An Approach for Performance Analysis of Discrete-Time Finite Capacity Open Queuing Network with Correlated arrivals

Naresh K. Singh, Sanjay K. Bose and Yatindra N. Singh
Department of Electrical Engineering,
Indian Institute of Technology Kanpur,
UP-208016, India

Abstract

Future high speed, broadband networks such as B-ISDN (Broadband Integrated Services Digital Network) support diverse applications like voice, video and data transfer. ATM (Asynchronous Transfer Mode) is considered to be the most promising transfer technology for implementing these networks. This paper presents a scheme, which may be used for the performance evaluation of ATM switching networks where an ATM switch is modeled as a discrete-time finite buffer queue. Approaches have been proposed for the performance analysis of discrete-time, finite buffer capacity open queuing networks by decomposition of each queue and then individually analyzing them. Cell arrivals to the network is modeled as a two-state Markov Modulated Bernoulli Process (MMBP). An approximation method for fitting a two-state MMBP to the departure processes of queues has been presented. The results obtained from this analytical algorithm have been shown to be acceptably close to those obtained through simulations.

1. Introduction

In ATM networks, the time axis is slotted and fixed length packets are used to transfer information. Modeling and performance prediction of these networks is a major issue for switch design and its operation. The traffic in these systems may be from a collection of data, voice, and video sources with varying characterizations and different Quality of Service (QoS) requirements. Models like the Markov Modulated Phase-type Process (MMPP) and the Markov Modulated Bernoulli Process (MMBP) have been proposed to capture the burstiness and correlated nature of this traffic. These models have also been used in the literature for queuing analysis [1-5].

Several authors [6-8] have considered the performance of an ATM switch by modeling it as a discrete-time queue. Such networks of discrete-time queues are generally not exactly analyzable but approximate techniques for studying them may be developed. It may be possible in some cases to analyze the performance of these networks by isolating the queues and then analyzing them individually. Tandem networks of discrete-time queues have been analyzed by this approach in [5, 9].

We have extended the approach of [5] for the analysis of general open networks of discrete-time, finite buffer capacity queues. In some ATM switch architectures, a cell may be re-transmitted several times due to possible collision with other cells. In this case, the total transmission time is typically modeled by a geometric distribution. with parameter s. This means that with the probability s the cells will be retransmitted. We have also assumed this model in our analytical approach to model the service time. Burstiness of the source and its correlation in successive arrival of cells need to be considered for the typical traffic expected in future high-speed networks. We have done this by modeling the number of cell arrivals by a two-state Markov Modulated Bernoulli Process (2-MMBP). Since in a general queuing network, partitioning of the paths is common, probabilistic splitting of a 2-MMBP are modeled and approximated by other 2-MMBPs with different, appropriately computed, parameters. An approach for approximating the output process of a queue as a 2-MMBP when it is fed with two or more 2-MMBP arrival processes has also been developed. The subsequent sections of the paper are organized as follows. In Sec.2, we provide a brief description of the 2-MMBP and explain the method for approximating the random splitting of this process as 2-MMBP. In Sec.3, the approach for merging of two 2-MMBP in a queue has been given. These merging and splitting operations will be needed to analyze a network of discrete-time, finite capacity queues. Sec.4 applies these to analyze some examples of discrete-time open queuing networks with arbitrary configurations. The results obtained by our proposed approach and those obtained through simulations are presented for these examples. In Sec.5 summary of the present work is given.

2. Two-State Markov Modulated Bernoulli Process and Its Random Splitting

2.1 Two-State Markov Modulated Bernoulli process (2-MMBP) [11]

A 2-state MMBP is characterized by its transition probability matrix P and a diagonal matrix Λ of arrival probabilities, as given below.

$$P = \begin{bmatrix} p & (1-p) \\ (1-q) & q \end{bmatrix}, \text{ and } \Lambda = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \qquad (1)$$

where, $0<p, q<1$ and $0<\alpha,\beta<1$. In a 2-MMBP, there is a geometrically distributed period of time during which an arrival occurs in Bernoulli fashion with a specific probability α; the system is said to be in state 1 during this period. The system may move from state 1 to another state, i.e. state 2, where arrivals will occur in a Bernoulli fashion with a different probability β; the system will then spend a geometrically distributed period of time in this state before moving to state 1. These periods alternate continuously. Given that the process is in state-1 (state-2) in slot n, it will remain in the same state in the next slot (n+1) with probability p (q), or will change to state-2 (state-1) with probability 1-p (1-q). The mean, E[T], and the squared coefficient of variation, C_{sq}^2 , of the interarrival times of cells for the process are given by the expressions [11].

$$E[T] = \frac{1}{r} = \frac{2-p-q}{(1-q)a + (1-p)b} \qquad (2)$$

$$C_{sq}^2 = \frac{2r[(2-p-q)^2 + \{(1-p)a + (1-q)b\}(p+q-1)]}{(2-p-q)[(1-q)a + (1-p)b + ab(p+q-1)]} - r - 1 \qquad (3)$$

The coefficients of autocorrelation for interarrival times and of the number of cells per slot are the other two moments, which are of interest [11]. The auto-correlation coefficient of the interarrival times of cells for lag 1, ψ_1, is given by-

$$y_1 = \frac{ab(a-b)^2(1-p)(1-q)(p+q-1)^2}{C_{sq}^2(2-p-q)^2[a(1-q) + b(1-p) + ab(p+q-1)]^2} \qquad (4)$$

The autocorrelation coefficient ϕ(k) of the number of arrivals per slot (‘ 0 ' or ' 1’) of a 2-MMBP for lag k, is given by-

$$f(k) = \frac{(1-p)(1-q)(a-b)^2(p+q-1)^k}{[(1-q)a + (1-p)b][(1-q)(1-a) + (1-p)(1-b)]} \qquad (5)$$

2.2 Probabilistic Splitting of a 2-MMBP

The departure process of discrete-time, finite buffer capacity queue with a 2-MMBP arrival process, can itself be approximated by another 2-MMBP. This has been shown in [5]. We have used this in our approach to analyze discrete-time open queuing networks in which each of the external arrival processes are assumed to be 2-MMBP. We also make similar approximations for the derived processes obtained from a 2-MMBP by randomly routing its generated cells to two or more branching routes through a probabilistic splitting of the original process. We have approximated the processes in each route as a 2-MMBP with different (appropriately computed) parameters, which are obtained by satisfying certain conditions. In Fig. 1, we have shown an example of (N+1) queues connected in a specific fashion where such an approximation will be useful.

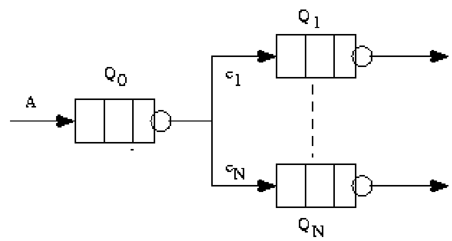

Figure 1: Probabilistic Splitting of 2-MMBP along N Routes

Assuming the arrival process to Q_0 to be a 2-MMBP with parameters p, q, α, β, the output process of the queue can be approximated, as described in [5], by 2-MMBP with parameters p', q', α', β'. The processes after the random splitting are approximated with the same state transition probabilities as before the splitting. Cell generation now depends on the cell arrival probabilities before partitioning the process and on the routing probabilities, c_i , i=1,2…N. The sum of all the routing probabilities must be equal to one i.e. $\sum_{i=1}^{N} c_i = 1$. The process for the route i, i.e. the arrival process to Q_i, may now be defined by its own state transition probability matrix P_i and diagonal matrix of arrival probabilities Λ_i as -

$$P_i = \begin{bmatrix} p & (1-p) \\ (1-q) & q \end{bmatrix} \text{ and } \Lambda_i = \begin{bmatrix} ac_i & 0 \\ 0 & bc_i \end{bmatrix} \qquad (6)$$

The four moments i.e. Mean, Squared coefficient of variation, autocorrelation coefficient of interarrival or departure times and autocorrelation coefficient of number of cells per slot for the processes on i^{th} route are given by the same expressions as in (2), (3), (4) and (5) by replacing four parameters of 2-MMBP with as derived for 2-MMBP on i^{th} route.

2.3 Verification of Analytical Approximations through Simulations

We have verified the approximation for fitting 2-MMBPs on each route after probabilistically splitting the process by considering the probabilistic splitting of a 2-MMBP into three branches as shown in Fig. 2. The original parameter values of 2-MMBP before splitting are p=0.9, q=0.45, α=0.99, β=0.56 and routing probabilities considered are c_1=0.1, c_2=0.4, c_3=0.5. In Table 1, the approximated parameters for the 2-MMBP after splitting have been given for the three branches. Tables 2 present the theoretical Vs simulation results for the various moments for each of the three branches, respectively.

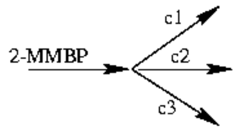

Figure 2: Splitting of 2-MMBP in three Branches

Branches	p_i	q_i	α_i	β_I
Route 1	0.9	0.45	0.099	0.056
Route 2	0.9	0.45	0.396	0.224
Route 3	0.9	0.45	0.495	0.280

Table 1:Approximated parameters of 2-MMBPs for three Branches

	Route -1		**Route - 2**		**Route - 3**	
	Theoretical values	**Simulation Results**	**Theoretical values**	**Simulation Results**	**Theoretical values**	**Simulation Results**
Rate(ρ)	9.23846e-2	9.23313e-2 ±9.38211e-5	3.69538e-1	3.69420e-1 ±1.53283e-4	4.6192e-1	4.61871e-1 ±1.62971e-4
C^2_{sqt}	9.10333e-1	9.09418e-1 ±2.07107e-3	6.40400e-1	6.403428e-1 ±6.66420e-4	5.50155e-1	5.50490e-1 ±5.06913e-4
E[$T_{n-1}T_n$]	1.17171e+2	1.173550e+2 ±2.75951e-1	7.32703e+0	7.33014e+0 ±6.56209e-3	4.69057e+0	4.69089e+0 ±3.53338e-3
$y(1)$	4.67286e-5	3.38439e-4 ±1.07779e-3	8.88163e-4	5.13412e-4 ±4.99444e-4	1.52677e-3	1.20869e-3 ±4.82459e-4
E[Y_n]	9.23846e-2	9.23269e-2 ±9.37907e-5	3.69539e-1	3.69417e-1 ±1.53185e-4	4.61923e-1	4.61869e-1 ±1.62841e-4
C^2_{sqn}	9.82431e+0	9.83161e+0 ±1.10085e-2	1.70608e+0	1.70699e+0 ±1.12269e-3	1.16486e+0	1.16513e+0 ±7.63034e-4
E[$Y_{n-1}Y_n$]	8.6192e-3	8.61195e-3 ±3.18299e-5	1.37907e-1	1.37837e-1 ±1.40387e-4	2.15479e-1	2.15459e-1 ±1.66689e-4
$f(1)$	1.00471e-3	1.04115e-3 ±3.12387e-4	5.78552e-3	5.86580e-3 ±3.35922e-4	8.47358e-3	8.59277e-3 ±2.89246e-4

Table 2: Theoretical Vs Simulation Results for three Branches

The various moments in Table 2 are C^2_{sqt} ,the squared coeff. of Var. of interarrival time, E[$T_{n-1}T_n$], the correlation between T_{n-1} and T_n , where T_n is time between n^{th} and $(n-1)^{th}$ arrival, E[Y_n], mean number of cells per slot, C^2_{sqn} , squared coeff. of Var. of number of cells per slot, and E[$Y_{n-1}Y_n$], the correlation between Y_{n-1} and Y_n. One can observe that the theoretical results are very close to the results obtained from simulations and are also well within the confidence intervals shown for a confidence level of 96%.

3. Merging of 2-MMBPs in a Queue [12]

Fig. 3 shows two 2-MMBPs (with possibly different parameters) jointly providing the arrivals to a queue. For the departure process from this queue, we evaluate the four moments, i.e. mean, squared coefficient of variation c_d^2 , coefficient of autocorrelation of inter-departure time $y_d(k)$, and coefficient of auto-correlation of number of cells per slot $f_d(k)$. As described in [5] these four moments can then be used to model the merged departure process approximately as a 2-MMBP.

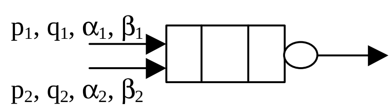

Figure 3:Merging of Two 2-MMBPs in a Queue

The slot lengths are fixed throughout the entire network. Hence in each slot the state of both 2-MMBPs may fall in one of the four combinations of states i.e. state (i, j) with i, j =1, 2. These 2-MMBPs are independent so the combination of the two 2-MMBPs can be considered as a single four-state Markov chain which can now generate either 0, 1, or 2 cells per slot. We have replaced the two 2-MMBPs by a four state Markov chain as shown in Fig. 4.

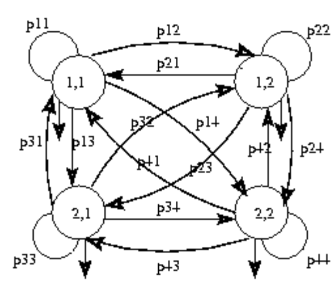

Figure 4: Four State Markov Chain.

The state transition matrix P of this process becomes

$$P = \begin{bmatrix} p_1 p_2 & p_1(1-p_2) & (1-p_1)p_2 & (1-p_1)(1-p_2) \\ p_1(1-q_2) & p_1 q_2 & (1-p_1)(1-q_2) & (1-p_1)q_2 \\ (1-q_1)p_2 & (1-q_1)(1-p_2) & q_1 p_2 & q_1(1-p_2) \\ (1-q_1)(1-q_2) & (1-q_1)q_2 & q_1(1-q_2) & q_1 q_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \tag{7}$$

Now our aim is to analyze a discrete-time single server queue of buffer capacity K with 4-state Markov arrival process and geometrically distributed service times. Let $P_d = P_{wd} + P_{wod}$ be the transition probability matrix of the queue with P_{wd} and P_{wod} as the transition probability matrices with a departure and without a departure.

The Z- transform of interdeparture process, $D(z)$, is the product of $I(z)$ and $S(z)$ where I and S are the server idle period and service time. The generating functions of I and S can be determined as in [12]. The mean interdeparture time $E[t_n]=1/\rho_d$, variance and the squared coefficient of variation of interdeparture times may be found by differentiating $D(z)$ as obtained in [12].Using $D(z)$, $I(z)$, P_{wd}, P_{wod}, one can determine expression for $E[t_n t_{n+k}]$ as in [12].The autocorrelation coefficient of the interdeparture times of cells for the queue with a 4-state Markov arrival process for lag k, $y_d(k)$, can now be obtained from the expression-

$$y_d(k) = \frac{E[t_n t_{n+k}] - E^2[t_n]}{Var[t_n]} \tag{8}$$

Let Y_n be a random variable representing the number of departures at the n^{th} slot, where Y_n can be either 0 or 1. The autocorrelation coefficient of number of cells per slot for lag k, $f_d(k)$, may now be obtained as -

$$f_d(k) = \frac{E[Y_n Y_{n+k}] - E^2[Y_n]}{Var[Y_n]} \tag{9}$$

where $E[Y_n] = E[Y_n^2] = X\bar{l}_d$ and the correlation between Y_n and Y_{n+k}, $E[Y_n Y_{n+k}]$, is given as in [12] by-

$$E[Y_n Y_{n+k}] = X P_{wd} P_d^{k-1} \bar{l}_d \tag{10}$$

with X as stationary probability vector for queue and

$\bar{l}_d = [0,0,0,0,(1-s),(1-s),.........,(1-s)]^T$

3.3 Characterization of the Departure Process by 2-MMBP

We have used the algorithm proposed in [5] for computation of four parameters of 2-MMBP from four computed moments. The approximated 2-MMBP is characterized by the four parameters, p_{est}, q_{est}, α_{est}, and β_{est}. We match the computed four moments of the departure process e.g. ρ_d, C_d^2, autocorrelation coefficient $\phi_d(1)$, and $\Psi_d(1)$ with that of the 2-MMBP respectively in order to obtain p_{est}, q_{est}, α_{est}, and β_{est} .

3.4 Verification of Analytical Approximations through Simulations

Simulation results have been obtained to examine the feasibility of the proposed approach. For the departure process of a queue theoretical values of various moments have been computed and verified by simulation.

S.No.	**p**	**q**	**α**	**β**	**σ**	**K**
1.	0.65	0.7	0.11	0.006	0.2	5
	0.75	0.45	0.99	0.25		
2.	0.9995	0.9999	0.88	0.01	0.15	8
	0.85	0.8	0.5	0.99		
3.	0.85	0.74	0.6	0.45	0.09	15
	0.99	0.8	0.45	0.5		
4.	0.89	0.89	0.001	0.45	0.005	20
	0.79	0.90	1.0	0.09		
5.	0.89	0.89	0.001	0.45	0.1	20
	0.79	0.90	1.0	0.09		
6.	0.89	0.89	0.001	0.45	0.2	20
	0.79	0.90	1.0	0.09		
7.	0.89	0.89	0.001	0.45	0.5	20
	0.79	0.90	1.0	0.09		

Table 3: Parameters of Arrival Processes and that of the Queue

As shown in Fig 5, we have considered a single discrete time queue in which two 2-MMBPs act as arrival processes to a queue. In Table 3 we have shown the parameters of the arrival processes. Seven different cases are considered. In Table 4 the calculated moments of the departure process and the simulation results for the same moments have been presented.

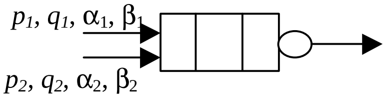

Figure 5: merging of two 2-MMBPs in a Queue

The results with simulations are obtained by running each simulation run for one million events. We find that our results are very close to the results obtained from simulations. If we go on increasing the value of σ, the mean interdeparture time and queue length must also increase. The same effect is observed in Table 4.

S. No.		ρ_d	C^2_d	$E[t_n t_{n+1}]$	$\Psi_d(1)$	$E[Y_n]$	$E[Y_n Y_{n+1}]$	$\phi_d(1)$	**Mean Queue Length**
1.	T*	7.6384e-1	2.4759e-1	1.7147e+0	1.6994e-3	7.6384e-1	5.8653e-1	1.7119e-2	3.0220e+0
	S*	7.6362e-1	2.4762e-1	1.7156e+0	1.4311e-3	7.6361e-1	5.8621e-1	1.7178e-2	3.0223e+0
2.	T	7.4078e-1	3.3039e-1	1.8643e+0	6.9691e-2	7.4078e-1	5.6622e-1	9.0908e-2	2.9661e+0
	S	7.4542e-1	3.2428e-1	1.8410e+0	6.9437e-2	7.4541e-1	5.7287e-1	9.0369e-2	3.1664e+0
3.	T	9.0906e-1	9.1405e-2	1.2103e+0	1.4177e-3	9.0906e-1	8.2654e-1	1.8789e-3	1.2315e+1
	S	9.0916e-1	9.1226e-2	1.2099e+0	1.3181e-3	9.0918e-1	8.2680e-1	2.4625e-3	1.2284e+1
4.	T	6.0904e-1	1.2849e+0	3.1572e+0	1.3316e-1	6.0909e-1	4.8589e-1	4.8278e-1	1.3913e+0
	S	6.0867e-1	1.2856e+0	3.1615e+0	1.3306e-1	6.0866e-1	4.8537e-1	4.8237e-1	1.3834e+0
5.	T	6.0887e-1	1.1483e+0	3.1009e+0	1.3027e-1	6.0887e-1	4.5879e-1	3.6982e-1	2.1173e+0
	S	6.0819e-1	1.1514e+0	3.1099e+0	1.3039e-1	6.0818e-1	4.5811e-1	3.7021e-1	2.1089e+0
6.	T	6.0732e-1	9.6400e-1	3.0286e+0	1.2144e-1	6.0732e-1	4.2789e-1	2.4765e-1	3.5209e+0
	S	6.0672e-1	9.6808e-1	3.0370e+0	1.2159e-1	6.0671e-1	4.2745e-1	2.4868e-1	3.5084e+0
7.	T	4.9413e-1	5.2549e-1	4.1106e+0	6.9629e-3	4.9413e-1	2.4540e-1	4.9662e-3	1.4862e+1
	S	4.9412e-1	5.2524e-1	4.1111e+0	7.0149e-3	4.9411e-1	2.4516e-1	4.0521e-3	1.4856e+1

Table 4: Theoretical Vs Simulation Results for Moments

T* - Theoretical results
S* - Simulation results

4. The General Algorithm for Approximate Analysis of a Network of Discrete Time-Queues

This has been done under the following assumptions -

- All the external arrival processes are 2-MMBP
- All the queues are of finite buffer capacity.
- Cells are served as FIFO.
- No immediate feedback.
- Time slots are of equal length.
- Service time is geometrically distributed.

Algorithm 4

Step 0: First of all consider only feed forward flows of the traffic in the networks. Start with any queue in the network to which the arrival processes are external known processes.
Step 1: Approximate the departure process of the queue using algorithms [12] with another 2-MMBP by appropriately computing the parameters of 2-MMBP.
Step 2: If the departure process is feeding N number of queues with routing probabilities c_i's, where $\sum_{i=1}^{N} c_i = 1$, then approximate the process after splitting departed 2-MMBP in each routing branch by 2-MMBP using proposed algorithm for splitting [12].
Step 3: Start again with the same queue as taken in step 0 by considering the feed backs from the queues. Do iteratively steps 1, 2 and 3 until the parameters of 2-MMBP for input to each queue become approximately constant.
Step 4: Isolate each queue from the network together with known arrival processes.
Step 5: Analyze each queue individually.

4.1 Verification of Analytical Approximations through Simulations

The network considered for analysis, shown in Fig. 6, consist of three nodes. Each node has an input from 2-MMBP source and each of them feeds the other two nodes in the network. In Tables 5 and 6 we have given the parameters of the external arrival processes and of the nodes in the network respectively. Each queue is decomposed and analyzed individually. In Table 7 the theoretical and simulation results for Q1, Q2, Q3 respectively have been presented.

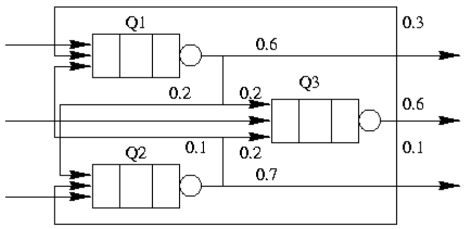

Figure 6: A Three-Node Network

Process No.	p_i	q_i	α_i	β_i
1	0.700	0.900	0.55	0.650
2	0.999	0.999	1.00	0.008
3	0.990	0.880	0.60	0.450

Table 5: External Arrival Processes

Node	σ	K
1	0.050	15
2	0.100	16
3	0.089	25

Table 6: Node Parameters

Parameters	Queue-1		Queue-2		Queue-3	
	Theoretical Values	Simulation Result	Theoretical Values	Simulation Result	Theoretical Values	Simulation Result
Throughput (ρ_d)	9.38919e-1	9.238172e-1 ±4.67314e-4	5.98989e-1	6.032432e-1 ±8.55418e-3	8.88478e-1	8.85953e-1 ±6.87678e-4
Squ. Coeff. of Var. (c^2_{dt})	6.61029e-2	1.030169e-1 ±1.04369e-3	1.23977e+0	1.23051e+0 ±1.33715e-2	1.24144e-1	1.29853e-1 ±1.11745e-3
Mean_Q Length	7.85669e+0	7.74869e+0 ±4.16152e-2	8.11202e+0	8.21839e+0 ±2.17318e-1	1.01332e+1	1.05162e+1 ±1.51117e-1
Mean_Q Delay	8.36779e+0	8.40704e+0 ±5.26169e-2	1.35428e+1	1.36747e+1 ±2.63935e-1	1.14051e+1	1.19265e+1 ±2.05261e-1
Cell Loss Probability	0.010200	0.010188 ±2.54302e-4	0.231533	0.231168 ±3.98390e-3	0.007042	0.007005 ±3.79572e-4

Table 7: Parameters for all three queues

From the table one can see that the results from proposed approaches are very close to the results obtained from simulations and are also well within the confidence intervals shown for a 96% confidence level.

5. Summary

We have developed a method for analyzing the performance of general network of discrete-time finite buffer capacity queues. This method may be useful in the analysis of an ATM network where the ATM switching nodes are modeled as discrete-time queues with finite buffers. Our proposed approach is implemented by extending the techniques available in the literature for the analysis of discrete-time finite buffer queues. The proposed algorithm has been used for the analysis of discrete-time open queuing networks with finite buffer capacity queues under rejection blocking. This is done by isolating and modeling each switching node by a discrete-time finite buffer queue. Isolation of the queues has been carried out after computing the relevant details of the arrival process to each queue.

6. Bibliography

[1]. H. Heffes and D. M. Lucatoni, "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Performance". *IEEE JSAC, Vol. 4, No.6, pp. 856-868, Sept., 1986.*

[2]. V. Ramaswami and G. Latouche, "Modelling Packets Arrivals from Asynchronous Input Lines", *ITC-12, pp.721-727, 1989.*

[3]. I. Stavrakakis, "Analysis of a Statistical Multiplexer under a General Input Traffic Model", *IEEE INFOCOM'90, pp. 1220-1225, 1990.*

[4]. Jean-Yves Le Boudec, "An Efficient Solution Method for Markov Models of ATM Links with Loss Priorities", *in IEEE JSAC, Vol. 9. No. 3,pp. 408-417, April 1991.*

[5]. Dooyeong Park, H. G. Perros and H. Yamashita, "Approximate Analysis of Discrete-Time Tandem Queuing Networks with Bursty and Correlated Input Traffic and Customer Loss", *Operation research letters 15(1994) 95-104.*

[6]. K. Laevens and H. Bruneel, "Discrete-time Queuing Models with Feedback for Input-Buffered ATM Switches", *in Performance evaluation 27&28, pp. 71-87,1996.*

[7]. T. Takine, T. Suda and T. Hasegawa, "Cell Loss and Output Process Analysis of a Finite-Buffer Discrete-Time ATM Queuing Systems with Correlated Arrivals", *in IEEE INFOCOM'93, pp. 1259-1267.*

[8]. Nelson L.S. Fonseca & Jhon A. Silvester, "Modelling the Output Process of an ATM Multiplexer

with Markov Modulated Arrivals", *in IEEE ICC'94, Vlo. 2, pp.721-725, 1994.*
[9]. Dooyeong Park, H. G. Perros, "Approximate Analysis of Discrete-time Tandem Queuing Networks with Customer Loss", *in IEEE, GLOBCOM'92, 1503-1507, Olando, FL, 1992.*
[10]. Jean-Yves Le Boudec, "An Efficient Solution Method for Markov Models of ATM Links with Loss Priorities", *IEEE JSAC, vol. 9, no. 3,April 1991.*
[11]. ATM Networks: Performance issues" (*Artech House Publ., MA, 1993*) by Raif O. Onvural.
[12]. Naresh K. Singh, *"Discrete-Time Analysis of Finite Buffer Capacity Open Queuing Networks"*, M.Tech Thesis, IIT Kanpur, March 1999.