b_i,\, b_{i+1},\, ...,\, b_{N-1}, 1)$ will be

$$D_{th} = \frac{\sigma(all\, bits\, are\, '0')I_{sig}(all\, bits\, are\, '1') + \sigma(all\, bits\, are\, '1')I_{sig}(all\, bits\, are\, '0')}{\sigma(all\, bits\, are\, '0') + \sigma(all\, bits\, are\, '1')} \tag{3.29}$$

The probability of error for a particular bit pattern of interfering channels is

$$\begin{aligned} P_e(b_i,\, b_{i+1},\, ...,\, b_N) &= \frac{1}{2^{N-i-1}} \sum_{b_i,\, b_{i+1},\, ...,\, b_N} \left[\frac{1}{2} \left\{ \frac{1}{2} erfc \left(\frac{I_{sig}(b_i,\, ...,\, b_{N-1}, 1) - D_{th}}{\sqrt{2}\sigma(b_i,\, b_{i+1},\, ...,\, b_{N-1}, 1)} \right) \right. \right. \\ &\left. \left. + \frac{1}{2} erfc \left(\frac{D_{th} - I_{sig}(b_i,\, b_{i+1}, ...,\, b_{N-1}, 0)}{\sqrt{2}\sigma(b_i,\, b_{i+1},\, ...,\, b_{N-1}, 0)} \right) \right\} \right] \end{aligned} \tag{3.30}$$

3.3 Example

Consider a set of N wavelength corresponding to N number of branches ($N = 1,\, 2,\, 3, ...$). There are n number of user per branch ($n = 2^i$; where i is an integer) and data rate of each channel is $2.5\ Gbits/sec$. If the minimum bit rate required per user and active users at any time are assumed to be $100\ Mbits/sec$ and 75% respectively than maximum number of users per branch comes out to be 32. Hence the value of n is fixed in our analysis. Typical value of parameters used for computation in the system analysis is given in Table 3.2.

The total number of users after placing amplifier need to be computed. Since number of users per branch is already fixed to 32, the total number of users is determined by the number of supported branches .Hence only number of supported branches needs to be computed.

3.3.1 Without Amplifiers

Network without any amplifier can supported only 2 branches (64 users). Bit error rate at the user of each branch for different extinction ratios is given in Table 3.3.

3.3.2 Ideal Amplifiers

A network without amplifier can support only two branches, so there are three position to place first amplifier. Number of supported branches with respect to each position is given in table 3.4. Amplifier at position I can support maximum number of branches.

Maximum allowed transmitter power	P_T	$0\,dBm$
Desired bit error rate		10^{-9}
Insertion loss of single 2×2 coupler	L_i	$0.5\,dB$
Insertion loss of splice	L_w	$0.5\,dB$
Fiber attenuation coefficient	α	$0.2\,dB/Km$
Distance between branches	L	$5\,Km$
Insertion loss for dropping signal in OADM	L_{12}	$0.5\,dB$
Insertion loss for passing signal in OADM	L_m	$0.7\,dB$
Receiver load resistance	R_L	$100\,\Omega$
Operating wavelength	λ	$1.55\,\mu m$
Optical Bandwidth	B_o	$25\,GHz$
Electrical bandwidth	B_e	$2.5\,GHz$
Receiver temperature	T	300^oK
Electron charge	e	$1.602 \times 10^{-19}\,C$
Quantum efficiency of PD	η	1
Responsivity of photodiode	R_0	1.28
Number of user per branch	n	32

Table 3.2: Values of various parameters

Extinction ratio $\rightarrow$	Pe			
Number of branches$\downarrow$	0.00	0.05	0.10	0.15
1	1.037×10^{-39}	8.122×10^{-33}	3.856×10^{-27}	1.899×10^{-22}
2	3.920×10^{-19}	6.074×10^{-16}	2.596×10^{-13}	3.917×10^{-11}

Table 3.3: Bit error without optical amplifier

This is due to the fact that ASE produced by the amplifier at position I attenuated more than ASE produced by the amplifier at position II and III (see Fig. 3.3). This ASE limit the supported number of branches.

Extinction ratio → Pos. of amp ↓	Number of supported branches/Pe			
	0.00	0.05	0.10	0.15
I	$19/8.868 \times 10^{-11}$	$18/4.674 \times 10^{-18}$	$18/4.793 \times 10^{-15}$	$18/1.489 \times 10^{-12}$
II	$18/8.885 \times 10^{-21}$	$17/4.710 \times 10^{-18}$	$17/4.823 \times 10^{-15}$	$17/1.787 \times 10^{-12}$
III	$17/8.911 \times 10^{-11}$	$16/4.763 \times 10^{-18}$	$16/4.868 \times 10^{-15}$	$16/1.508 \times 10^{-12}$

Table 3.4: Number of supported branches after placing first ideal amplifier in subscriber access network

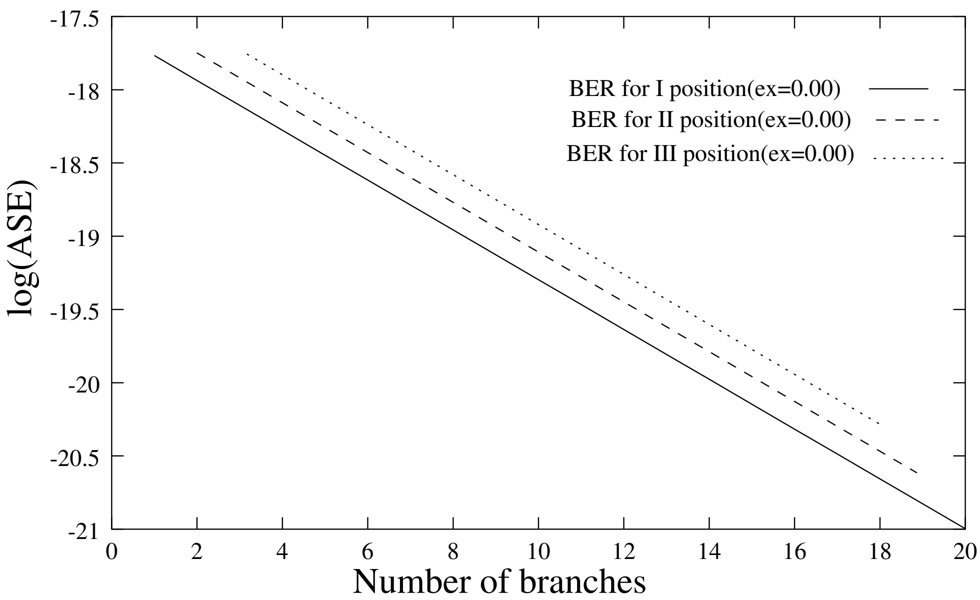

Figure 3.3: Graph between Number of branches and ASE per user after placing first amplifier at I position in subscriber access network

After placing the first amplifier at position I network supports 19 branches. The table ___ gives the probability of error in branches 17, 18 and 19 after the placement of first amplifier. There are 20 possibilities available to place the second amplifier but position I is already occupied by first amplifier, therefore, II amplifier can be placed at the remaining 19 positions. Number of branches supported by the network for remaining 19 positions is computed and shown in table 3.6.

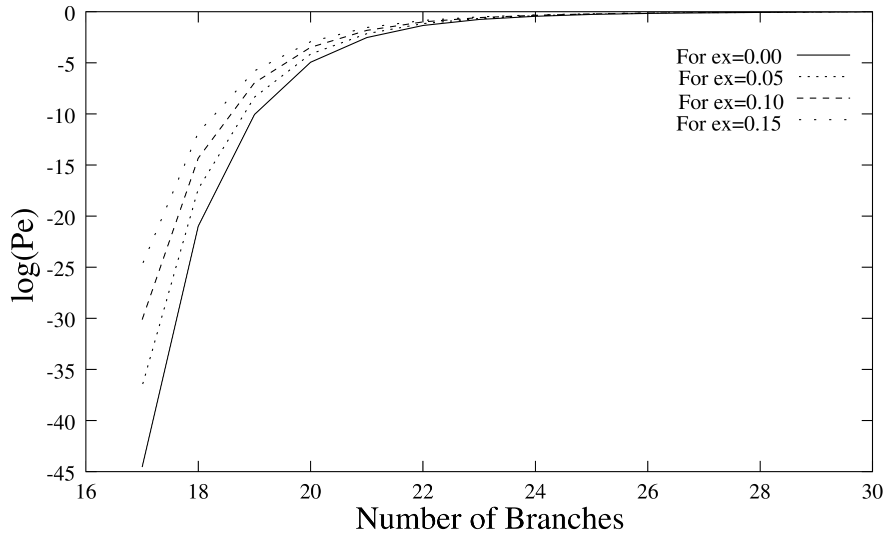

Figure 3.4: Variation of BER with No of branches after placing first amplifier at I position in subscriber access network

The number of supported branches increases initially with the second amplifier being placed farther and farther away from first amplifier up to position V afterward number of supported branches decreases. The initial increase in number of supported branches is attributed to the fact that ASE due to first amplifier is amplified by second amplifier and this accumulation of ASE is larger for II , III and IV positions. Thereafter the decrease in number of supported branches is understood in a way similar to the first amplifier case.

Similarly placement of third and fourth are computed. Supported number of branches and BER at the users of respective branch is given in tables 3.7 and 3.8. There is no substantial increase in supported number of branches for placement of fifth and sixth amplifier so these results are not given in detail (see Table 3.9). After sixth amplifier there is no increase in number of branches.

No. of branches	Pe			
	0.00	0.05	0.10	0.15
$1-16$ BER is very small				
17	2.803×10^{-45}	2.363×10^{-37}	7.439×10^{-31}	1.745×10^{-25}
18	1.041×10^{-21}	4.674×10^{-18}	4.793×10^{-15}	1.489×10^{-12}
19	8.868×10^{-11}	$\otimes$	$\otimes$	$\otimes$
20	$\otimes$	$\otimes$	$\otimes$	$\otimes$

Table 3.5: Bit error rate with one optical amplifier in subscriber access network

$\otimes \Rightarrow$ Shows Pe > 10^{-9}or amplifier can not be placed

Extinction ratio $\rightarrow$	No. of supported branches/Pe per user of respective branch			
Pos. of amp $\downarrow$	0.00	0.05	0.10	0.15
I	$22/5.947 \times 10^{-22}$	$22/3.846 \times 10^{-16}$	$22/5.846 \times 10^{-11}$	$21/2.474 \times 10^{-19}$
II	$26/4.863 \times 10^{-12}$	$25/1.457 \times 10^{-23}$	$25/3.745 \times 10^{-14}$	$25/8.588 \times 10^{-10}$
III	$27/3.826{\times}10^{-13}$	$26/6.326 \times 10^{-22}$	$26/4.642 \times 10^{-17}$	$26/6.735 \times 10^{-13}$
IV	$28/5.746 \times 10^{-12}$	$27/4.756 \times 10^{-15}$	$26/5.467 \times 10^{-21}$	$26/5.873 \times 10^{-17}$
V	$29/3.174 \times 10^{-11}$	$28/5.893{\times}10^{-14}$	$27/5.874 \times 10^{-18}$	$27/1.342 \times 10^{-13}$
VI	$28/4.523 \times 10^{-10}$	$27/3.625 \times 10^{-16}$	$27/4.744 \times 10^{-13}$	$26/4.736 \times 10^{-21}$
VII	$27/4.759 \times 10^{-21}$	$27/2.937 \times 10^{-12}$	$26/3.753 \times 10^{-23}$	$26/2.645 \times 10^{-16}$
$VIII$	$27/6.384 \times 10^{-16}$	$26/5.138 \times 10^{-26}$	$26/9.635 \times 10^{-20}$	$26/3.567 \times 10^{-13}$
IX	$27/3.363 \times 10^{-13}$	$26/3.489 \times 10^{-21}$	$26/4.735 \times 10^{-16}$	$26/7.377 \times 10^{-10}$
X	$27/5.659 \times 10^{-11}$	$26/3.714 \times 10^{-18}$	$26/5.836 \times 10^{-12}$	$25/6.639 \times 10^{-20}$
XI	$26/1.283 \times 10^{-23}$	$26/4.834 \times 10^{-13}$	$25/3.734 \times 10^{-22}$	$25/4.574 \times 10^{-15}$
XII	$26/2.748 \times 10^{-19}$	$25/8.914 \times 10^{-25}$	$25/7.567 \times 10^{-19}$	$25/5.835 \times 10^{-13}$
$XIII$	$26/1.476 \times 10^{-16}$	$25/6.923 \times 10^{-20}$	$25/3.866 \times 10^{-14}$	$24/5.473 \times 10^{-26}$
XIV	$26/3.837 \times 10^{-12}$	$25/4.736 \times 10^{-16}$	$25/5.955 \times 10^{-11}$	$24/1.473 \times 10^{-22}$
XV	$25/3.272 \times 10^{-20}$	$25/5.737 \times 10^{-11}$	$24/4.845 \times 10^{-23}$	$24/4.414 \times 10^{-17}$
XVI	$25/1.263 \times 10^{-17}$	$24/3.937 \times 10^{-19}$	$24/7.451 \times 10^{-16}$	$24/5.528 \times 10^{-11}$
$XVII$	$25/2.645 \times 10^{-13}$	$24/7.354 \times 10^{-14}$	$24/6.447 \times 10^{-09}$	$23/3.556 \times 10^{-18}$
$XVIII$	$24/4.634 \times 10^{-14}$	$23/5.876 \times 10^{-21}$	$23/3.980 \times 10^{-16}$	$23/4.975 \times 10^{-13}$
XIX	$23/5.827 \times 10^{-19}$	$23/8.271 \times 10^{-15}$	$23/6.974 \times 10^{-11}$	$22/4.987 \times 10^{-20}$
XX	$23/6.826 \times 10^{-13}$	$\otimes$	$\otimes$	$\otimes$

Table 3.6: Number of supported branches and BER after placing second ideal amplifier in subscriber access network

3.3.3 Amplifiers with Average Gain Saturation

Supported number of branches after placing first amplifier are given in table 3.10. The number of supported branches are less than ideal amplifier case due gain saturation (Since increased power reduces the gain and hence supported number of branches). But trend of decrease in number of supported branches with change in position is not expected since optical power falling at amplifier on II and III position is less thereby providing large gain (due to gain saturation). It seems that number of supported branches shall be more in II and III case. But as result shows that effect of ASE overpower the effect of gain saturation.

Extinction ratio →	No. of supported branches/Pe per user of respective branch			
Pos. of amp ↓	0.00	0.05	0.10	0.15
II	$31/6.635 \times 10^{-12}$	$30/4.645 \times 10^{-19}$	$30/3.853 \times 10^{-15}$	$30/5.028 \times 10^{-10}$
III	$32/7.534 \times 10^{-19}$	$32/5.423 \times 10^{-16}$	$32/5.136 \times 10^{-11}$	$31/2.462 \times 10^{-22}$
IV	$32/6.746 \times 10^{-23}$	$32/5.160 \times 10^{-19}$	$32/9.476 \times 10^{-14}$	$31/9.236 \times 10^{-25}$
VI	$33/3.645 \times 10^{-14}$	$32/3.064 \times 10^{-22}$	$32/5.972 \times 10^{-17}$	$32/4.723 \times 10^{-12}$
VII	$33/4.947 \times 10^{-20}$	$33/1.594 \times 10^{-17}$	$33/3.346 \times 10^{-11}$	$32/8.325 \times 10^{-22}$
$VIII$	$34/4.653 \times 10^{-13}$	$34/8.945 \times 10^{-09}$	$33/4.746 \times 10^{-23}$	$33/5.126 \times 10^{-17}$
IX	$34/1.748 \times 10^{-17}$	$34/2.672 \times 10^{-12}$	$33/5.038 \times 10^{-19}$	$33/7.434 \times 10^{-14}$
X	$34/3.864 \times 10^{-11}$	$33/3.913 \times 10^{-23}$	$33/6.934 \times 10^{-15}$	$33/6.74 \times 10^{-10}$
XI	$33/1.423 \times 10^{-25}$	$33/4.915 \times 10^{-19}$	$33/3.847 \times 10^{-12}$	$32/9.117 \times 10^{-23}$
XII	$33/3.634 \times 10^{-21}$	$33/1.274 \times 10^{-15}$	$32/8.341 \times 10^{-24}$	$32/3.723 \times 10^{-18}$
$XIII$	$33/3.846 \times 10^{-16}$	$33/3.827 \times 10^{-12}$	$32/4.093 \times 10^{-20}$	$32/5.933 \times 10^{-16}$
XIV	$33/4.623 \times 10^{-11}$	$32/7.297 \times 10^{-24}$	$32/3.825 \times 10^{-16}$	$32/4.196 \times 10^{-11}$
XV	$32/5.463 \times 10^{-23}$	$32/4.825 \times 10^{-18}$	$32/3.620 \times 10^{-12}$	$31/4.038 \times 10^{-19}$
XVI	$32/6.422 \times 10^{-18}$	$32/7.354 \times 10^{-14}$	$31/3.074 \times 10^{-21}$	$31/4.143 \times 10^{-17}$
$XVII$	$32/5.634 \times 10^{-12}$	$31/1.635 \times 10^{-21}$	$31/3.346 \times 10^{-17}$	$31/3.048 \times 10^{-13}$
$XVIII$	$31/3.756 \times 10^{-19}$	$31/3.648 \times 10^{-14}$	$31/2.732 \times 10^{-09}$	$30/4.573 \times 10^{-21}$
XIX	$31/1.264 \times 10^{-14}$	$30/2.825 \times 10^{-23}$	$30/6.230 \times 10^{-18}$	$30/6.937 \times 10^{-15}$
XX	$31/5.493 \times 10^{-10}$	$30/2.940 \times 10^{-19}$	$30/3.542 \times 10^{-15}$	$30/8.034 \times 10^{-11}$
XXI	$30/6.532 \times 10^{-21}$	$30/7.215 \times 10^{-15}$	$30/1.328 \times 10^{-10}$	$29/2.564 \times 10^{-23}$
$XXII$	$30/5.534 \times 10^{-17}$	$30/3.138 \times 10^{-13}$	$29/3.623 \times 10^{-20}$	$29/5.037 \times 10^{-17}$
$XXIII$	$30/3.645 \times 10^{-12}$	$29/2.846 \times 10^{-21}$	$29/8.746 \times 10^{-16}$	$29/6.037 \times 10^{-12}$
$XXIV$	$29/4.736 \times 10^{-20}$	$29/4.923 \times 10^{-17}$	$29/4.129 \times 10^{-13}$	$28/4.623 \times 10^{-20}$
XXV	$29/4.645 \times 10^{-16}$	$29/5.646 \times 10^{-11}$	$28/3.738 \times 10^{-18}$	$28/4.139 \times 10^{-13}$
$XXVI$	$29/5.463 \times 10^{-13}$	$28/3.846 \times 10^{-20}$	$28/2.453 \times 10^{-15}$	$27/5.278 \times 10^{-21}$
$XXVII$	$29/3.485 \times 10^{-09}$	$28/6.893 \times 10^{-14}$	$27/3.349 \times 10^{-26}$	$27/6.486 \times 10^{-17}$
$XXVIII$	$28/2.823 \times 10^{-18}$	$28/7.923 \times 10^{-12}$	$27/2.984 \times 10^{-24}$	$27/3.732 \times 10^{-19}$
$XXIX$	$29/4.534 \times 10^{-19}$	$28/5.893 \times 10^{-14}$	$\otimes$	$\otimes$
XXX	$29/3.174 \times 10^{-11}$	$\otimes$	$\otimes$	$\otimes$

Table 3.7: Number of branches supported and BER after placing third ideal amplifier in subscriber access network

Extinction ratio →	No. of supported branches/Pe per user of respective branch			
Pos. of amp ↓	0.00	0.05	0.10	0.15
II	$33/1.190 \times 10^{-14}$	$33/3.917 \times 10^{-12}$	$32/3.137 \times 10^{-25}$	$32/7.901 \times 10^{-18}$
III	$33/1.684 \times 10^{-20}$	$33/2.888 \times 10^{-17}$	$33/6.216 \times 10^{-11}$	$32/7.395 \times 10^{-23}$
IV	$33/7.047 \times 10^{-27}$	$33/9.552 \times 10^{-21}$	$33/5.194 \times 10^{-17}$	$33/6.270 \times 10^{-14}$
VI	$34/6.020 \times 10^{-15}$	$34/7.318 \times 10^{-12}$	$33/3.801 \times 10^{-27}$	$33/7.381 \times 10^{-21}$
VII	$34/4.847 \times 10^{-17}$	$34/6.823 \times 10^{-12}$	$33/9.324 \times 10^{-28}$	$33/3.845 \times 10^{-22}$
$VIII$	$34/5.152 \times 10^{-18}$	$34/5.136 \times 10^{-11}$	$33/4.473 \times 10^{-29}$	$33/4.861 \times 10^{-23}$
X	$34/2.916 \times 10^{-23}$	$34/2.167 \times 10^{-18}$	$34/2.944 \times 10^{-12}$	$33/6.930 \times 10^{-20}$
XI	$34/3.952 \times 10^{-29}$	$34/9.926 \times 10^{-23}$	$34/4.671 \times 10^{-18}$	$34/6.621 \times 10^{-14}$
XII	$35/3.311 \times 10^{-15}$	$35/8.741 \times 10^{-10}$	$34/7.011 \times 10^{-24}$	$34/3.802 \times 10^{-18}$
$XIII$	$35/5.515 \times 10^{-22}$	$35/4.896 \times 10^{-17}$	$35/8.664 \times 10^{-11}$	$34/6.845 \times 10^{-16}$
XIV	$36/6.482 \times 10^{-18}$	$36/5.871 \times 10^{-14}$	$35/6.105 \times 10^{-21}$	$35/8.892 \times 10^{-17}$
XV	$35/7.971 \times 10^{-24}$	$35/3.586 \times 10^{-19}$	$35/5.955 \times 10^{-16}$	$35/2.508 \times 10^{-12}$
XVI	$35/4.055 \times 10^{-19}$	$35/6.061 \times 10^{-14}$	$34/3.481 \times 10^{-28}$	$34/6.652 \times 10^{-21}$
$XVII$	$35/9.370 \times 10^{-14}$	$35/6.968 \times 10^{-10}$	$34/2.736 \times 10^{-20}$	$34/9.793 \times 10^{-17}$
$XVIII$	$35/3.240 \times 10^{-09}$	$34/5.220 \times 10^{-23}$	$34/1.863 \times 10^{-17}$	$34/1.924 \times 10^{-11}$
XIX	$34/5.791 \times 10^{-28}$	$34/3.356 \times 10^{-21}$	$34/7.803 \times 10^{-14}$	$33/9.633 \times 10^{-29}$
XX	$34/3.090 \times 10^{-21}$	$34/7.909 \times 10^{-17}$	$34/6.847 \times 10^{-12}$	$33/5.026 \times 10^{-22}$
XXI	$34/4.339 \times 10^{-16}$	$34/2.431 \times 10^{-11}$	$33/6.718 \times 10^{-22}$	$33/9.839 \times 10^{-16}$
$XXII$	$34/8.744 \times 10^{-13}$	$33/6.109 \times 10^{-24}$	$33/8.121 \times 10^{-17}$	$33/2.741 \times 10^{-13}$
$XIII$	$34/5.265 \times 10^{-09}$	$33/1.952 \times 10^{-19}$	$33/7.428 \times 10^{-15}$	$32/4.899 \times 10^{-26}$
XIV	$33/1.734 \times 10^{-23}$	$33/5.956 \times 10^{-17}$	$33/2.169 \times 10^{-12}$	$32/4.002 \times 10^{-21}$
XV	$33/3.196 \times 10^{-20}$	$33/2.593 \times 10^{-14}$	$32/6.715 \times 10^{-28}$	$32/2.619 \times 10^{-19}$
XVI	$33/6.923 \times 10^{-15}$	$33/2.208 \times 10^{-10}$	$32/7.401 \times 10^{-21}$	$32/5.863 \times 10^{-15}$
$XXVII$	$33/3.361 \times 10^{-11}$	$32/7.544 \times 10^{-26}$	$32/8.913 \times 10^{-20}$	$32/8.743 \times 10^{-14}$
$XXVIII$	$32/4.216 \times 10^{-29}$	$32/4.691 \times 10^{-22}$	$32/3.308 \times 10^{-17}$	$32/6.477 \times 10^{-09}$
$XXIX$	$32/3.189 \times 10^{-24}$	$32/3.948 \times 10^{-19}$	$32/5.399 \times 10^{-12}$	$31/4.041 \times 10^{-24}$
XXX	$32/5.316 \times 10^{-18}$	$32/2.406 \times 10^{-14}$	$31/8.316 \times 10^{-22}$	$31/5.641 \times 10^{-18}$
$XXXI$	$32/1.397 \times 10^{-14}$	$32/9.038 \times 10^{-11}$	$31/8.862 \times 10^{-17}$	$31/9.563 \times 10^{-11}$
$XXXII$	$32/2.960 \times 10^{-10}$	$31/6.124 \times 10^{-23}$	$31/5.870 \times 10^{-20}$	$31/6.202 \times 10^{-17}$
$XXXIII$	$33/4.189 \times 10^{-18}$	$32/3.883 \times 10^{-17}$	$32/1.683 \times 10^{-14}$	$32/2.128 \times 10^{-12}$
$XXXIV$	$33/3.269 \times 10^{-17}$	$33/3.913 \times 10^{-15}$	$33/2.414 \times 10^{-12}$	$33/1.698 \times 10^{-09}$
$XXXV$	$34/3.178 \times 10^{-11}$	$\otimes$	$\otimes$	$\otimes$

Table 3.8: Number of branches supported and BER after placing forth ideal amplifier in subscriber access network

No. of amplifier	Supported branches after placing amplifier at optimum position
5^{th}	37
6^{th}	38
7^{th}	38

Table 3.9: Number of branches supported with number of amplifier

Number of supported branches after placing second, third and fourth amplifiers is given in table 3.11, table 3.12 and table 3.13 respectively and optimum position is shown in bold letters. Effect of ASE can be clearly seen in table 3.13 as there is no increase in number of supported branches after placing amplifier at XV position. Here signal power is very less as compared to ASE.

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
I	$8/4.973 \times 10^{-24}$	$8/9.179 \times 10^{-10}$	$7/5.992 \times 10^{-19}$	$7/3.607 \times 10^{-15}$
II	$6/2.067 \times 10^{-22}$	$5/8.721 \times 10^{-21}$	$5/2.156 \times 10^{-15}$	$5/4.482 \times 10^{-12}$
III	$5/1.271 \times 10^{-29}$	$4/1.325 \times 10^{-27}$	$4/5.743 \times 10^{-19}$	$4/2.104 \times 10^{-14}$

Table 3.10: Number of branches supported and BER per user after placing first amplifier(with saturation) in subscriber access network

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
II	$11/4.892 \times 10^{-22}$	$11/4.347 \times 10^{-18}$	$11/9.354 \times 10^{-14}$	$10/3.607 \times 10^{-25}$
III	$12/5.924 \times 10^{-23}$	$12/8.721 \times 10^{-17}$	$12/2.156 \times 10^{-13}$	$11/4.482 \times 10^{-21}$
IV	$12/8.304 \times 10^{-17}$	$12/1.325 \times 10^{-12}$	$11/5.743 \times 10^{-19}$	$11/5.237 \times 10^{-14}$
V	$12/7.893 \times 10^{-11}$	$11/4.542 \times 10^{-25}$	$11/6.253 \times 10^{-15}$	$11/7.350 \times 10^{-09}$
VI	$11/6.795 \times 10^{-27}$	$11/6.937 \times 10^{-21}$	$11/4.892 \times 10^{-14}$	$10/3.653 \times 10^{-23}$
VII	$11/5.128 \times 10^{-21}$	$11/7.924 \times 10^{-15}$	$10/9.512 \times 10^{-22}$	$10/6.649 \times 10^{-16}$
$VIII$	$11/7.297 \times 10^{-17}$	$11/8.354 \times 10^{-12}$	$10/4.394 \times 10^{-17}$	$10/1.937 \times 10^{-12}$
IX	$11/7.297 \times 10^{-11}$	$10/3.507 \times 10^{-21}$	$\otimes$	$\otimes$

Table 3.11: Number of branches supported and BER per user after placing second amplifier(with saturation) in subscriber access network

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
II	$12/8.937 \times 10^{-17}$	$12/4.795 \times 10^{-14}$	$11/6.217 \times 10^{-24}$	$11/4.286 \times 10^{-18}$
IV	$13/7.823 \times 10^{-15}$	$13/8.237 \times 10^{-10}$	$12/7.150 \times 10^{-18}$	$12/9.326 \times 10^{-11}$
V	$14/9.467 \times 10^{-14}$	$14/7.253 \times 10^{-09}$	$13/8.634 \times 10^{-20}$	$13/2.478 \times 10^{-15}$
VI	$13/7.249 \times 10^{-25}$	$13/3.147 \times 10^{-19}$	$13/9.645 \times 10^{-14}$	$12/6.345 \times 10^{-23}$
VII	$13/5.464 \times 10^{-20}$	$13/5.127 \times 10^{-14}$	$12/9.234 \times 10^{-26}$	$12/5.471 \times 10^{-18}$
$VIII$	$13/6.463 \times 10^{-16}$	$13/2.479 \times 10^{-10}$	$12/8.446 \times 10^{-19}$	$12/6.834 \times 10^{-13}$
IX	$13/3.284 \times 10^{-10}$	$12/4.446 \times 10^{-22}$	$12/1.526 \times 10^{-16}$	$11/8.464 \times 10^{-24}$
X	$12/5.743 \times 10^{-19}$	$12/8.463 \times 10^{-15}$	$12/7.209 \times 10^{-11}$	$11/8.462 \times 10^{-19}$
XI	$12/5.643 \times 10^{-12}$	$11/6.274 \times 10^{-23}$	$11/7.233 \times 10^{-17}$	$11/3.147 \times 10^{-12}$
XII	$12/7.423 \times 10^{-16}$	$12/5.934 \times 10^{-14}$	$12/7.028 \times 10^{-10}$	$11/4.482 \times 10^{-21}$
$XIII$	$12/5.924 \times 10^{-23}$	$12/8.721 \times 10^{-17}$	$12/8.721 \times 10^{-17}$	$\otimes$

Table 3.12: Number of branches supported and BER per user after placing third amplifier(with saturation) in subscriber access network

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
II	$13/3.271 \times 10^{-17}$	$13/9.762 \times 10^{-12}$	$12/8.986 \times 10^{-22}$	$12/5.793 \times 10^{-17}$
IV	$14/8.773 \times 10^{-10}$	$13/3.783 \times 10^{-17}$	$13/2.828 \times 10^{-11}$	$12/9.426 \times 10^{-23}$
VI	$14/9.642 \times 10^{-15}$	$13/7.932 \times 10^{-21}$	$13/5.454 \times 10^{-18}$	$13/4.719 \times 10^{-12}$
VII	$14/2.881 \times 10^{-22}$	$13/1.503 \times 10^{-28}$	$13/4.171 \times 10^{-23}$	$13/6.213 \times 10^{-19}$
$VIII$	$14/6.884 \times 10^{-27}$	$14/7.675 \times 10^{-21}$	$14/7.270 \times 10^{-15}$	$14/4.883 \times 10^{-10}$
IX	$14/5.147 \times 10^{-29}$	$14/4.793 \times 10^{-15}$	$14/9.346 \times 10^{-10}$	$13/9.744 \times 10^{-28}$
X	$14/2.416 \times 10^{-16}$	$14/7.974 \times 10^{-11}$	$13/8.742 \times 10^{-23}$	$13/3.719 \times 10^{-20}$
XI	$14/8.533 \times 10^{-12}$	$13/9.145 \times 10^{-21}$	$13/1.349 \times 10^{-16}$	$13/8.858 \times 10^{-14}$
XII	$13/3.675 \times 10^{-23}$	$13/7.343 \times 10^{-16}$	$13/5.154 \times 10^{-12}$	$12/4.752 \times 10^{-09}$
$XIII$	$13/4.148 \times 10^{-17}$	$13/4.357 \times 10^{-14}$	$13/3.304 \times 10^{-12}$	$13/1.602 \times 10^{-09}$
XIV	$13/9.025 \times 10^{-24}$	$13/2.657 \times 10^{-20}$	$13/8.634 \times 10^{-20}$	$13/2.478 \times 10^{-18}$
XV	$14/9.467 \times 10^{-14}$	$14/7.253 \times 10^{-09}$	$\otimes$	$\otimes$

Table 3.13: Number of branches supported and BER per user after placing forth amplifier(with saturation) in subscriber access network

3.3.4 AMPLIFIERS WITH AVERAGE GAIN SATURATION AND GAIN FLUCTUATION

Computed results of amplifier placement at various positions of amplifiers with average gain saturation and gain fluctuation are shown in tables 3.14, 3.15, 3.16 and 3.17. Here the gain crosstalk substantially affect the number of supported branches. There is no increase in number of supported branches after third amplifier.

Extinction ratio →	No. of supported branches/Pe per user of respective branch			
Pos. of amp ↓	0.00	0.05	0.10	0.15
I	$7/1.465 \times 10^{-10}$	$6/1.036 \times 10^{-19}$	$6/4.527 \times 10^{-14}$	$6/4.651 \times 10^{-11}$
II	$6/5.876 \times 10^{-16}$	$6/6.475 \times 10^{-12}$	$5/1.074 \times 10^{-15}$	$5/1.374 \times 10^{-10}$
III	$6/5.350 \times 10^{-14}$	$5/1.330 \times 10^{-15}$	$4/2.976 \times 10^{-23}$	$4/3.463 \times 10^{-12}$

Table 3.14: Number of branches supported and BER per user after placing first amplifier with average gain saturation and gain fluctuation in subscriber access network

Extinction ratio →	No. of supported branches/Pe per user of respective branch			
Pos. of amp ↓	0.00	0.05	0.10	0.15
II	$8/9.980 \times 10^{-18}$	$7/1.523 \times 10^{-21}$	$7/1.037 \times 10^{-15}$	$7/3.796 \times 10^{-11}$
III	$9/1.957 \times 10^{-23}$	$8/5.229 \times 10^{-28}$	$8/4.806 \times 10^{-22}$	$8/6.277 \times 10^{-17}$
IV	$10/1.536 \times 10^{-19}$	$9/5.428 \times 10^{-32}$	$9/5.369 \times 10^{-27}$	$9/9.047 \times 10^{-23}$
V	$9/5.436 \times 10^{-17}$	$8/7.817 \times 10^{-24}$	$8/4.958 \times 10^{-19}$	$8/2.013 \times 10^{-14}$
VI	$9/6.932 \times 10^{-14}$	$8/1.036 \times 10^{-19}$	$8/4.527 \times 10^{-14}$	$8/4.651 \times 10^{-11}$
VII	$8/8.163 \times 10^{-16}$	$7/8.834 \times 10^{-19}$	$7/8.644 \times 10^{-15}$	$6/4.651 \times 10^{-11}$
$VIII$	$8/1.465 \times 10^{-10}$	$\otimes$	$\otimes$	$\otimes$

Table 3.15: Number of branches supported and BER per user after placing second amplifier with average gain saturation and gain fluctuation in subscriber access network

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
II	$9/1.417 \times 10^{-21}$	$9/6.017 \times 10^{-18}$	$9/4.426 \times 10^{-14}$	$9/5.032 \times 10^{-09}$
III	$9/2.960 \times 10^{-28}$	$9/9.465 \times 10^{-22}$	$9/9.553 \times 10^{-17}$	$9/3.102 \times 10^{-13}$
V	$10/5.229 \times 10^{-34}$	$10/5.327 \times 10^{-23}$	$10/5.514 \times 10^{-19}$	$10/1.772 \times 10^{-14}$
VI	$11/4.351 \times 10^{-19}$	$10/5.428 \times 10^{-32}$	$10/5.369 \times 10^{-27}$	$10/9.047 \times 10^{-23}$
VII	$12/5.775 \times 10^{-15}$	$11/5.875 \times 10^{-27}$	$11/8.634 \times 10^{-20}$	$11/8.643 \times 10^{-16}$
$VIII$	$12/8.421 \times 10^{-11}$	$11/4.764 \times 10^{-21}$	$11/2.645 \times 10^{-14}$	$10/5.643 \times 10^{-22}$
IX	$11/1.536 \times 10^{-19}$	$10/9.634 \times 10^{-25}$	$10/9.723 \times 10^{-10}$	$9/6.841 \times 10^{-21}$
X	$10/6.834 \times 10^{-16}$	$9/5.428 \times 10^{-32}$	$9/5.369 \times 10^{-27}$	$9/9.047 \times 10^{-23}$
XI	$10/1.536 \times 10^{-19}$	$\otimes$	$\otimes$	$\otimes$

Table 3.16: Number of branches supported and BER per user after placing third amplifier with average gain saturation and gain fluctuation in subscriber access network

Extinction ratio → Pos. of amp ↓	No. of supported branches/Pe per user of respective branch			
	0.00	0.05	0.10	0.15
II	$10/6.826 \times 10^{-12}$	$9/6.328 \times 10^{-22}$	$9/3.945 \times 10^{-17}$	$9/5.344 \times 10^{-13}$
III	$10/4.847 \times 10^{-16}$	$10/4.348 \times 10^{-12}$	$9/4.786 \times 10^{-24}$	$9/6.345 \times 10^{-19}$
V	$10/5.783 \times 10^{-20}$	$10/3.193 \times 10^{-14}$	$9/2.975 \times 10^{-31}$	$9/2.455 \times 10^{-24}$
VI	$11/3.836 \times 10^{-24}$	$11/5.927 \times 10^{-17}$	$10/8.236 \times 10^{-23}$	$10/8.236 \times 10^{-18}$
$VIII$	$12/7.283 \times 10^{-16}$	$12/6.249 \times 10^{-11}$	$11/4.293 \times 10^{-20}$	$11/4.947 \times 10^{-15}$
IX	$12/5.238 \times 10^{-24}$	$12/4.732 \times 10^{-18}$	$12/1.734 \times 10^{-12}$	$11/5.748 \times 10^{-22}$
X	$12/4.383 \times 10^{-20}$	$12/1.745 \times 10^{-15}$	$11/7.462 \times 10^{-10}$	$10/4.129 \times 10^{-26}$
XI	$12/7.294 \times 10^{-14}$	$11/4.012 \times 10^{-26}$	$11/3.823 \times 10^{-18}$	$10/3.740 \times 10^{-22}$
XII	$11/5.345 \times 10^{-23}$	$11/5.875 \times 10^{-27}$	$11/8.634 \times 10^{-20}$	$11/8.643 \times 10^{-16}$
$XIII$	$12/4.352 \times 10^{-19}$	$\otimes$	$\otimes$	$\otimes$

Table 3.17: Number of branches supported and BER per user after placing forth amplifier with average gain saturation and gain fluctuation in subscriber access network

Chapter 4

AMPLIFIERS IN STAR TOPOLOGY

Star topology supports maximum number of users as compared to other topologies viz. bus , ring and tree. So placement of amplifier is important in it. There are only two places where amplifiers can be put; a) after the transmitter and b) before the receiver. The effect of placing the amplifiers at both these position on number of users is already been studied [2]. In this chapter, the effect of placing the amplifier at both position simultaneously is investigated. Further, amplifiers placements in multi-star topology (Fig.4.1) and many combinations of multi-stars are tried. Only the results of optimized placement (where the number of users are maximum) are shown in this chapter.

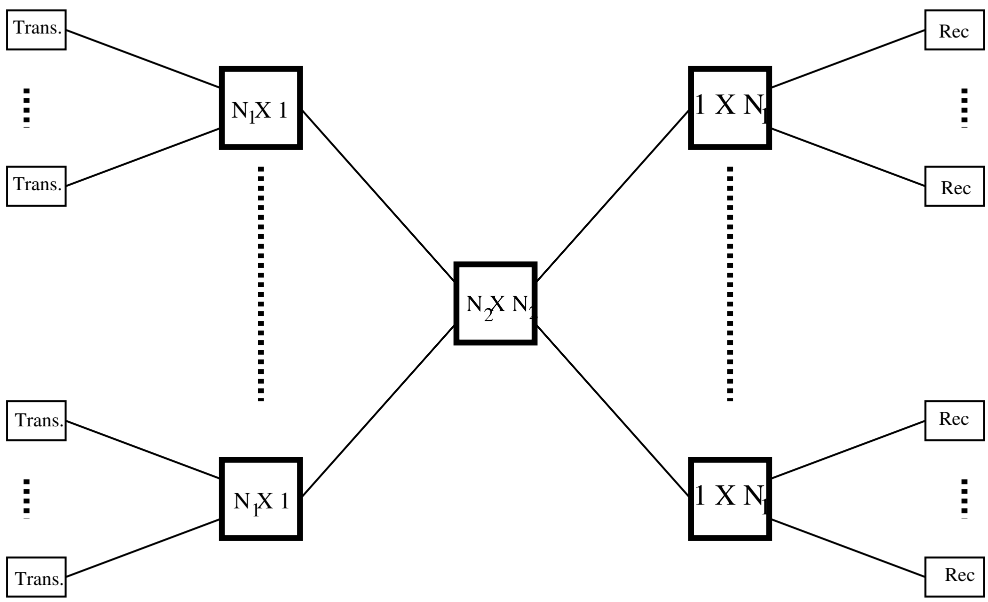

Figure 4.1: Multi-star Topology

4.1 SYSTEM DESCRIPTION

In multi-star topology, there are multiple stages of star couplers (SCs). First and last SC is connected with transmitter and receiver respectively of all the users. Every user is provided a wavelength in $1.5\,\mu m$ band. Receivers use tunable optical filters to receive the transmission from desired transmitter.

All the star couplers are made by using $3\,dB$ 2×2 couplers. Generally $N \times N$, $1 \times N$ and $N \times 1$ star couplers will have $\log_2 N$ stages and $\frac{N}{2} \log_2 N$, 2×2 couplers. The insertion loss L_I for $N \times N$, $1 \times N$ and $N \times 1$ star couplers is given by

$$L_I = L_i \log_2 N.$$

4.2 EVALUATION OF MAXIMUM NUMBER OF USER

Evaluation of maximum number of users is done exactly in the similar way as done for the subscriber access network in the chapter 3. The difference with the formulation of last chapter lies in the losses suffers by the signal and same number of wavelengths for all the amplifiers. This happens due to change in topology and equal division of signal power by star coupler.

4.2.1 SYSTEM LOSS MODEL

The losses between transmitter and receiver in multi-star topology without any amplifier is

$$\begin{aligned} L_{TR}(dB) &= (S+1)\alpha L + iSL_{cv} + (L_{I_1} + L_{I_2} + ... + L_{I_S}) + 2(S+1)L_{sp} + L_f \\ &\quad +10\log_{10}(N_1) + 10\log_{10}(N_2) + ... + 10\log_{10}(N_S) \end{aligned} \tag{4.1}$$

where L_{cv} represents the coupler variability and S represents the number of stages in the multi-star topology. The amplifiers are placed in the topology as shown in 4.2. The losses between the transmitter and first amplifier which is placed before i^{th} stage SC can be given by

$$\begin{aligned} L_{TA_i}(dB) &= \alpha i L + (i-1)L_{cv} + \left(L_{I_1} + L_{I_2} .... + L_{I_{i-1}}\right) + 2(i-1)L_{sp} \\ &\quad +10\log_{10}(N_1) + 10\log_{10}(N_2) + ... + 10\log_{10}(N_{i-1}) \end{aligned} \tag{4.2}$$

and the losses between two consecutive amplifiers placed before j^{th} and k^{th} stage SCs respectively is given by

$$\begin{aligned} L_{A_jA_k}(dB) &= 2(k-j)\alpha L + (k-j)L_{cv} + \left(L_{I_j} + L_{I_{j+1}} + ... + L_{I_{k-1}}\right) + 2(k-j)L_{sp} \\ &\quad +10\log_{10}(N_j) + 10\log_{10}(N_{j+1}) + ... + 10\log_{10}(N_{k-1}) \end{aligned} \tag{4.3}$$

Losses between receiver and nearest amplifier is given by placed before t^{th} stage SC having total S stages is given by

$$\begin{aligned} L_{A_tR}(dB) &= (S-t+1)\alpha L + L_f + (N-t)L_{cv} + \left(L_{I_t} + L_{I_{t+1}} + ... + L_{I_s}\right) + 2(S-t+1)L_{sp} \\ &\quad +10\log_{10}(N_t) + 10\log_{10}(N_{t+1}) + ... + 10\log_{10}(N_S) \end{aligned} \tag{4.4}$$

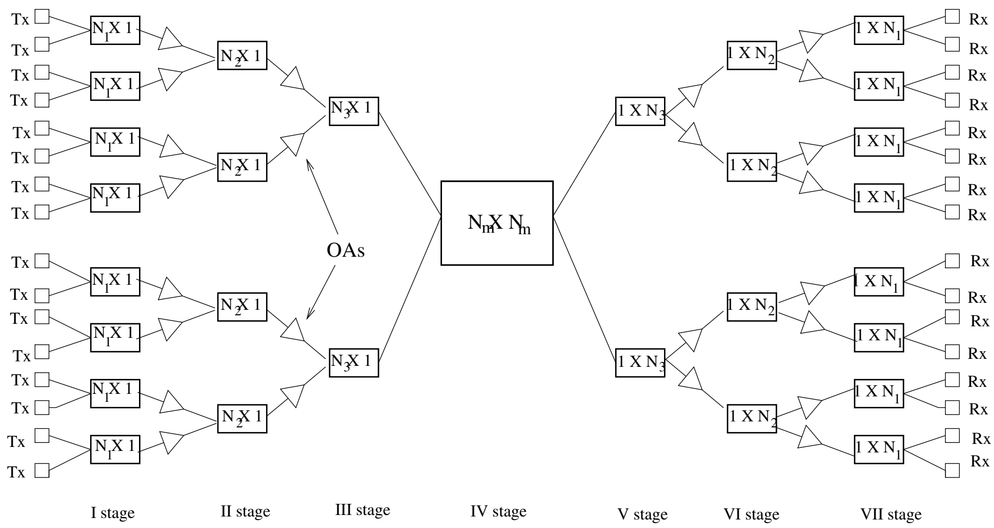

Figure 4.2: Multi-star topology with Optical amplifiers

4.2.2 TOPOLOGY WITH AMPLIFIERS

Amplifier is placed in topology as shown in Fig 4.2. There are total s stages in the topology and there is a $N_m \times N_m$ coupler in the center of the topology. There are

a amplifiers between each transmitter and central coupler and also in between central coupler and each receiver. Therefore, there will be $2a$ amplifiers in between each transmitter and receiver. First a amplifiers before coupler and last a amplifiers after central couplers are placed to form mirror image symmetry w.r.t central coupler (Fig.4.2). In part of topology that is before main coupler, 1^{st} amplifier is placed before i^{th} stage SC and 2^{nd} amplifier is placed before j^{th} SC and so on. Last amplifier is placed before t^{th} stage SC. Similarly, amplifiers are placed after the main coupler in such a way that symmetry is maintained. ASE at the output of amplifiers will be given by

$$
\begin{aligned}
S_{sp_1} &= n_{sp}(G_1 - 1)h\nu \\
S_{sp_2} &= S_{sp_1} L_{A_i A_j} G_2 N_i N_{i+1} \ldots N_{j-1} + n_{sp}(G_2 - 1)h\nu \\
S_{sp_3} &= S_{sp_2} L_{A_j A_k} G_3 N_j N_{j+1} \ldots N_{k-1} + n_{sp}(G_2 - 1)h\nu \\
&\vdots \\
S_{sp_a} &= S_{sp_a} L_{A_s A_t} G_a N_s N_{s+1} \ldots N_{t-1} + n_{sp}(G_a - 1)h\nu \\
S_{sp_{a+1}} &= S_{sp_a} G_{a+1} L_{A_t A_{m+i}} N_t N_{t+1} \ldots N_{\frac{S-1}{2}} N_m + n_{sp}(G_{a+1} - 1)h\nu \\
S_{sp_{a+2}} &= S_{sp_{a+1}} L_{A_{m+i} A_{m+j}} G_{a+2} + n_{sp}(G_{a+2} - 1)h\nu \\
S_{sp_{a+3}} &= S_{sp_{a+2}} L_{A_{m+j} A_{m+k}} G_{a+3} + n_{sp}(G_{a+3} - 1)h\nu \\
&\vdots \\
S_{sp_{2a}} &= S_{sp_{2a-1}} L_{A_{m+s} A_{m+t}} G_{2a} + n_{sp}(G_{2a} - 1)h\nu
\end{aligned}
\tag{4.5}
$$

here G_1 G_2 .... G_a are the gains of amplifiers. The input to each amplifier is given by

$$
\begin{aligned}
P_{in_1}(b) &= P_s(b)L_{TA_i} \\
P_{in_2}(b) &= P_{in_1}(b)G_1L_{A_iA_j} + S_{sp_1}L_{A_iA_j} \\
P_{in_3}(b) &= P_{in_2}(b)G_2L_{A_jA_k} + n_{sp}(G_2-1)h\nu L_{A_jA_k} \\
&\vdots \\
P_{in_a}(b) &= P_{in_{a-1}}(b)G_{a-1}L_{A_sA_t} + n_{sp}(G_{a-1}-1)h\nu L_{A_sA_t} \\
P_{in_{a+1}}(b) &= P_{in_a}(b)L_{A_tA_{m+i}}G_a + n_{sp}(G_{a-1}-1)h\nu L_{A_tA_{m+i}} \\
P_{in_{a+2}}(b) &= P_{in_{a+1}}(b)G_{a+1}L_{A_{m+i}A_{m+j}} + n_{sp}(G_{a-1}-1)h\nu L_{A_{m+i}A_{m+j}} \\
P_{in_{a+3}}(b) &= P_{in_{a+2}}(b)G_2L_{A_{m+j}A_{m+k}} + n_{sp}(G_{a+2}-1)h\nu L_{A_{m+j}A_{m+k}} \\
&\vdots \\
P_{in_{2a}}(b) &= P_{in_{2a-1}}(b)G_{2a-1}L_{A_{m+s}A_{m+t}} + n_{sp}(G_{2a-1}-1)h\nu L_{A_{m+s}A_{m+t}}. \quad (4.6)
\end{aligned}
$$

Evaluation of number of user can be done by the methods described in sections 3.2.2, 3.2.3 for ideal OA and OA with gain saturation using equations 4.6,4.5for input power to amplifier and ASE respectively. There is a little difference between the method described in 3.2.4 for OA with gain fluctuation. Since there is no drop of any signal in between the network so pattern of bits of channels does not matter and analysis become simple as described bellow.

The probability of N_1 channels out of $N-1$ interfering having bit '1' is given by

$$P_{N_1} = \begin{pmatrix} N-1 \\ N_1 \end{pmatrix} \left[\frac{1}{2}\right]^{N-1} \quad (4.7)$$

The a^{th} amplifier input power for bit b in the desired channels is

$$P_{in_a(amp)}(b) = P_{in}(b) + N_1P_{in}(1) + (N-1-N_1)P_{in}(0) \quad (4.8)$$

The signal power and ASE noise psd at the photodetector is given by

$$P_R(b, N_1) = P_{in_{a-1}}(b)G_a(b, N_1)L_{A_tR} \quad (4.9)$$

and

$$S_{spr}(i, N_1) = S_{sp_a} L_{A_t R} \tag{4.10}$$

The signal current and noise variance corresponding to bit b are

$$I_{sig}(b, N_1) = R_0 P_R(b, N_1) \tag{4.11}$$

and

$$\begin{aligned} \sigma^2(b, N_1) &= 2eR_0 \left[P_R(b, N_1) + S_{spr}(b, N_1)B_0\right] B_e + 4R_0^2 P_R(b, N_1) S_{spr}(b, N_1) B_e \\ &+ R_0^2 \left[S_{spr}(b, N_1)\right]^2 \left[2B_0 B_e - B_e^2\right] + \frac{4kTB_e}{R_L} \end{aligned} \tag{4.12}$$

Both $I_{sig}(0)$ and $I_{sig}(1)$ will have N level depending upon the value of N_1. The threshold corresponding to highest level $I_{sig}(0)$ and lowest level $I_{sig}(1)$ will be

$$D_{th} = \frac{\sigma(0,0) I_{sig}(1, N-1) + \sigma(1, N-1) I_{sig}(0,0)}{\sigma(0,0) + \sigma(1, N-1)} \tag{4.13}$$

The probability of error under the condition that N_1 interfering channels having bit '1' is

$$P_e(N_1) = \frac{1}{2}\left[\frac{1}{2} erfc\left(\frac{I_{sig}(1, N_1) - D_{th}}{\sqrt{2}\sigma(1, N_1)}\right) + \frac{1}{2} erfc\left(\frac{D_{th} - I_{sig}(0, N_1)}{\sqrt{2}\sigma(0, N_1)}\right)\right] \tag{4.14}$$

The average probability P_e will be

$$P_e = \sum_{N_1=0}^{N-1} P_{N_1} P_e(N_1) \tag{4.15}$$

4.3 EXAMPLE

The value which are used in this example is given in the table 4.1.

Result of placement of amplifier as shown in figure 4.3 is given in table 4.2. These results shows the increase number of user as compared to the case when only one amplifier is placed at a time [2]. This increase can be explained as gain of both the

Responsitivity of receiver	R_0	1.18
Length of fiber used to star coupler	L	1 Km
Attenuation coefficient of fiber	α	0.2 dB/Km
Unsaturated amplifier gain	G_0	1800
Saturation power level	P_{sat}	-6 dBm
Optical filter bandwidth	B_o	10 GHz
Insertion loss of 2×2 SC	L_i	0.5 dB
Load Resistance	R_L	100 Ω
Spontaneous emissions factor	n_{sp}	2.0
Electrical bandwidth o receiver	B_e	1 GHz
Operating wavelength	λ	1.55 μm
Splice loss	L_{sp}	0.5 dB

Table 4.1: Values of parameter used for analysis of star topology

amplifiers boost the power to minimize the affect of ASE. But number of users dramatically decreases in the case of amplifiers with gain fluctuation. This is because of multiple channels present in second amplifier which increases the crosstalk.

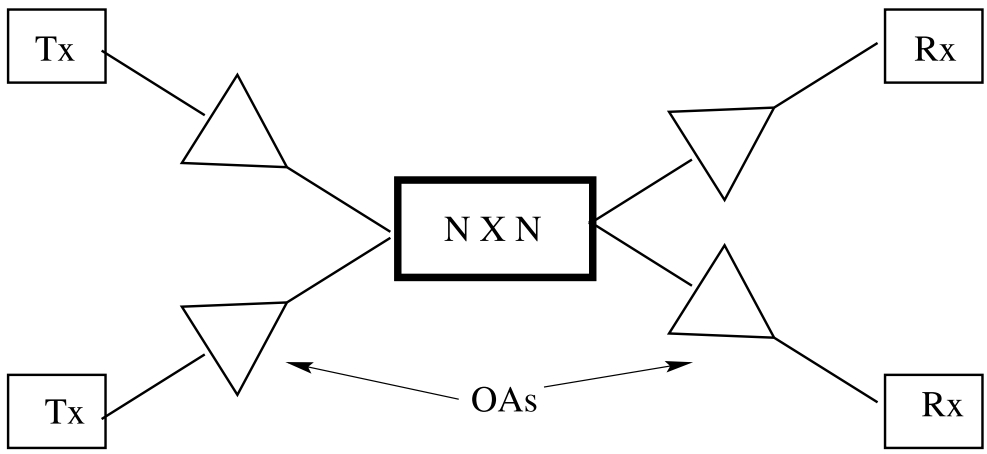

Figure 4.3: Star topology after placing amplifier

The results of placement of amplifiers in the multi-star shown in fig 4.4 are given in Tables 4.3, 4.4 and 4.5 for ideal OA, OA with saturation and OA with gain fluctuation respectively. Results show number of users increases considerably in this case as ASE produced by each amplifier is further divided by next stage star coupler.

Extinction Ratio	No. of supported user			
$\rightarrow$	0.00	0.05	0.10	0.15
Ideal Amp	16	16	16	16
OA with saturation	16	16	8	4
OA with gain fluctuation	4	4	4	2

Table 4.2: Number of supported users topology given in fig 4.3

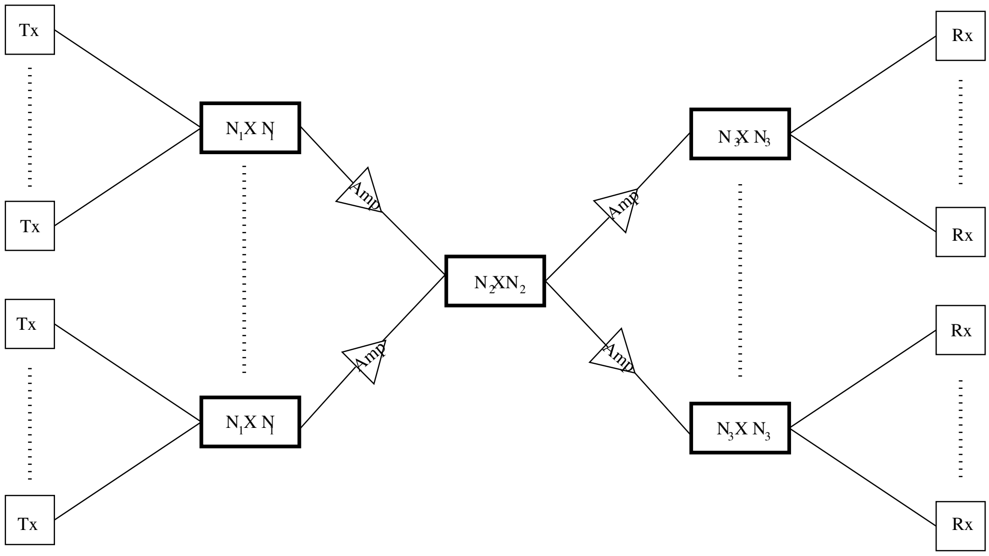

Figure 4.4: Multi-star topology after placing amplifier

Extinction ratio=0.00			Extinction ratio=0.05			Extinction ratio=0.10			Extinction ratio=0.15		
No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users
Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC	
2	8	16	2	8	16	2	8	16	2	4	8
4	8	32	4	8	32	4	8	32	4	4	16
8	4	32	8	8	64	8	4	32	8	4	32
16	4	64	16	4	64	16	4	64	16	2	32
32	4	128	32	4	128	32	2	64	32	2	64
64	2	128	64	2	128	64	2	128	64	2	128
128	2	256	128	2	256	128	2	256	-	-	-

Table 4.3: Number of supported users using ideal amplifier in topology given in fig 4.4

Extinction ratio=0.00			Extinction ratio=0.05			Extinction ratio=0.10			Extinction ratio=0.15		
No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users
Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC	
2	64	128	2	32	64	2	16	32	2	16	32
4	16	64	4	8	32	4	4	16	4	4	16
8	4	32	8	2	16	-	-	-	-	-	-

Table 4.4: Number of supported users using amplifier with saturation in topology given in fig 4.4

Extinction ratio=0.00			Extinction ratio=0.05			Extinction ratio=0.10			Extinction ratio=0.15		
No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users	No. of branches in		Total no. of users
Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC		Ist SC	IInd SC	
2	32	64	2	32	64	2	8	16	2	8	16
4	8	32	4	4	16	4	4	16	4	2	8
8	4	32	8	2	16	-	-	-	-	-	-

Table 4.5: Number of supported users using amplifier with gain fluctuation in topology given in fig 4.4

Chapter 5

CONCLUSION AND FUTURE WORK

Placement of amplifier in a subscriber access network and multi-star topology is studied to maximize the number of users. Three cases , ideal amplifier , amplifier with gain saturation and amplifiers with gain saturation and gain fluctuations are considered. The effect of extinction ratios is also taken into consideration. It is seen that with ideal OAs, the subscriber access network supports 6 amplifiers. This is reduced to 3 in case of amplifiers with saturation and remains 3 in case of amplifiers with gain fluctuation. The number of user is mainly affected by gain saturation and ASE. In new subscriber access network most of the time ASE overpowers the effect of saturation. Number of user is limited by ASE only. Effect of gain saturation can be seen in case of gain saturated amplifiers as the number of users is decreased because of reduction in received power level. The effect of extinction ratio on number of users is moderate in case of amplifier with gain fluctuation only.

In case of star topology, a multi-star topology is evolved to maximize the number of users. Here, the effect of gain saturation overpowers the effect of ASE noise and number of stages in multi-star topology is only limited by the amplified signal power. This is opposite of the trend observed in subscriber access network case.

There are few assumption made for the work done in this thesis. Effect of Polarisation sensitivity in amplifier is not considered. Gain is also taken to be independent of wavelength. Filter is assumed to be ideal and bandwidth of star couplers is taken as infinite. Effect of removal of these assumptions can be studied in future. Proposed study can also be done for other network topologies.

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