Stability and Reliability in QoS Environment under Adversarial Queuing Model

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Abstract—To assure Quality of Service (QoS) for multiclass traffic over a packet routing network, we implement multiclass queuing policy. This setup builds a QoS environment. Stability and reliability of a (multiclass) queuing network are functional requirements. They are also contributing factors towards QoS that comprises of Band width, Delay, Jitter and Reliability parameters. Overload traffic conditions always make the network unstable. In under load conditions a queuing policy may make a network unstable and/or unreliable. In this paper we investigate the stability and reliability of SP/FIFO (Strict Priority/First In First Out) multiclass packet scheduling networks. We use framework of Adversarial Queuing Model (AQM) as an investigation tool. Many studies have focused on stability and reliability of multiclass routing networks, but little effort has been devoted under AQM.

We consider small and simple multiclass networks to bring out valuable results. These networks are two class single station network and two class two station acyclic network. We obtain a counter intuitive result where a network is stable but unreliable. We give conditions for stability and reliability of two class single station and two class two station acyclic networks. We also prove for two class two station acyclic network to have zero condition at beginning and end of each round of packet injections, the adversary must inject at a restricted rate. These results can be used to predict and control network behavior.

Index Terms—Stability; Reliability; Multiclass; QoS; Adversarial queuing model

I. INTRODUCTION

A packet routing network facilitates end to end delivery of information by breaking it into small pieces called packets. These packets are delivered from source to destination by moving them over the network using store and forward methodology. The most proliferated and subscribed global system of computer networks is Internet. Internet handles traffic by packet routing. Protocol designed for forwarding packets on internet is called Internet Protocol (IP). The users of digital world take benefit of the communication services and information resources over internet through various applications. These applications may demand single or combination of various contents like text, voice, video and images. Applications requiring combination of multiple contents are termed multimedia applications. The multimedia applications like HDTV, video conferencing and merchandise have a stringent QoS requirement. QoS is built from essential parameters of Delay, Jitter, Reliability and Band width that an application desires for its content packets. Packets desiring same QoS constitute a class. These classes of packets can be effectively handled by giving them some priority ordering and implementing a priority queue at a forwarding station. A priority queue has a buffer for each priority class in the queue. As packets wait in the queue, a scheduling policy is chosen to pick and pass packets through the station one by one. IP networks give best effort service for which the most commonly used scheduling policy is FIFO. On top of this SP/FIFO can be used to assure QoS. SP/FIFO queuing policy significantly resembles the approaches being considered for QoS provisioning in Differentiated Services [1, 2, 3, 4] for internet. Differentiated services or DiffServ is a computer networking architecture that specifies a simple mechanism for classifying and managing network traffic and providing QoS on modern IP networks. DiffServ can, for example, be used to provide low-latency to critical network traffic such as voice or streaming media while providing simple best-effort service to non-critical services such as web traffic or file transfers. The SP/FIFO policy on a priority queue allows control over delay to a class of packets by giving quick service to highest priority class at any time. This ensures improved latency to a priority class in accordance with its QoS demand.

Overload condition is characterized by higher input rate vis-a-vis output rate of packets in a network. It is easy to appreciate that overload conditions cause instability in networks. A network can also become unstable in under load conditions due to a queuing policy. For last two decades under load instabilities due to queuing policies have been shown to exist [5, 6, 7]. Despite a network remaining stable the queuing policy implemented to make forwarding decision may render the network to hold certain packets for unbounded times and result in unreliable delivery of such packets. In this sense the network becomes unreliable. Reliability in packet delivery can be lost due to many other reasons too such as connection break, noise and overload. This paper deals reliability issues due to starvation of packets in priority queues.

In this paper we investigate stability and reliability of SP/FIFO multiclass packet scheduling networks depicting QoS environment. The gathered insight into multiclass networks using Adversarial Queuing Theory (AQT) can be utilized in practical situations. The previous work on multiclass queuing networks by Kelly [8], Kumar and Seidman [9], Lu and Kumar [10], Rebko and Stolyar [11] and Bramson [12, 13] are based on stochastic input traffic and service times. Further, they have used fluid model of network as a tool of study. Thus the analysis suffers from limitations of assumptions of traffic modeling and becomes less general as compared to AQM. The
AQM makes as many few assumptions about the input traffic and service times in the network as possible [5, 6, 14] which makes it more general. Marsan et. al. [15] discussed AQM for multiclass queueing networks with SP/FIFO policy but used tools of stochastic and fluid models for their results. Our set up differs in approach for the analysis that is based purely on AQM and uses most simple multiclass networks. Hence, analysis has become simpler.

In this paper we have constructed single station and two station multiclass networks. For multiclass packet classification we have taken case of simplest multiclass i.e two class classification of packets, for obtaining results. Then we extended the results over multiclass and simple path or chain. Results are presented in form of theorems and corollaries. Results include the following.

(i) Two class Single station network is stable and reliable against all adversaries of rate strictly below 1.

(ii) An adversary of rate 1 can make two class single station network unreliable, while keeping the network stable.

(iii) In case of two class two station acyclic network to have zero condition at beginning and end of each round, the adversary must inject at a restricted rate.

(iv) Two class two station network is always stable for an adversary of rate strictly below 1.

The paper is organized as follows. In section II we discuss details of AQM. In section III we present description of SP/FIFO policy in our context and section IV provides definitions and notations. Section V and VI we discuss two class single station network and two class two station network respectively. Finally we have concluded the paper with observations and future directions for further work.

II. ADVERSARIAL QUEUEING MODEL

Modeling of arrival and departure process of traffic in a network needs to be realistic and practical for study. Earlier all packet arrival and even departure processes had probabilistic approach. So the results hinged upon particular stochastic assumptions. AQM was developed by Borodin et. al. for robust modeling of input sources as it precludes any stochastic assumptions of traffic arrivals and departures and makes as many few assumptions as possible about traffic pattern, thus making it more general [5, 6, 14]. It resembles the complex real scenario traffic injections in a network. The essence of AQM is injection rate of the adversary as single and most striking parameter of the adversary. Adversarial Queuing Theory (AQT) or AQM was proposed

In AQT, a network is a directed graph \( G(V, E) \). \( V \) is set of vertices or nodes of the network. \( E \) is set of directed edges connecting two nodes in the network. Each edge at its tail has a queue-server pair called station (figure 1).

Packet are injected in the network by an adversary \( A \). The time proceeds in discrete steps of equal sizes. The path of injected packets is given along with packet itself similar to source routing followed today in most networks. A path is a connected sequence of edges in a network. When a path alone is a network we term it as chain. The adversary injects in any one time slot a set of edge disjoint paths called path packing.

![figure 1. Station](image)

All paths considered are simple paths. A simple path is a path that precludes traversal of an edge in it more than once and contains no cycles at all. Adversary injects path packings in the network in a restricted manner with a allowed rate of at most one path packing per time step. An adversary is said to be of rate \( r: 0 < r < 1 \), if over a time window of \( w \) time steps that is larger than large enough constant, the adversary injects at most \([rw]\) path packings in the network. Such an adversary is called AQT adversary [5].

An edge is traversed by only one packet in a time step. Contention between two or more packets contesting to cross an edge in the same time step is broken by a greedy and non preemptive queuing policy Q. First In First Out (FIFO), Nearest To Go (NTG) and Longest In System (LIS) etc are some examples of queuing policies. A protocol is greedy protocol if it always forwards a packet if there are waiting packets in the queue. For a non preemptive queuing policy a packet being forwarded is never halted to allow another contesting packet to get forwarded, irrespective of any priority. We consider contention breaking time to be zero. As packets wait in queue they experience delay. Delay of a packet is the time spent by the packet waiting in the queue. We describe a system for analysis with a triple \((G, A, Q)\). We will consider both path packings and any set of simple paths for the purpose of investigation as suitable.

As noted by Rosen [7] there are two types of adversaries in AQM, AQT adversary and Bursty adversary. Rosen [7] terms bursty adversary as leaky bucket adversary. Bursty adversary is an adversary of rate \( r \) if it injects at most \([rw]\) packets over time window of \( w \) time steps. \( w_o \in N \) is called burst parameter of the adversary. It is observed that both adversaries have equal powers at rate strictly below one, but at rate \( 1 \) bursty adversary is more powerful than AQT adversary [7]. It is worth noting that any adversary can inject at most rate \( r \) packets at any edge, by choosing the edge in each request continuously.
III. STRICT PRIORITY/FIRST IN FIRST OUT POLICY

FIFO policy follows that a packet which arrives first wins for transmission. It is most simple and natural policy in the sense that packets are transmitted over an edge in the order as they arrive in the queue of the edge. It is most prevalent as a scheduling policy in present day networks and that motivates its deeper analysis. Its widespread implementation is attributable to the advantages it offers inherently. It is easy to implement being most simple. It requires only local information for scheduling contrary to other policies like Furthest To Go (FTG) that requires knowledge of path length ahead. FIFO is very fast policy since time needed for scheduling decision is insignificant. On the other hand analysis of FIFO is difficult being extremely non-discriminative in nature. FIFO is extremely unbiased in forwarding a packet as compared to other policies like Shortest In System (SIS) that favors the youngest packet over older packets irrespective of order of arrival. This leaves no tell tail sign for analysis that can cause stability or in stability in a network. As an example, in SIS it is easily seen that continuous arrival of younger packets can cause accumulation of older packets in the network adding towards instability. FIFO gives no such hints making its analysis difficult.

Priority queue offers priority to some class of packets. For this, priority queuing implements at a station a stack of parallel buffers in the queue. One buffer corresponds to only one class of a packet. The buffers or equivalently classes in the queue have priority over each other. This awarding of priority can be strict in nature. Strict priority implies that no two buffers have same priority and packets are always extracted from one of the non empty highest priority buffers in the queue. The priority buffers on a queue allow control over delay to a class of packets by giving quick service to highest priority class of packet at any time to ensure better latency to it. The construction of SP/FIFO policy implies that packets are always extracted from one of the non empty highest priority buffers in the queue and the extraction of packet from a buffer is in FIFO manner. Figure 2 shows our model of station. We take the delay in server shift from one buffer to another to be zero. A packet maintains its class and priority at each station in a network.

IV. DEFINITIONS, NOTATIONS AND PRELIMINARIES

Definition 1. Stable System. A system is said to be stable when total number of packets remains bounded in the system as system runs for arbitrary long period of time.

Definition 2. Reliable System. A system is said to be reliable if every packet in the system experiences bounded delay. The reliable nature of network is called reliability.

At this point it is important to note that instability implies unreliability inherently in a packet routing network. Under instability unbounded number of packets demand unbounded time to deliver all packets present in the network. Therefore unreliability is incorporated in instability. However there can be a case where system remains stable but unreliable. We highlight ahead some situations where this can happen in multiclass queuing. As a repercussion if stability of a system is proved, issue of reliability must be validated separately.

Notations:
1. $N^m_l$ - $l$ class and $m$ station acyclic network, $l$, $m \in \mathbb{N}$.
2. $b_i$ - $i$th buffer corresponding to $i$th class in a queue, $i \in \mathbb{N}$.

Preliminaries:
1. We consider only acyclic network $G$ of stations.
2. For any two buffers $b_i$ and $b_j$, if $i < j$ then $b_i$ has strict priority over $b_j$ in the queue.

V. TWO CLASS SINGLE STATION NETWORK $N_2^1$

Consider two class single station network $N_2^1$ in figure 3. It has a single station with two priority buffers to handle two different classes of packets. The state of network at any time step $t \in \mathbb{Z}^+$ is given by vector $B_t = [q_1, q_2]$, where $q_1$ and $q_2 \in \mathbb{Z}^+$ give numbers of packets in buffer $b_1$ and $b_2$. This network is most trivial multiclass network. However, it highlights very important properties with respect to adversarial injection of packets alone at any station and hence deserves merit for investigation.

Theorem 1. $(N_2^1, A(0 \leq r < 1), SP/FIFO)$ is always stable and reliable.

Proof: Let the initial condition of $N_2^1$ be $B_0 = [q_1, q_2]$: $q_1, q_2 \neq 0$. Above claim proved for non zero initial condition will hold for zero initial condition naturally. We will consider both AQ and bursty adversary.

AQ Adversary:
Stability. Since rate of adversarial injection of packets is strictly less than 1 and there are no inputs from any other edges as no such edges exist, hence there is no accumulation of packets in the queue. Accumulation of packet will occur above rate one. Owing to this fact the total number of packets...
in the network can never exceed the number of packets already present in the network equal to \(|B_0| = (q_1 + q_2)\). Thus total number of packets remain bounded in the network at any time by \(|B_0| = (q_1 + q_2)\), thus system is stable.

Reliability. We note, a non zero initial condition of \(b_2\) with all subsequent arrivals at buffer \(b_1\) presents the worst case scenario for making transmission of all packets most unlikely. This results from suppression in transmission of lower class packets resting in lower priority buffer \(b_2\). But the rate of net arrival at the station is less than 1, hence there exist vacant time slots over an observation time window (figure 4) when there are no packet arrivals. During these vacant time slots the excess packets due to non zero initial conditions can be transmitted. Total vacant time slots required are equal to \(|B_0| = (q_1 + q_2)\) to clear the packets. These vacant time slots start becoming available latest by \([rw]\) and all of them become available latest by \([rw] + q_1 + q_2\) time steps, for window of observation \(w\), to make delivery of all packets in reliable manner. There by making the system \((N^1_2, A(0 \leq r < 1), SP/FIFO)\) both stable and reliable.

Bursty Adversary:

Below rate 1 any bursty adversary has equivalence with some AQT adversary, [7]. Hence the proof.

**Theorem 2.** \((N^1_2, AQT(r = 1), SP/FIFO)\) is always stable, but can become unreliable.

**Proof:** It can be easily seen that \(N^1_2\) always remains stable against rate 1 AQT adversary, as number of packets in the network remain bounded by total number of packets present initially in the system at all times. And there is no packet accumulation since rate of injection is 1 equal to the server capacity. The root of reliability problem lies in the initial condition of the network \(N^1_2\). Initial conditions \(B_0 = (0, 0)\) or \(B_0 = (q_i, 0)\) allow the system reliability even against AQT adversary of rate 1. But in case of non zero initial conditions \(B_0 = [0, q_2] : q_2 \geq 1\) and \(B_0 = [q_1, q_2] : q_1, q_2 \geq 1\), AQT adversary of rate 1 can make some packets experience unbounded delay in delivery. AQT adversary can choose to inject continuously at rate 1 only the packets of class 1 type and engage the server at buffer \(b_1\) for arbitrarily long period. In such situation all packets of buffer \(b_2\) suffer unbounded delay and system \((N^1_2, AQT(r = 1), SP/FIFO)\) becomes unreliable.

**Corollary 3.** For non zero initial conditions of a lower class buffers and continuous injections in higher classes \((N^1_1, AQT(r = 1), SP/FIFO)\) is stable but unreliable.

We must watch out for non zero initial conditions of lower class buffers to avoid unreliability in a multiclass network.

**Theorem 4.** \((N^1_2, Bursty(r = 1), SP/FIFO)\) it is always stable, but can become unreliable.

**Proof:** Let \(w_0\) be the burst parameter of the bursty adversary. Let the initial condition of \(N^1_2\) be zero i.e. \(B_0 = [0, 0]\). This condition is most suitable to avoid any chances of delay to the accumulated packets ab initio in the network. But adversary can bring state of network after some time duration \(t\) to \(B_1 = [0, w_0]\) by bursty injection and continue to inject in buffer \(b_2\) at rate 1. Thus packets in buffer \(b_2\) are made to experience unbounded delay. Similarly it can be proved for any non zero initial configuration as well. However system always contains bounded number of packets at any time, hence stable.

**Corollary 5.** \((N^1_2, Bursty(r = 1), SP/FIFO)\) is always stable but unreliable with bursty injections across any lower class buffers and continuous injections into higher class buffers.

We must employ a feasible mechanism to service lower class buffers time to time to avoid unreliability in a multiclass network.

**VI. TWO CLASS TWO STATION NETWORK \(N^2_2\)**

In this section we consider acyclic network of two classes and two stations as given in figure 5. The state of the system at any time \(t\) is given by a vector \(B_t = [q_1, q_2, q_3, q_4]\). The variables \(q_1, q_2, q_3, q_4 \in Z^+\) give number of packets in buffers.
$b_1$, $b_2$, $b_3$, $b_4$ respectively at any time $t$. The stability analysis in this section is only with respect to AQT Adversary. Any network property proved for AQT adversary of rate strictly below 1 stands for bursty adversary with rate strictly below due to equivalence and any network property is disproved for AQT adversary of rate 1 is also disproved for bursty adversary of rate 1, bursty adversary being more powerful with rate 1, [7]. We will study $N_2^2$ using round wise arrival and departure operations of packets. Each round is a contiguous set of time steps. The approach has been used in [6, 14] to prove a few results, but no formal proof for equivalence of time step wise and round wise operations is given. We state the equivalence and prove it. Round wise operation provides a easy tool over tedious time step wise analysis, hence the choice.

**Theorem 6.** Time step wise arrival and departure operations of packets in a network under AQM are equivalent to round wise operations, against an adversary of rate $r$: $0 \leq r \leq 1$.

**Proof:** Base case: For one time slot $\Delta t = 1$ maximum average arrival at a station is $k + r$, where $k$ is number of incoming edges to the station and maximum average departure is 1. We avoid ceiling function for ease of calculation and consider rounds with large enough number of time steps to offset the effect of loosing some constants by not applying ceiling.

**Induction Step:** For $\Delta t = n$ there are $n(k + r)$ arrivals and $n$ departures.

Finally, for $\Delta t = n + 1$ we find it is true that there are $(n + 1)(k + r)$ arrivals and $(n + 1)$ departures. Hence the proof.

We allow the AQT adversary of rate $r$ to inject packets of path types $c_1c_2$ and $c_2c_2$ on $S_1$ and $S_2$ respectively. This injection violates path-packing constraint and is more general akin to realistic injections. Certainly this kind of adversarial injection is more stressful for the network than path packing injections. Path packing injections become akin to $N_2^2$ case at each station that is already investigated. The operations of arrival and departure of packets is broken into rounds. First round is of $k$ time steps. $k$ is large enough positive integer to avoid ceiling functions for ease of calculations. $n^{th}$ round is of $kr^{(n-1)}$ time steps. State of the network at the beginning of round $n$ is given by $B_n^* = [q_1, q_2, q_3, q_4]$ and at the end of the round by $B_n = [q_1,q_2,q_3,q_4]$.

**Theorem 7.** Starting with zero initial condition $N_2^2$ to have zero condition at end of every round, AQT adversary must inject with rate $r \leq 0.5$ under SP/FIFO policy.

**Proof:** Initial condition is $B_0 = [0, 0, 0, 0]$. Round 1 is of $k$ time steps. So, the beginning state of first round is $B_1^* = [kr, 0, kr, 0]$ or $[kr, 0, kr, 0]$ or $[0, kr, 0, kr, 0, kr]$. Total injected packets are $2kr$. To allow all packets to get transmitted time required is at least $2kr$. Therefore $k \geq 2kr \Rightarrow r \leq 0.5$. For any subsequent round the condition is $2kr^n \leq kr^{(n-1)}$, hence $r \leq 0.5$.

**Corollary 8.** Starting with zero initial condition a chain of $n$ number of multiclass stations rests in zero state at end of every round when AQT adversary injects with rate $r \leq \frac{1}{n}$ under SP/FIFO policy.

We can construct a multiclass networks that do not keep pending work load at end of any round of packet injections.

**Theorem 9.** ($N_2^2$, AQT(0 $\leq r < 1$), SP/FIFO) is always stable and reliable.

**Proof:** Consider zero initial condition, $B_0 = [0, 0, 0, 0]$. Let round 1 be of $k$ time steps. We take $k$ to be large enough positive integer.

If all injections are for same class, class 1, then $B_1^* = [kr, 0, kr, 0]$.

$\Rightarrow B_1 = [0, 0, 2kr - k, 0]$;

subsequently, $B_2 = [kr, 0, 2kr - k + kr^2, 0]$, at most.

$\Rightarrow B_3 = [0, 0, kr - k + 2kr^2 + kr^3 + \ldots + 2kr^n, 0]$, at most.

Hence, for $n^{th}$ round, $B_n = [0, 0, kr - k + kr^2 + kr^3 + \ldots + 2kr^n - k, 0]$.

For $r = 1$, $\lim_{n \to \infty} |B_n| = \lim_{n \to \infty} (kr + kr^2 + \ldots + kr^n + kr^n - k) = \infty$

So, the system is certainly unstable at rate $r = 1$.

For, $r < 1$, $\lim_{n \to \infty} |B_n| = \lim_{n \to \infty} (kr + kr^2 + \ldots + kr^n + kr^n - k) = \frac{kr}{1-r} - k$.

Hence, for the system to be stable, $\lim_{n \to \infty} |B_n| < \infty$.

$\Rightarrow (\frac{kr}{1-r} - k) < \infty$

Hence $0 \leq r < 1$.

The proof on similar lines can be given for any non zero initial condition and injections over different classes.

**Corollary 10.** $N_n^m$ chain is always stable for an AQT adversary of rate strictly below 1 under SP/FIFO policy.

**VII. CONCLUSION**

The paper provides an analysis of stability and reliability in realistic QoS environment under adversarial queuing model. We have developed insight of the multiclass traffic behavior based on priority treatment in a network. In stable networks nonempty initial conditions and bursty injections of packets are root cause of unreliability. We also obtained conditions of both stability and reliability for multiclass chain networks. An
interesting observation was for a chain to have start and end states empty after every round of packet injection at restricted rate. The rate depended on the chain size. Our results verify practical approach adopted to handle multiclass services over internet today.

An important open issue for study is stability-reliability analysis for more complex multiclass networks like cycles. Stability-reliability study of other multiclass queueing networks is also an interesting open area.

REFERENCES