IET Communications

Research Article

Max–min fairness based linear transceiver–relay design for MIMO interference relay channel

ISSN 1751-8628
Received on 23rd January 2017
Accepted on 2nd March 2017
E-First on 26th June 2017
doi: 10.1049/iet-com.2017.0034
www.ietdl.org

Vindheshwari P. Singh[1] ✉, Ajit Kumar Chaturvedi[1]
[1]*Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India*
✉ *E-mail: vprakash@iitk.ac.in*

Abstract: In this work, the authors investigate a multi-user multi-input multi-output (MIMO) interference relay system, where several transmitter nodes simultaneously communicate with their respective receiver nodes through half-duplex MIMO amplify and forward (AF) relay nodes. For this system configuration, they address the problem of linear transceiver–relay design to achieve max–min fairness among all the users' data streams. Assuming perfect channel state information, this problem is formulated as the maximisation of the minimum signal-to-interference-plus-noise ratio (SINR) per stream among all the users subject to transmit power constraints at the transmitter and relay nodes. Since the formulated problem of jointly optimising all the transmitters, relays and receivers is non-convex, a globally optimal solution cannot be easily obtained. Therefore, they propose a quasi-optimal iterative algorithm to jointly optimise the transmitter precoders, relay precoding matrices and receiver filters for all the users and relays such that the minimum per stream received SINR is improved at each iteration. Simulation results show that the proposed algorithm improves fairness among data-streams of all users and outperforms existing strategies in terms of minimum user rate and bit error rate.

1 Introduction

An interference amplify and forward (AF) relay system models the setting where multiple transmitter nodes communicate with their respective receiver nodes aided by a number of AF relays. Relay-assisted cooperative communication has been adopted as a potential technology for current/future wireless communication standards, e.g. LTE-advanced and WiMAX owing to its benefits such as throughput improvement, enhanced coverage and increase in reliability [1, 2]. Employing multiple antennas in a multiuser system not only provides diversity and multiplexing gain but also assists in managing co-channel interference [3]. The benefits of multi-input multi-output (MIMO) systems can be achieved in a cooperative communication system by employing multiple antennas at the transmitter, receiver and relay nodes. Compared with other relaying strategies, AF relaying is attractive because of its simple implementation and lower precoding delay [4]. Half-duplex relaying is the most practical in which transmission from transmitter node to receiver node across two hops is completed in two distinct time/frequency slots.

The problem of multi-user interference management in a single-hop MIMO interference channel has been a subject of intense research in recent years. A K user MIMO interference system with N antennas per node almost surely has $KN/2$ sum degrees of freedom (DoF) for time varying and/or frequency selective channel [5]. The DoF provide a capacity approximation which becomes accurate as signal-to-noise ratio (SNR) approaches infinity. It is also known as maximum multiplexing gain of a network. The achievability of the optimal DoF in a single-hop interference channel is based on the interference alignment (IA) scheme, which uses exponentially large symbol extensions in time/frequency domain to achieve DoF outerbound [6]. The authors in [7] showed that relays cannot increase the maximum achievable DoF of a fully-connected interference system where channels are time-varying and/or frequency selective. However, relays in an interference network can reduce the number of symbol extensions required for relay-aided IA at the receiver nodes [8, 9]. Implementing large symbol extensions over time/frequency dimensions is not always practical, e.g. over constant or quasi-static interference channels. The maximum achievable DoF in a K user MIMO interference system, with IA over spatial dimensions, does not scale linearly with K and is in fact upper-bounded by the sum of transmit and receive antennas at a user [10]. The use of relays in such networks can provide additional freedoms to manage inter user interference, thus achieving higher DoF [11].

Several works have been reported in the literature [12–23], which consider an interference AF relay system without direct link between transmitter and receiver pairs. The authors in [12, 13] develop zero-forcing and linear minimum mean square error-based transmitter–receiver beamformers at the relay nodes to suppress all the interference at the receiver nodes. The authors in [14] present a technique, in which transmitter and relay nodes cooperate to remove the co-channel interference at the receiver nodes. These works assume the availability of a sufficient number of single-antenna relays or sufficient antennas at a multiple-antenna relay to completely suppress the interference at the receiver nodes.

Optimisation of relay weights in a general interference AF relay system targeting sum power minimisation with signal-to-interference-plus-noise ratio (SINR) constraints at each receiver node is considered in [15–17] for single antenna relays. The relay matrix design using SINR constrained power minimisation and minimum SINR maximisation criteria in a multi-point to multi-point transmission via multiple MIMO relays is considered in [18]. However the schemes in [15–18] consider only single antenna transmitter and receiver nodes. Exploiting full potential of a multi-user relay network with multiple antennas at all the nodes, requires jointly designing the beamforming matrices at all the transmitters, relays and receivers. Most of the prior works [15–18], which consider single antenna transmitter and receiver nodes, cannot be readily extended to this general MIMO interference relay system.

There are few works on joint transceiver–relay design for a MIMO interference AF relay system with multiple antennas at all the nodes. Specifically, criteria such as sum interference and noise leakage minimisation [19], sum mean-squared-error minimisation [20, 22], total transmit power minimisation with SINR constraints at each receiver node [21] have been considered. The sum leakage minimisation based design is sub-optimal at low to intermediate SNR values since it does not consider the desired signal and receiver noise. Moreover, the feasibility of relay-aided IA may require a large number of relays. The sum-MSE based designs sacrifice user fairness since they may switch off the data streams with bad channel states. Total transmit power minimisation with SINR constraints based design enhances fairness but it cannot be

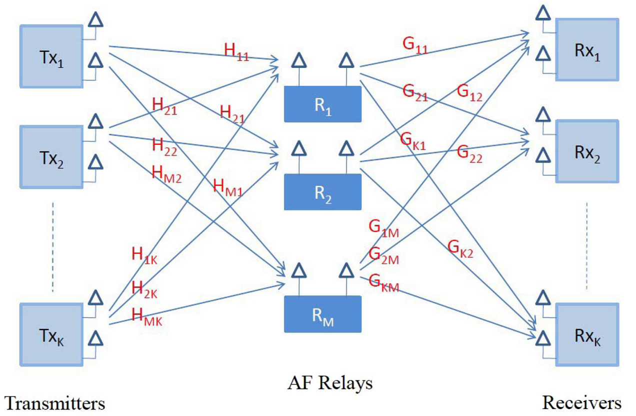

Fig. 1 *Interference AF relay system with K users (transmitter–receiver pairs) and M AF relays*

applied in case there are constraints on maximum power at the transmitter and relay nodes. In addition, this work considers only single data-stream per user.

In this work, we consider a MIMO interference AF relay system where all the transmitter, receiver and relay nodes have multiple antennas. In addition, each transmitter transmits multiple streams using spatial multiplexing. We do not consider any direct links between the transmitter and receiver nodes. A practical application scenario of the considered system model is the downlink transmission in a macro cellular network, where a single cell-edge mobile station (MS) per-cell receives message from its corresponding base station (BS) assisted by fixed relay stations. Due to limited backhaul, BSs are not allowed to jointly process the data and only exchange of channel state information is permitted. The main contributions of this work are summarised as follows:

- We formulate a novel joint transceiver and relay design problem that aims at maximising the minimum per-stream received SINR among all the users subject to transmit power constraints at each transmitter and relay node. To the best of our knowledge, max–min SINR based design for jointly optimising the transmitter, receiver and relay nodes in this general MIMO interference AF relay system with multiple antennas at all the nodes is still an open and challenging problem.
- Since the formulated transceiver design problem is non-convex, we present a quasi-optimal iterative algorithm based on block coordinate update (BCU) method where each iteration solves three sub-problems for updating the transmitter precoders, receiver filters and relay precoding matrices, respectively. We reformulate the transmitter precoders and relay matrices design sub-problems as quasi-convex optimisation problems which can be solved using the classical bisection method and convex feasibility checking.
- However, in this case, each iteration would require solving a sequence of convex feasibility problems (second-order cone programming – SOCP) each for the transmitter precoders and relay precoding matrices, which is computationally expensive. Hence, we propose a novel computationally efficient transceiver design algorithm based on the inexact BCU method where the update applied for transmit precoders and relay matrices is not exact maximisation of the objective function. In particular, we inexactly solve the transmitter precoders and relay matrices design sub-problems facilitated by solving their corresponding inverse problems (QoS constrained power minimisation) such that minimum SINR per stream improves at each iteration.
- Finally, we present simulation results to show that (a) the proposed inexact BCU based algorithm achieves similar averaged performance as the conventional exact BCU based algorithm at a significantly reduced computational complexity and (b) it outperforms the existing algorithms in terms of the minimum user-rate and bit error rate (BER) performance.

The remaining of this paper is organised as follows. The system model of a MIMO interference AF relay system is presented in Section 2. The transceiver–relay design problem formulation is described in Section 3. In Section 4, we develop and analyse our proposed transceiver design algorithm. In Section 5, we present the simulation results and Section 6 concludes the paper.

Notations: Lower boldface letters are used for vectors and uppercase boldface letters are used for matrices. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $\mathbb{E}\{\cdot\}$, $\mathrm{tr}\{\cdot\}$ and $\otimes$ denote transpose, conjugate transpose, mathematical expectation, trace and Kronecker product. $\boldsymbol{I}_N$ is the $N \times N$ identity matrix and $\boldsymbol{0}_N$ is the $N \times N$ zero matrix. $\mathrm{vec}(\boldsymbol{A})$ denotes the vectorisation operator and $\mathrm{vec}^{-1}(\boldsymbol{x})$ denotes the inverse vectorisation operator. $\mathscr{K} \triangleq \{1, 2, \ldots, K\}$, $\mathscr{M} \triangleq \{1, 2, \ldots, M\}$ and $\mathscr{D}_k \triangleq \{1, 2, \ldots, d_k\}$ represent the index sets for K users, M relays and d_k streams for kth user, respectively.

2 System model

We consider a MIMO interference AF relay system as illustrated in Fig. 1. There are K users, comprised of K transmitter–receiver pairs, and M half-duplex AF relays. Each transmitter communicates with only its respective receiver with the aid of relays. The direct links between transmitter and receiver nodes are not considered by the receiver nodes. The transmitter and receiver nodes of kth user have $N_{s,k}$ and $N_{d,k}$ antennas, respectively, and mth relay node has $N_{\mathrm{r},m}$ antennas. Due to half-duplex relaying, the communication between transmitter and receiver nodes takes two distinct time slots. In the first time slot, all the transmitter nodes send data to the relay nodes and relays forward the linearly processed received signals to the receiver nodes in the second time slot. We consider quasi-static flat-fading on both the hops, i.e. channel on all links in each hop is static for a block of transmission. This is the same model as in prior works on algorithms design for the MIMO interference AF relay system [19–23]. Synchronous transmission is assumed on each hop.

Let transmitter k transmits d_k data streams $\boldsymbol{s}_k = [s_k^{(1)}, \ldots, s_k^{(d_k)}]^{\mathrm{T}} \in \boldsymbol{C}^{d_k \times 1}$ to receiver k where $d_k \leq \min\{N_{s,k}, N_{d,k}\}$ and $\sum_{k=1}^{K} d_k \leq \sum_{m=1}^{M} N_{\mathrm{r},m}$. We assume that $\mathbb{E}(\boldsymbol{s}_k \boldsymbol{s}_k^{\mathrm{H}}) = \boldsymbol{I}_{d_k}$ and $\mathbb{E}(\boldsymbol{s}_k \boldsymbol{s}_j^{\mathrm{H}}) = \boldsymbol{0}$ for $j \neq k$. Transmitter k precodes its data streams using a transmitter precoder matrix $\boldsymbol{V}_k = [\boldsymbol{v}_k^{(1)}, \ldots, \boldsymbol{v}_k^{(d_k)}] \in C^{N_{s,k} \times d_k}$. The transmit signal at the kth transmitter is written as

$$\boldsymbol{x}_{s,k} = \boldsymbol{V}_k \boldsymbol{s}_k = \sum_{l=1}^{d_k} \boldsymbol{v}_k^{(l)} s_k^{(l)} \tag{1}$$

The signal received at the mth relay node is written as

$$\boldsymbol{y}_{\mathrm{r},m} = \sum_{k=1}^{K} \boldsymbol{H}_{m,k} \boldsymbol{V}_k \boldsymbol{s}_k + \boldsymbol{z}_{\mathrm{r},m} \tag{2}$$

where $\boldsymbol{H}_{m,k} \in C^{N_{\mathrm{r},m} \times N_{s,k}}$ is the channel coefficient matrix between the kth transmitter and mth relay and $\boldsymbol{z}_{\mathrm{r},m} \in C^{N_{r,m} \times 1}$ is the additive Gaussian noise vector at relay m with mean zero and covariance $\mathbb{E}(\boldsymbol{z}_{\mathrm{r},m} \boldsymbol{z}_{\mathrm{r},m}^{\mathrm{H}}) = \sigma_{\mathrm{r}}^2 \boldsymbol{I}$. The mth relay multiplies its received signal $\boldsymbol{y}_{\mathrm{r},m}$ by a precoding matrix $\boldsymbol{U}_m \in C^{N_{\mathrm{r},m} \times N_{\mathrm{r},m}}$ and forwards $\boldsymbol{x}_{\mathrm{r},m}$ to the receiver nodes, where

$$\boldsymbol{x}_{\mathrm{r},m} = \boldsymbol{U}_m \boldsymbol{y}_{\mathrm{r},m} = \sum_{k=1}^{K} \boldsymbol{U}_m \boldsymbol{H}_{m,k} \boldsymbol{V}_k \boldsymbol{s}_k + \boldsymbol{U}_m \boldsymbol{z}_{\mathrm{r},m} \tag{3}$$

The received signal at receiver k is written as

$$\begin{aligned}\boldsymbol{y}_{d,k} &= \sum_{m=1}^{M} \boldsymbol{G}_{k,m}\boldsymbol{x}_{\mathrm{r},m} + \boldsymbol{z}_{d,k} \\ &= \sum_{m=1}^{M} \boldsymbol{G}_{k,m}\boldsymbol{U}_m\left(\sum_{j=1}^{K} \boldsymbol{H}_{m,j}\boldsymbol{V}_j\boldsymbol{s}_j + \boldsymbol{z}_{\mathrm{r},m}\right) + \boldsymbol{z}_{d,k}\end{aligned} \tag{4}$$

where $\boldsymbol{G}_{k,m} \in C^{N_{d,k}\times N_{\mathrm{r},m}}$ is the channel coefficient matrix between the mth relay and kth receiver and $\boldsymbol{z}_{d,k} \in C^{N_{d,k}\times 1}$ is the additive Gaussian noise vector at receiver k with mean zero and covariance $\mathbb{E}(\boldsymbol{z}_{d,k}\boldsymbol{z}_{d,k}^{\mathrm{H}}) = \sigma_d^2\boldsymbol{I}$. Let us define, $k \in \mathscr{K}$

$$\begin{aligned}\tilde{\boldsymbol{H}}_k &\triangleq [\boldsymbol{H}_{1,k}^{\mathrm{T}}, \ldots, \boldsymbol{H}_{M,k}^{\mathrm{T}}]^{\mathrm{T}} \in C^{N_R\times N_{s,k}} \\ \tilde{\boldsymbol{G}}_k &\triangleq [\boldsymbol{G}_{k,1}, \ldots, \boldsymbol{G}_{k,M}] \in C^{N_{d,k}\times N_R} \\ \tilde{\boldsymbol{U}} &\triangleq \mathrm{blkdiag}(\boldsymbol{U}_1, \ldots, \boldsymbol{U}_M) \in C^{N_R\times N_R} \\ \tilde{\boldsymbol{z}}_{\mathrm{r}} &\triangleq [\boldsymbol{z}_{\mathrm{r},1}^{\mathrm{T}}, \ldots, \boldsymbol{z}_{\mathrm{r},M}^{\mathrm{T}}]^{\mathrm{T}} \in C^{N_R\times 1}\end{aligned} \tag{5}$$

where $N_R \triangleq \sum_{m=1}^{M} N_{\mathrm{r},m}$ and $\tilde{\boldsymbol{U}}$ is a block diagonal matrix. Using (5), (4) can be rewritten as

$$\begin{aligned}\boldsymbol{y}_{d,k} &= \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\left(\sum_{j=1}^{K} \tilde{\boldsymbol{H}}_j\boldsymbol{V}_j\boldsymbol{s}_j + \tilde{\boldsymbol{z}}_{\mathrm{r}}\right) + \boldsymbol{z}_{d,k} \\ &= \sum_{j=1}^{K} \boldsymbol{T}_{k,j}\boldsymbol{V}_j\boldsymbol{s}_j + \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_{\mathrm{r}} + \boldsymbol{z}_{d,k}\end{aligned} \tag{6}$$

where $\boldsymbol{T}_{k,j} = \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{H}}_j$ is the effective channel matrix between the jth transmitter and kth receiver. Receiver k applies a linear receive filter $\boldsymbol{W}_k = [\boldsymbol{w}_k^{(1)}, \ldots, \boldsymbol{w}_k^{(d_k)}] \in C^{N_{d,k}\times d_k}$. The estimate of lth data-stream at kth receiver is written as

$$\begin{aligned}\tilde{s}_k^{(l)} = (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{y}_{d,k} &= \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)}\boldsymbol{s}_k^{(l)}}_{\text{desired signal}} \\ &+ \underbrace{\sum_{\substack{p=1\\p\neq l}}^{d_k} (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(p)}\boldsymbol{s}_k^{(p)}}_{\text{inter stream interference}} \\ &+ \underbrace{\sum_{\substack{j=1\\j\neq k}}^{K}\sum_{p=1}^{d_j} (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,j}\boldsymbol{v}_j^{(p)}\boldsymbol{s}_j^{(p)}}_{\text{inter user interference}} \\ &+ \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_r}_{\text{relay enhanced noise}} + \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{z}_{d,k}}_{\text{local receiver noise}}\end{aligned} \tag{7}$$

The transmit power of the kth transmitter is written as

$$p_{s,k} = \mathrm{tr}(\mathbb{E}\{\boldsymbol{x}_{s,k}\boldsymbol{x}_{s,k}^{\mathrm{H}}\}) = \mathrm{tr}(\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}) = \sum_{l=1}^{d_k} (\boldsymbol{v}_k^{(l)})^{\mathrm{H}}\boldsymbol{v}_k^{(l)} \tag{8}$$

and the transmit power of the mth relay is written as

$$\begin{aligned}p_{\mathrm{r},m} &= \mathrm{tr}(\mathbb{E}\{\boldsymbol{x}_{\mathrm{r},m}\boldsymbol{x}_{\mathrm{r},m}^{\mathrm{H}}\}) \\ &= \sum_{k=1}^{K} \mathrm{tr}(\boldsymbol{U}_m\boldsymbol{H}_{m,k}\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{H}_{m,k}^{\mathrm{H}}\boldsymbol{U}_m^{\mathrm{H}}) + \sigma_{\mathrm{r}}^2\mathrm{tr}(\boldsymbol{U}_m\boldsymbol{U}_m^{\mathrm{H}})\end{aligned} \tag{9}$$

Remark 1: We assume that the perfect CSI of all links is present at a central node, which determines the matrices for all the nodes and send these to the respective nodes. Although this strict requirement would require large CSI acquisition overhead and is challenging in practice, the proposed algorithm can provide a framework for developing algorithms which consider more realistic CSI assumptions. Moreover, the algorithm proposed in this work can provide a benchmark for developing distributed algorithms which require only local CSI and robust algorithms which require only statistical CSI. The same assumption has been considered in related works [19–23].

3 Problem formulation

We formulate the max–min fairness based linear transceiver–relay optimisation problem for MIMO interference AF relay system. Specifically, we design transmitter precoders $\{\boldsymbol{V}_k\}_{k=1}^K$, relay precoding matrices $\{\boldsymbol{U}_m\}_{m=1}^M$ and receiver filters $\{\boldsymbol{W}_k\}_{k=1}^K$ for all the users and relays to maximise the minimum per-stream received SINR, subject to constraints on the maximum transmit power at the transmitter and relay nodes. Treating interference as noise, the SINR of the received data-stream $\tilde{s}_k^{(l)}$ is written as (see (10))

The data-rate for lth data-stream at kth user is written as

$$R_{k,l} = \frac{1}{2}\log_2(1 + \mathrm{SINR}_{k,l}) \tag{11}$$

where the 1/2 factor is due to the two distinct time slots used for end-to-end transmission between transmitter–receiver pairs. The max–min fair optimisation problem (see Remark 2) is formulated as

$$\begin{aligned}\mathscr{P}:\quad &\max_{\substack{\{\boldsymbol{V}_k\}_{k=1}^K, \{\boldsymbol{U}_m\}_{m=1}^M\\ \{\boldsymbol{W}_k\}_{k=1}^K}} \min_{\substack{l\in\mathscr{D}_k\\k\in\mathscr{K}}} \mathrm{SINR}_{k,l} \\ &\text{s.t.}\quad p_{s,k} \le P_{s,k},\quad k \in \mathscr{K} \\ &\qquad\ p_{\mathrm{r},m} \le P_{\mathrm{r},m},\quad m \in \mathscr{M}\end{aligned} \tag{12}$$

where $P_{s,k}$ and $P_{\mathrm{r},m}$ are the maximum powers allowed at the kth transmitter node and mth relay node, respectively.

Let $\tilde{P}_s = \min(P_{s,1}, \ldots, P_{s,K}, P_{\mathrm{r},1}, \ldots, P_{\mathrm{r},M})$, $P_{s,k} = \alpha_k\tilde{P}_s$ where $\alpha_k \ge 1$; $k \in \mathscr{K}$ and $P_{\mathrm{r},m} = \beta_m\tilde{P}_s$ where $\beta_m \ge 1$; $m \in \mathscr{M}$. –

Using an auxiliary variable $\gamma = \min_{\substack{l\in\mathscr{D}_k\\k\in\mathscr{K}}} \mathrm{SINR}_{k,l}$, the optimisation problem (12) can be rewritten as

$$\begin{aligned}\mathscr{P}:\quad &\max_{\substack{\{\boldsymbol{V}_k\}_{k=1}^K, \{\boldsymbol{U}_m\}_{m=1}^M\\ \{\boldsymbol{W}_k\}_{k=1}^K}} \gamma \\ &\text{s.t.}\quad \mathrm{SINR}_{k,l} \ge \gamma,\quad l \in \mathscr{D}_k,\quad k \in \mathscr{K} \\ &\qquad\ p_{s,k} \le \alpha_k\tilde{P}_s,\quad k \in \mathscr{K} \\ &\qquad\ p_{\mathrm{r},m} \le \beta_m\tilde{P}_s,\quad m \in \mathscr{M}\end{aligned} \tag{13}$$

Remark 2: In applications, where some data-streams have different QoS requirements, we can write $\mathrm{SINR}_{k,l}/\mu_{k,l}$ in place of $\mathrm{SINR}_{k,l}$ in (13), where $\mu_{k,l}$ are constants depending on the priority of the lth data-stream of kth user. This optimisation problem is called minimum weighted per-stream SINR maximisation, which ensures weighted fairness among all data streams in the network.

$$\mathrm{SINR}_{k,l} = \frac{|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)}|^2}{\sum_{\substack{p=1\\p\neq l}}^{d_k} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(p)}|^2 + \sum_{\substack{j=1\\j\neq k}}^{K}\sum_{p=1}^{d_j} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,j}\boldsymbol{v}_j^{(p)}|^2 + \sigma_r^2\|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\|^2 + \sigma_d^2\|(\boldsymbol{w}_k^{(l)})\|^2} \tag{10}$$

4 Proposed solution

The SINR expression (10) is not a concave function of the design variables $\{\boldsymbol{W}_k\}_{k=1}^K\}$, $\{\boldsymbol{V}_k\}_{k=1}^K\}$ and $\{\boldsymbol{U}_m\}_{m=1}^M\}$ as design variables are present in the numerator as well as in the denominator of (10). Therefore, the transceiver–relay optimisation problems in (12) and (13) are non-convex and we cannot easily find a globally optimal solution. Therefore, we present a quasi-optimal iterative algorithm based on BCU method for finding computationally efficient high quality solution. The design variables are partitioned into three blocks, namely transmitter precoders $\{\{\boldsymbol{V}_k\}_{k=1}^K\}$, receiver filters $\{\{\boldsymbol{W}_k\}_{k=1}^K\}$ and relay precoding matrices $\{\{\boldsymbol{U}_m\}_{m=1}^M\}$. In each iteration of the proposed algorithm, we solve three sub-problems for updating the receiver filters, transmitter precoders and relay precoding matrices in a cyclic order.

4.1 Receiver filter design

With fixed $\{\boldsymbol{V}_k^*\}_{k=1}^K$ and $\{\boldsymbol{U}_m^*\}_{m=1}^M$, receiver beamforming vectors for each data-stream $\boldsymbol{w}_k^{(l)}$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ is obtained by solving the following optimisation problem $\mathscr{P}_w$:

$$\mathscr{P}_w: \max_{\{\{\boldsymbol{w}_k^{(l)}\}_{l=1}^{d_k}\}_{k=1}^K} \min_{\substack{l \in \mathscr{D}_k \\ k \in \mathscr{K}}} \mathrm{SINR}_{k,l} \tag{14}$$

Since the received SINR of the lth data-stream of kth user depends only on $\boldsymbol{w}_k^{(l)}$, the receiver beamforming vector $\boldsymbol{w}_k^{(l)}$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ can be determined independently in parallel by solving the following optimisation problem such that it maximises the SINR of that data-stream:

$$\boldsymbol{w}_k^{(l)} = \underset{\boldsymbol{x} \in C^{N_{d,k} \times 1}}{\operatorname{argmax}} \ \mathrm{SINR}_{k,l} \tag{15}$$

It is well-known that optimal solution $\boldsymbol{w}_k^{(l)^*}$ to problem (16) is the linear MMSE receive beamformer given as

$$\boldsymbol{w}_k^{(l)^*} = \frac{\hat{\boldsymbol{w}}_k^{(l)}}{\| \hat{\boldsymbol{w}}_k^{(l)} \|} \tag{16}$$

where

$$\hat{\boldsymbol{w}}_k^{(l)} = \left(\sum_{j=1}^{K} \sum_{p=1}^{d_j} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)} (\boldsymbol{v}_j^{(p)})^{\mathrm{H}} \boldsymbol{T}_{k,j}^{\mathrm{H}} + \tilde{\boldsymbol{R}}_k \right)^{-1} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)} \tag{17}$$

and $\tilde{\boldsymbol{R}}_k = \sigma_r^2 \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \tilde{\boldsymbol{U}}^{\mathrm{H}} \tilde{\boldsymbol{G}}_k^{\mathrm{H}} + \sigma_d^2 \boldsymbol{I}_{N_{d,k}}$ is the covariance of noise $\tilde{z}_k = \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \tilde{z}_r + z_{d,k}$.

4.2. Transmitter precoders design

With fixed $\{\boldsymbol{W}_k^*\}_{k=1}^K$ and $\{\boldsymbol{U}_m^*\}_{m=1}^M$, transmitter beamforming vectors for each data-stream $\boldsymbol{v}_k^{(l)}$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ is obtained by solving the following optimisation problem $\mathscr{P}_v$:

$$\begin{aligned} \mathscr{P}_v: \quad & \max_{\{\{\boldsymbol{v}_k^{(l)}\}_{l=1}^{d_k}\}_{k=1}^K} \quad \gamma \\ \text{s.t.} \quad & \mathrm{SINR}_{k,l} \geq \gamma, \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K} \\ & p_{s,k} \leq \alpha_k \tilde{P}_s, \quad \forall k \in \mathscr{K} \\ & p_{\mathrm{r},m} \leq \beta_m \tilde{P}_s, \quad \forall m \in \mathscr{M} \end{aligned} \tag{18}$$

From (10), the received SINR of the lth data-stream of kth user is written as

$$\mathrm{SINR}_{k,l} = \frac{|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)}|^2}{\sum_{(j,p) \neq (k,l)} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)}|^2 + d_{k,l}} \tag{19}$$

where $d_{k,l} = \sigma_{\mathrm{r}}^2 \| (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \|^2 + \sigma_d^2 \| (\boldsymbol{w}_k^{(l)}) \|^2$; $l \in \mathscr{D}_k$, $k \in \mathscr{K}$.

From (9), the transmit power at mth relay is written as

$$p_{\mathrm{r},m} = \sum_{k=1}^{K} \sum_{p=1}^{d_k} (\boldsymbol{v}_k^{(p)})^{\mathrm{H}} \boldsymbol{H}_{m,k}^{\mathrm{H}} \boldsymbol{U}_m^{\mathrm{H}} \boldsymbol{U}_m \boldsymbol{H}_{m,k} \boldsymbol{v}_k^{(p)} + t_m \tag{20}$$

where $t_m = \sigma_{\mathrm{r}}^2 \mathrm{tr}(\boldsymbol{U}_m \boldsymbol{U}_m^{\mathrm{H}})$.

From (19) and (20), we see that the argument $\boldsymbol{v}_k^{(l)}$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ of the optimisation problem $\mathscr{P}_v$ is defined up to a phase-scaling on the right, i.e. if $\boldsymbol{v}_k^{(l)}; l \in \mathscr{D}_k, k \in \mathscr{K}$ is optimal then $\boldsymbol{v}_k^{(l)} \mathrm{e}^{\mathrm{j}\phi_{k,l}}$ is also optimal, where $\phi_{k,l}$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ are arbitrary phases. This can be verified by checking that neither the objective nor the constraints are changed by the phase of $\boldsymbol{v}_k^{(l)}; l \in \mathscr{D}_k, k \in \mathscr{K}$. Therefore, we can restrict the transmitter beamformer in which $(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)} \geq 0$; $l \in \mathscr{D}_k, k \in \mathscr{K}$, i.e. real part is non-negative and imaginary part is zero. Let us define, $\forall l \in \mathscr{D}_k$ and $\forall k \in \mathscr{K} -$.

$$\begin{aligned} \boldsymbol{T} &\triangleq \begin{bmatrix} \boldsymbol{T}_{1,1} & \boldsymbol{T}_{1,2} & \dots & \boldsymbol{T}_{1,K} \\ \boldsymbol{T}_{2,1} & \boldsymbol{T}_{2,2} & \dots & \boldsymbol{T}_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{T}_{K,1} & \boldsymbol{T}_{K,2} & \dots & \boldsymbol{T}_{K,K} \end{bmatrix} \\ \boldsymbol{V} &\triangleq \mathrm{blkdiag}(\boldsymbol{V}_1, \boldsymbol{V}_2, \dots, \boldsymbol{V}_K) \\ \boldsymbol{A}_k &\triangleq \left[\boldsymbol{0}_{N_{d,k} \times \sum_{j=1}^{k-1} N_{d,j}}, \boldsymbol{I}_{N_{d,k}}, \boldsymbol{0}_{N_{d,k} \times \sum_{j=k+1}^{K} N_{d,j}} \right] \\ \boldsymbol{B}_k &\triangleq \left[\boldsymbol{0}_{N_{s,k} \times \sum_{j=1}^{k-1} N_{s,j}}, \boldsymbol{I}_{N_{s,k}}, \boldsymbol{0}_{N_{s,k} \times \sum_{j=k+1}^{K} N_{s,j}} \right] \\ \tilde{\boldsymbol{B}}_k &= \boldsymbol{B}_k^{\mathrm{T}} \boldsymbol{B}_k \\ \boldsymbol{C}_k &\triangleq \left[\boldsymbol{0}_{d_k \times \sum_{j=1}^{k-1} d_j}, \boldsymbol{I}_{d_k}, \boldsymbol{0}_{d_k \times \sum_{j=k+1}^{K} d_j} \right] \\ \boldsymbol{e}_{k,l} &\triangleq [\boldsymbol{0}_{l-1 \times 1}^{\mathrm{T}}, 1, \boldsymbol{0}_{d_k - l \times 1}^{\mathrm{T}}]^{\mathrm{T}} \end{aligned} \tag{21}$$

Using the definitions of (21) and applying $\mathrm{vec}(\boldsymbol{ABC}) = (C^{\mathrm{T}} \otimes \boldsymbol{A}) \mathrm{vec}(\boldsymbol{B})$ [24], we rewrite (19) as (see (22))

Similarly, using (21), the transmit power at mth relay node is rewritten as

$$p_{\mathrm{r},m} = \sum_{k=1}^{K} \| [\boldsymbol{I}_{d_k} \otimes \boldsymbol{U}_m \boldsymbol{H}_{m,k}] \mathrm{vec}(\boldsymbol{B}_k \boldsymbol{V} C_k^{\mathrm{T}}) \|^2 + t_m \tag{23}$$

Since $(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)} \geq 0$; $l \in \mathscr{D}_k, k \in \mathscr{K}$, the problem $\mathscr{P}_v$ (18) is rewritten as

$$\mathrm{SINR}_{k,l} = \frac{\left| [\boldsymbol{e}_{k,l}^{\mathrm{T}} C_k \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k] \mathrm{vec}(\boldsymbol{V}) \right|^2}{\sum_{(j,p) \neq (k,l)} \| [\boldsymbol{e}_{j,p}^{\mathrm{T}} C_j \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_j] \mathrm{vec}(\boldsymbol{V}) \|^2 + d_{k,l}} \tag{22}$$

$$\begin{aligned} \mathscr{P}_v: \quad & \max_{\{V\}} \quad \gamma \\ \text{s.t.} \quad & \| \boldsymbol{\mu}_k^l \| \le \tau_k^l; \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K} \\ & \| \mathrm{vec}(\boldsymbol{B}_k \boldsymbol{V} C_k^{\mathrm{T}}) \| \le \sqrt{\alpha_k \tilde{P}_s}, \quad \forall k \in \mathscr{K} \\ & \| \boldsymbol{\theta}_m \| \le \sqrt{\beta_m \tilde{P}_s}, \quad \forall m \in \mathscr{M} \end{aligned} \tag{24}$$

where the following vector forms have been used:

$$\begin{aligned} \boldsymbol{\mu}_k^l &\triangleq \begin{bmatrix} \sqrt{d_{k,l}} \\ [C_1 \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_1] \mathrm{vec}(\boldsymbol{V}) \\ \vdots \\ [C_K \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_K] \mathrm{vec}(\boldsymbol{V}) \end{bmatrix} \\ \tau_k^l &\triangleq \sqrt{1 + \frac{1}{\gamma}} [\boldsymbol{e}_{k,l}^{\mathrm{T}} C_k \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k] \mathrm{vec}(\boldsymbol{V}) \\ \boldsymbol{\theta}_m &\triangleq \begin{bmatrix} \sigma_{\mathrm{r}} \sqrt{t_m} \\ [\boldsymbol{I}_{d_1} \otimes \boldsymbol{U}_m H_{m,1}] \mathrm{vec}(\boldsymbol{B}_1 \boldsymbol{V} C_1^{\mathrm{T}}) \\ \vdots \\ [\boldsymbol{I}_{d_K} \otimes \boldsymbol{U}_m H_{m,K}] \mathrm{vec}(\boldsymbol{B}_K \boldsymbol{V} C_K^{\mathrm{T}}) \end{bmatrix} \end{aligned} \tag{25}$$

The transmitter precoder design sub-problem $\mathscr{P}_v$ for a given γ is an SOCP problem [25], which can be efficiently solved using interior point algorithms [26].

However, $\mathscr{P}_v$ in variables γ and $\boldsymbol{V}$, is a quasi-convex problem [26], which can be solved to global optimality using bisection search over γ and convex feasibility checking over $\boldsymbol{V}$. This feasibility checking problem is an SOCP problem. Therefore the exact solution of problem (18) requires solving a sequence of SOCP problems, which has a high computational cost. This motivates us to find a more practical method to solve problem $\mathscr{P}_v$ (18) with moderate computational complexity. By this motivation, we solve the problem $\mathscr{P}_v$ (18) by solving the QoS constrained power minimisation problem $\mathscr{Q}_v$. We propose a novel per-node power minimisation problem $\mathscr{Q}_v$ with SINR constraints at each receiver written as

$$\begin{aligned} \mathscr{Q}_v: \quad & \min_{\{\{v_k^l\}_{l=1}^{d_k}\}_{k=1}^K, \tilde{P}_v \in R_+} \quad \tilde{P}_v \\ \text{s.t.} \quad & \mathrm{SINR}_{k,l} \ge \tilde{\gamma}, \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K} \\ & p_{s,k} \le \alpha_k \tilde{P}_v; \quad k \in \mathscr{K} \\ & p_{\mathrm{r},m} \le \beta_m \tilde{P}_v; \quad m \in \mathscr{M} \end{aligned} \tag{26}$$

Table 1 Procedure for solving $\mathscr{P}_v(\tilde{P}_s)$ (18) through $\mathscr{Q}_v(\tilde{\gamma})$ (26) with fixed $\{\boldsymbol{U}_m^*\}_{m=1}^M, \{\boldsymbol{W}_k^*\}_{k=1}^K$ Initialise $\{\boldsymbol{V}_k^*\}_{k=1}^K$ such that it satisfy transmit power constraints

Repeat

1. **S1:** Compute minimum SINR $\gamma_{\min} = \min_{\substack{l \in \mathscr{D}_k \\ k \in \mathscr{K}}} \mathrm{SINR}_{k,l}(\{\boldsymbol{V}_k^*\}_{k=1}^K, \{\boldsymbol{U}_m^*\}_{m=1}^M, \{\boldsymbol{W}_k^*\}_{k=1}^K)$
2. **S2:** Compute the transmitter precoder matrices $\{\boldsymbol{V}_k\}_{k=1}^K$ and $\tilde{P}_v$ with fixed $\{\boldsymbol{U}_m^*\}_{m=1}^M$ and $\{\boldsymbol{W}_k^*\}_{k=1}^K$ by solving $\mathscr{Q}_v(\tilde{\gamma})$ (26) with $\tilde{\gamma} = \gamma_{\min}$.
3. **S3:** Since $\{\tilde{P}_s, \{\boldsymbol{V}_k^*\}_{k=1}^K\}$ is a feasible solution for (26), $\tilde{P}_v \le \tilde{P}_s$.
   Update the transmitter precoders as $\boldsymbol{V}_k^* = \sqrt{(\tilde{P}_s/\tilde{P}_v)}\boldsymbol{V}_k; \ k \in \mathscr{K}$.

Until $\tilde{P}_v = \tilde{P}_s$

Problem $\mathscr{Q}_v$ (26) receives as input the target SINR value $\tilde{\gamma}$ for all data-streams of all users. $\mathscr{Q}_v$ attempts to minimise the maximum power among all transmitter and relay nodes. Similar to (24), $\mathscr{Q}_v$ can be reformulated as an SOCP. Let us first discuss the connection between the two problems $\mathscr{P}_v$ and $\mathscr{Q}_v$. For fixed relay precoding matrices and receiver filter matrices, the problem $\mathscr{P}_v$ is parameterised by $\tilde{P}$ and we use $\tilde{\gamma} = \mathscr{P}_v(\tilde{P})$ to denote its optimum value. Likewise problem $\mathscr{Q}_v$ is parameterised by $\tilde{\gamma}$ and we use $\tilde{P} = \mathscr{Q}_v(\tilde{\gamma})$ to denote its optimum value. Similar to the inverse relation between max–min SINR problem and QoS constrained power minimisation problems for the broadcast channel [27] and multicast channel [28], we have the following results.

Claim 1: The problem $\mathscr{P}_v$ (19) and problem $\mathscr{Q}_v$ (26) are related as follows:

$$\tilde{\gamma} = \mathscr{P}_v(\mathscr{Q}_v(\tilde{\gamma})) \tag{27}$$

$$\tilde{P} = \mathscr{Q}_v(\mathscr{P}_v(\tilde{P})) \tag{28}$$

Claim 2: The optimum objective values of the problem $\mathscr{Q}_v(\tilde{\gamma})$ and problem $\mathscr{P}_v(\tilde{P})$ are monotonically non-decreasing in $\tilde{\gamma}$ and $\tilde{P}$, respectively,

$$\gamma_1 > \gamma_2 \Rightarrow \mathscr{Q}_v(\gamma_1) > \mathscr{Q}_v(\gamma_2) \tag{29}$$

$$P_1 > P_2 \Rightarrow \mathscr{P}_v(P_1) > \mathscr{P}_v(P_2) \tag{30}$$

Proof: We first prove (27) by contradiction. Let $\{\boldsymbol{V}_k\}_{k=1}^K$ and P_1 are the optimal solution and associated optimal value for a feasible instance of $\mathscr{Q}_v(\gamma_1)$, where γ_1 is the target SINR value. For problem $\mathscr{P}_v(P_1)$, $\{\boldsymbol{V}_k\}_{k=1}^K$ is a feasible solution with associated optimal value γ_1. Let $\{\tilde{\boldsymbol{V}}_k\}_{k=1}^K$ be another feasible solution for $\mathscr{P}_v(P_1)$ with associated optimal value $\gamma_2 > \gamma_1$. Then, we can find a scalar constant $c < 1$ such that $\{c\tilde{\boldsymbol{V}}_k\}_{k=1}^K$ is also feasible for $\mathscr{Q}_v(\gamma_1)$ with a smaller objective value than P_1, which is a contradiction for the optimality of $\{\boldsymbol{V}_k\}_{k=1}^K$ and P_1 for $\mathscr{Q}_v(\gamma_1)$. Similar arguments can be used to prove (28).
Next, we prove (29) by contradiction. Let $\{\boldsymbol{V}_k\}_{k=1}^K$ and P_1 are the optimal solution and associated optimal value for a feasible instance of $\mathscr{Q}_v(\gamma_1)$ and $\{\tilde{\boldsymbol{V}}_k\}_{k=1}^K$ and $P_2(>P_1)$ are the optimal solution and associated optimal value for a feasible instance of $\mathscr{Q}_v(\gamma_2)$ for $\gamma_2 < \gamma_1$. We can always find a scalar constant $c < 1$ so that $\{c\boldsymbol{V}_k\}_{k=1}^K$ is feasible for $\mathscr{Q}_v(\gamma_2)$ with a smaller objective value than $P_1(\le P_2)$. This contradicts the optimality of P_2 for $\mathscr{Q}_v(\gamma_2)$. Similar arguments can be used to prove (30). □

From (27) and (28), we see that the two problems $\mathscr{P}_v$ and $\mathscr{Q}_v$ are inverse problems. Therefore, we can find a solution for the problem $\mathscr{P}_v$ for a given $\tilde{P}_s$ by iteratively solving $\mathscr{Q}_v(\tilde{\gamma})$ for varying values of $\tilde{\gamma}$. When $\tilde{P}_s = \mathscr{Q}_v(\tilde{\gamma})$, then $\tilde{\gamma}$ will be the optimal solution for $\mathscr{P}_v(\tilde{P}_s)$. Thus $\mathscr{P}_v$ can be exactly solved either through bisection search and convex feasibility checking or through iteratively solving the inverse problem $\mathscr{Q}_v$. The procedure for solving $\mathscr{P}_v$ for a given $\tilde{P}_s$ through $\mathscr{Q}_v$ is outlined in Table 1.

From (29), we see that we can solve the transmitter precoder design sub-problem $\mathscr{P}_v$ (18) with fixed receiver filters and relay precoding matrices by iteratively solving the QoS constrained power minimisation problem $\mathscr{Q}_v$ (26). When $\tilde{P}_v = \tilde{P}_s$, $\{\boldsymbol{V}_k^*\}_{k=1}^K$ and $\gamma_{\min}$ will be the optimal solution and associated optimal value of $\mathscr{P}_v$ (18). However, this method will also require solving a sequence of SOCPs. Therefore, we inexactly update the transmitter precoders instead of exact maximisation of the minimum SINR over the transmitter precoder block. Specifically, in each iteration of the proposed algorithm, we perform the steps **S1**–**S3** of Table 1 only once to reduce the computational cost of updating the transmitter precoders.

We can further relax the relay power constraint of the problem $\mathcal{Q}_v$ to obtain $\mathcal{Q}_v^{\mathrm{r}}$ such that after upscaling the transmitter precoders, the relay powers satisfy the maximum relay power constraints, i.e. $p_{\mathrm{r},m} \le \beta_m \tilde{P}_s$; $m \in \mathcal{M}$. This is required for the feasibility of relay matrices design sub-problem. The problem $\mathcal{Q}_v^{\mathrm{r}}$ is written as

$$\begin{aligned} \mathcal{Q}_v^{\mathrm{r}}: \quad & \min_{\{\{\boldsymbol{v}_k^l\}_{l=1}^{d_k}\}_{k=1}^K, \tilde{P}_v \in R_+} \quad \tilde{P}_v \\ & \text{s.t.} \quad \mathrm{SINR}_{k,l} \ge \tilde{\gamma}, \quad l \in \mathcal{D}_k, \quad k \in \mathcal{K} \\ & p_{s,k} \le \alpha_k \tilde{P}_v; \quad k \in \mathcal{K} \\ & p_{\mathrm{r},m} \le \beta_m \tilde{P}_v + \left(1 - \frac{\tilde{P}_v}{\tilde{P}_s}\right) t_m; \quad m \in \mathcal{M} \end{aligned} \tag{31}$$

Remark 3: We can use either $\mathcal{Q}_v$ or $\mathcal{Q}_v^{\mathrm{r}}$ for exactly solving the problem $\mathcal{P}_v(\tilde{P}_s)$ because both the problems give same optimal solution and associated optimal value when $\tilde{P}_v = \tilde{P}_s$. Since we are inexactly solving the transmitter precoders design sub-problem $\mathcal{P}_v(\tilde{P}_s)$ through its inverse problem, $\mathcal{Q}_v^{\mathrm{r}}$ may provide more gain in one update of the transmitter precoders.

4.3 Relay precoding matrices design

With fixed $\{\boldsymbol{W}_k^*\}_{k=1}^K$ and $\{\boldsymbol{V}_k^*\}_{k=1}^K$, precoding matrix for each relay node $\boldsymbol{U}_m; m \in \mathcal{M}$ is obtained by solving the following optimisation problem $\mathcal{P}_u$:

$$\begin{aligned} \mathcal{P}_u: \quad & \max_{\{\boldsymbol{U}_m\}_{m=1}^M} \quad \gamma \\ & \text{s.t.} \quad \mathrm{SINR}_{k,l} \ge \gamma, \quad l \in \mathcal{D}_k, \quad k \in \mathcal{K} \\ & p_{\mathrm{r},m} \le \beta_m \tilde{P}_s, \quad m \in \mathcal{M} \end{aligned} \tag{32}$$

From (10), the SINR of the lth data-stream of kth user is written as (see (33))

The matrices $\boldsymbol{C}_{1,k}^l$; $l \in \mathcal{D}_k$ and $k \in \mathcal{K}$ and $\boldsymbol{C}_{2,k}^l$; $l \in \mathcal{D}_k$ and $k \in \mathcal{K}$ are Hermitian and positive semi-definite defined as

$$\boldsymbol{C}_{1,k}^l = \tilde{\boldsymbol{G}}_k^{\mathrm{H}} (\boldsymbol{w}_k^{(l)})(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tag{34}$$

$$\boldsymbol{C}_{2,k}^l = H_k (\boldsymbol{v}_k^{(l)})(\boldsymbol{v}_k^{(l)})^{\mathrm{H}} H_k^{\mathrm{H}} \tag{35}$$

Applying the following matrix equalities $\mathrm{tr}(\boldsymbol{A}^{\mathrm{H}} \boldsymbol{B} \boldsymbol{A} \boldsymbol{C}) = (\mathrm{vec}(\boldsymbol{A}))^{\mathrm{H}} (C^{\mathrm{T}} \otimes \boldsymbol{B}) \mathrm{vec}(\boldsymbol{A})$ [24] and introducing a new variable $\tilde{\boldsymbol{u}} = \mathrm{vec}(\tilde{\boldsymbol{U}})$, the SINR of the lth data-stream of kth user in (33) is rewritten as (see (36))

Furthermore, using $\mathrm{tr}(\boldsymbol{A}^{\mathrm{H}} \boldsymbol{B} \boldsymbol{A}) = (\mathrm{vec}(\boldsymbol{A}))^{\mathrm{H}} (I \otimes \boldsymbol{B}) \mathrm{vec}(\boldsymbol{A})$ [24] and introducing a new variable $\boldsymbol{u}_m = \mathrm{vec}(\boldsymbol{U}_m)$, transmit power of mth relay node (9) is rewritten as

$$\begin{aligned} p_{\mathrm{r},m} = & \boldsymbol{u}_m^{\mathrm{H}} \left(\left(\sum_{k=1}^K \boldsymbol{H}_{m,k} \boldsymbol{V}_k \boldsymbol{V}_k^{\mathrm{H}} \boldsymbol{H}_{m,k}^{\mathrm{H}} \right)^{\mathrm{T}} \otimes \boldsymbol{I}_{N_{\mathrm{r},m}} \right) \boldsymbol{u}_m \\ & + \boldsymbol{u}_m^{\mathrm{H}} \left(\sigma_{\mathrm{r}}^2 \boldsymbol{I}_{N_{\mathrm{r},m}} \otimes \boldsymbol{I}_{N_{\mathrm{r},m}} \right) \boldsymbol{u}_m \end{aligned} \tag{37}$$

As we have different variables in (36) and (37), let us define a new variable $\boldsymbol{u} \in C^{\tilde{N}_R}$, where $\tilde{N}_R \triangleq \sum_{m=1}^M N_{r,m}^2$ and

$$\boldsymbol{u} = \begin{bmatrix} \mathrm{vec}(\boldsymbol{U}_1) \\ \vdots \\ \mathrm{vec}(\boldsymbol{U}_M) \end{bmatrix}$$

We can write $\tilde{\boldsymbol{u}} = \boldsymbol{A}\boldsymbol{u}$ where $\boldsymbol{A} \in R^{N_R^2 \times \tilde{N}_R}$ is the matrix whose elements are either one or zero. We can easily construct $\boldsymbol{A}$ by noting the non-zero elements of $\tilde{\boldsymbol{u}}$. Further, we can write $\boldsymbol{u}_m = \boldsymbol{B}_m \boldsymbol{u}$, $m \in \mathcal{M}$ where $\boldsymbol{B}_m \in R^{N_{\mathrm{r},m}^2 \times \tilde{N}_R}$ defined as $\boldsymbol{B}_m = [\boldsymbol{B}_{m,1}, \ldots, \boldsymbol{B}_{m,M}]$ where $\boldsymbol{B}_{m,m} = \boldsymbol{I}_{N_{\mathrm{r},m}^2 \times N_{\mathrm{r},m}^2}$ and $\boldsymbol{B}_{m,n} = \boldsymbol{0}_{N_{\mathrm{r},m}^2 \times N_{\mathrm{r},m}^2}$, $n \in \mathcal{M}, n \ne m$. Using these transformations, SINR of the lth data-stream of kth user in (36) is rewritten as

$$\mathrm{SINR}_{k,l} = \frac{\boldsymbol{u}^{\mathrm{H}} \boldsymbol{C}_{3,k}^l \boldsymbol{u}}{\boldsymbol{u}^{\mathrm{H}} \boldsymbol{C}_{4,k}^l \boldsymbol{u} + \sigma_d^2 ||(\boldsymbol{w}_k^{(l)})||^2} \tag{38}$$

where $\boldsymbol{C}_{3,k}^l$; $l \in \mathcal{D}_k$ and $k \in \mathcal{K}$ and $\boldsymbol{C}_{4,k}^l$; $l \in \mathcal{D}_k$ and $k \in \mathcal{K}$ are Hermitian and positive semi-definite matrices defined as

$$\boldsymbol{C}_{3,k}^l \triangleq \boldsymbol{A}^{\mathrm{T}} (((\boldsymbol{C}_{2,k}^l)^{\mathrm{T}} \otimes \boldsymbol{C}_{1,k}^l) \boldsymbol{A} \tag{39}$$

$$\boldsymbol{C}_{4,k}^l \triangleq \boldsymbol{A}^{\mathrm{T}} \left(\left(\sum_{\substack{p=1 \\ p \ne l}}^{d_k} C_{2,k}^p + \sum_{\substack{j=1 \\ j \ne k}}^K \sum_{p=1}^{d_j} C_{2,j}^p + \sigma_{\mathrm{r}}^2 \boldsymbol{I} \right)^{\mathrm{T}} \otimes \boldsymbol{C}_{1,k}^l \right) \boldsymbol{A} \tag{40}$$

Similarly, the transmit power at mth relay node in (37) is rewritten as

$$p_{\mathrm{r},m} = \boldsymbol{u}^{\mathrm{H}} \boldsymbol{C}_{5,m} \boldsymbol{u} \tag{41}$$

where $\boldsymbol{C}_{5,m}$; $m \in \mathcal{D}_m$ is a Hermitian and positive semi-definite matrix defined as

$$\boldsymbol{C}_{5,m} \triangleq \boldsymbol{B}_m^{\mathrm{T}} \left(\left(\sum_{k=1}^K \boldsymbol{H}_{m,k} \boldsymbol{V}_k \boldsymbol{V}_k^{\mathrm{H}} \boldsymbol{H}_{m,k}^{\mathrm{H}} + \sigma_{\mathrm{r}}^2 \boldsymbol{I}_{N_{\mathrm{r},m}} \right)^{\mathrm{T}} \otimes \boldsymbol{I}_{N_{\mathrm{r},m}} \right) \boldsymbol{B}_m \tag{42}$$

Using (38) and (41), the problem (32) is rewritten as

$$\mathrm{SINR}_{k,l} = \frac{\mathrm{tr}(\tilde{\boldsymbol{U}}^{\mathrm{H}} \boldsymbol{C}_{1,k}^l \tilde{\boldsymbol{U}} \boldsymbol{C}_{2,k}^l)}{\sum_{\substack{p=1 \\ p \ne l}}^{d_k} \mathrm{tr}(\tilde{\boldsymbol{U}}^{\mathrm{H}} \boldsymbol{C}_{1,k}^l \tilde{\boldsymbol{U}} C_{2,k}^p) + \sum_{\substack{j=1 \\ j \ne k}}^K \sum_{p=1}^{d_j} \mathrm{tr}(\tilde{\boldsymbol{U}}^{\mathrm{H}} \boldsymbol{C}_{1,k}^l \tilde{\boldsymbol{U}} C_{2,j}^p) + \sigma_{\mathrm{r}}^2 \mathrm{tr}(\tilde{\boldsymbol{U}}^{\mathrm{H}} \boldsymbol{C}_{1,k}^l \tilde{\boldsymbol{U}}) + \sigma_d^2 ||(\boldsymbol{w}_k^{(l)})||^2} \tag{33}$$

$$\mathrm{SINR}_{k,l} = \frac{\tilde{\boldsymbol{u}}^{\mathrm{H}} ((\boldsymbol{C}_{2,k}^l)^{\mathrm{T}} \otimes \boldsymbol{C}_{1,k}^l) \tilde{\boldsymbol{u}}}{\sum_{\substack{p=1 \\ p \ne l}}^{d_k} \tilde{\boldsymbol{u}}^{\mathrm{H}} ((C_{2,k}^p)^{\mathrm{T}} \otimes \boldsymbol{C}_{1,k}^l) \tilde{\boldsymbol{u}} + \sum_{\substack{j=1 \\ j \ne k}}^K \sum_{p=1}^{d_j} \tilde{\boldsymbol{u}}^{\mathrm{H}} ((C_{2,j}^p)^{\mathrm{T}} \otimes \boldsymbol{C}_{1,k}^l) \tilde{\boldsymbol{u}} + \sigma_{\mathrm{r}}^2 \tilde{\boldsymbol{u}}^{\mathrm{H}} (\boldsymbol{I} \otimes \boldsymbol{C}_{1,k}^l) \tilde{\boldsymbol{u}} + \sigma_d^2 ||(\boldsymbol{w}_k^{(l)})||^2} \tag{36}$$

$$\mathscr{P}_u: \max_{\boldsymbol{u}} \quad \gamma$$

$$\text{s.t.} \quad \frac{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{C}_{3,k}^{l}\boldsymbol{u}}{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{C}_{4,k}^{l}\boldsymbol{u}+\sigma_d^2||(\boldsymbol{w}_k^{(l)})||^2} \geq \gamma; \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K}$$
$$\boldsymbol{u}^{\mathrm{H}}\boldsymbol{C}_{5,m}\boldsymbol{u} \leq \beta_m \tilde{P}_s; \quad m \in \mathscr{M} \tag{43}$$

Problem (43) is a fractional quadratic constraint quadratic program problem which can be approximated into a semi-definite programming (SDP) problem using semi-definite relaxation [19]. However, the resulting relaxed SDP problem substantially increases computational complexity of solving (43). Let $\boldsymbol{C}_{3,k}^{l} = \boldsymbol{c}_{3,k}^{l}(\boldsymbol{c}_{3,k}^{l})^{\mathrm{H}}$ because the matrix $\boldsymbol{C}_{3,k}^{l}$ is rank-one Hermitian and positive semi-definite matrix. The SINR constraint in problem (43) is rewritten as

$$\left\| \begin{matrix} \sigma_d||(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}|| \\ (\boldsymbol{C}_{4,k}^{l})^{1/2}\boldsymbol{u} \end{matrix} \right\| \leq \sqrt{\frac{1}{\gamma}}|\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}|; \quad l \in \mathscr{D}_k, \ k \in \mathscr{K} \tag{44}$$

It is clear that the non-convex constraints in (44) can be turned into convex if we have

$$|\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}| = \mathbb{R}\mathrm{e}\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}\}; \quad l \in \mathscr{D}_k, \ k \in \mathscr{K} \tag{45}$$

The main difficulty of using the phase rotation technique, which is applied in the case of transmitter precoders design, here is that $\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}\}$ for $l \in \mathscr{D}_k, k \in \mathscr{K}$ are coupled with each other via $\boldsymbol{u}$. Although we cannot make all of these real-valued, we can obtain a convex approximation of the SINR constraints as follows. Since we have

$$|\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}| \geq \mathbb{R}\mathrm{e}\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}\} \tag{46}$$

We can write the optimisation problem by strengthening the SINR constraints as (see (47))

The relay matrices design sub-problem $\mathscr{P}_u^c$ for a given γ is an SOCP problem, where $\mathscr{P}_u^c$ is a conservative form of $\mathscr{P}_u$ obtained by strengthening the SINR constraints. However, in $\mathscr{P}_u^c$, the joint optimisation over γ and $\boldsymbol{u}$ is a quasi-convex problem [26], which can be solved using bisection search over γ and convex feasibility checking over $\boldsymbol{u}$. This feasibility checking problem is an SOCP problem. The problem $\mathscr{P}_u^c$ is always feasible for optimisation variables obtained after transmit precoder design step as we force the $\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}\} = (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)} \geq 0$; $l \in \mathscr{D}_k, k \in \mathscr{K}$ in solving (26) to non-negative real.

Therefore, the exact solution of problem $\mathscr{P}_u^c$ (47) requires solving a sequence of SOCP problems, which has a high computational cost. This motivates us to find a more practical method to solve problem $\mathscr{P}_u$ (32) with moderate computational complexity. By this motivation, we solve the problem $\mathscr{P}_u$ (32) by solving the QoS constrained power minimisation problem $\mathscr{Q}_u$. The problem $\mathscr{Q}_u$ is written as

Table 2 Procedure for solving $\mathscr{P}_u$ (32) through $\mathscr{Q}_u$ (48) with fixed $\{\boldsymbol{V}_k^*\}_{k=1}^K, \{\boldsymbol{W}_k^*\}_{k=1}^K$. Initialise $\{\boldsymbol{U}_m^*\}_{m=1}^M$ such that it satisfy transmit power constraints

Repeat

1. **S1:** Compute minimum SINR $\gamma_{\min} = \min_{l \in \mathscr{D}_k, k \in \mathscr{K}} \mathrm{SINR}_{k,l}(\{\boldsymbol{V}_k^*\}_{k=1}^K, \{\boldsymbol{U}_m^*\}_{m=1}^M, \{\boldsymbol{W}_k^*\}_{k=1}^K)$
2. **S2:** Compute the relay precoding matrices $\{\boldsymbol{U}_m\}_{m=1}^M$ and $\tilde{P}_u$ with fixed $\{\boldsymbol{V}_k^*\}_{k=1}^K$ and $\{\boldsymbol{W}_k^*\}_{k=1}^K$ by solving the SOCP optimisation problem $\mathscr{Q}_u(\tilde{\gamma})$ (48) with $\tilde{\gamma} = \gamma_{\min}$.
3. **S3:** Since $\{\tilde{P}_s, \{\boldsymbol{U}_m^*\}_{m=1}^M\}$ is a feasible solution for (48), $\tilde{P}_u \leq \tilde{P}_s$.
   Update the relay matrices as $\boldsymbol{U}_m^* = \sqrt{(\tilde{P}_s/\tilde{P}_u)}\boldsymbol{U}_m$.

Until $\tilde{P}_u = \tilde{P}_s$

$$\mathscr{Q}_u: \min_{\{\boldsymbol{U}_m\}_{m=1}^M} \quad \tilde{P}_u$$
$$\text{s.t.} \quad \mathrm{SINR}_{k,l} \geq \gamma, \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K} \tag{48}$$
$$p_{\mathrm{r},m} \leq \beta_m \tilde{P}_u, \quad m \in \mathscr{M}$$

Similar to the transmitter precoder design sub-problem, the following results show the relationship between the problems $\mathscr{P}_u$ and $\mathscr{Q}_u$.

Claim 3: The problem $\mathscr{P}_u$ and problem $\mathscr{Q}_u$ are related as follows:

$$\tilde{\gamma} = \mathscr{P}_u(\mathscr{Q}_u(\tilde{\gamma})) \tag{49}$$

$$\tilde{P} = \mathscr{Q}_u(\mathscr{P}_u(\tilde{P})) \tag{50}$$

Claim 4: The optimum objective values of the problem $\mathscr{Q}_u(\tilde{\gamma})$ and problem $\mathscr{P}_v(\tilde{P})$ are monotonically non-decreasing in $\tilde{\gamma}$ and $\tilde{P}$, respectively,

$$\gamma_1 > \gamma_2 \Rightarrow \mathscr{Q}_u(\gamma_1) > \mathscr{Q}_u(\gamma_2) \tag{51}$$

$$P_1 > P_2 \Rightarrow \mathscr{P}_u(P_1) > \mathscr{P}_u(P_2) \tag{52}$$

Proof: Similar to the proof of claims 1 and 2. □

From (49) and (50), we see that the two problems $\mathscr{P}_u$ and $\mathscr{Q}_u$ are inverse problems. Therefore, we can find an optimal solution for the problem $\mathscr{P}_u$ for a given $\tilde{P}_s$ by iteratively solving $\mathscr{Q}_u(\tilde{\gamma})$ for varying values of $\tilde{\gamma}$. When $\tilde{P}_s = \mathscr{Q}_u(\tilde{\gamma})$, then $\tilde{\gamma}$ will be the optimal solution for $\mathscr{P}_u(\tilde{P}_s)$. The procedure for solving $\mathscr{P}_u$ for a given $\tilde{P}_s$ through $\mathscr{Q}_u$ is outlined in Table 2.

From (51), we see that we can solve the relay matrices design sub-problem $\mathscr{P}_u$ (32) with fixed receiver filters and transmitter precoders by iteratively solving the problem $\mathscr{Q}_u$. When $\tilde{P}_u = \tilde{P}_s$, $\{\boldsymbol{U}_m^*\}_{m=1}^M$ and $\gamma_{\min}$ will be the optimal solution and associated optimal value of (32). Since $\mathscr{P}_u$ and $\mathscr{Q}_u$ are non-convex, we instead solve $\mathscr{P}_u^c$ using $\mathscr{Q}_u^c$, which being an SOCP problem is efficiently solved using interior point methods. We can obtain $\mathscr{Q}_u^c$ by

$$\mathscr{P}_u^c: \max_{\boldsymbol{u}} \quad \gamma$$
$$\text{s.t.} \quad \left\| \begin{matrix} \sigma_d||(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}|| \\ (\boldsymbol{C}_{4,k}^{l})^{1/2}\boldsymbol{u} \end{matrix} \right\| \leq \sqrt{\frac{1}{\gamma}}\mathbb{R}\mathrm{e}\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{c}_{3,k}^{l}\}; \quad l \in \mathscr{D}_k, \quad k \in \mathscr{K} \tag{47}$$
$$\|(\boldsymbol{C}_{5,m})^{1/2}\boldsymbol{u}\| \leq \sqrt{\beta_m \tilde{P}_s}; \quad m \in \mathscr{M}$$

strengthening the SINR constraints of $\mathcal{Q}_u$ (48) similarly as we obtained $\mathcal{P}_u^c$ from $\mathcal{P}_u$ in (47). We note that a similar inverse relation exists between $\mathcal{P}_u^c$ and $\mathcal{Q}_u^c$ as well and $\mathcal{P}_u^c$ can be solved by iteratively solving the $\mathcal{Q}_u^c$. However, this will also require solving a sequence of SOCPs. Therefore, we inexactly update the relay matrices instead of exact maximisation of the minimum SINR over relay matrices block. Specifically, in each iteration of the proposed algorithm, we perform steps **S1**–**S3** in Table 2 only once to reduce the computational cost of updating the relay precoding matrices.

4.4 Convergence and complexity analysis

In this subsection, we present the complete algorithm and discuss its convergence and computational complexity. The proposed algorithm solves three design sub-problems namely receiver filters design, transmitter precoders design and relay precoding matrices design in a cyclic order in each iteration as summarised in Algorithm 1.

Theorem 1: The sequence of objective values generated by the proposed algorithm, Algorithm 1 is convergent.

Proof: We introduce the following compact notations for convenience: $\boldsymbol{W} \triangleq \{\boldsymbol{W}_k, \text{for } k \in \mathcal{K}\}$, $\boldsymbol{V} \triangleq \{\boldsymbol{V}_k, \text{for } k \in \mathcal{K}\}$ and $\boldsymbol{U} \triangleq \{\boldsymbol{U}_m, \text{for } m \in \mathcal{M}\}$. Let us define $\text{SINR}_{\min} = \min_{l \in \mathcal{D}_k, k \in \mathcal{K}} \text{SINR}_{k,l}$, which denotes the minimum per stream SINR among all the users, i.e. objective function in (12).
First, we show that the sequence $\{\text{SINR}_{\min}(\boldsymbol{W}^n, \boldsymbol{V}^n, \boldsymbol{U}^n)\}_{n=1}^{\infty}$ is monotonically non-decreasing.
In **Step 2**, for the given $\boldsymbol{V}^n$ and $\boldsymbol{U}^n$, the optimal solution of the optimisation problem (14) is given by (16). Hence $\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \boldsymbol{V}^n, \boldsymbol{U}^n) \geq \text{SINR}_{\min}(\boldsymbol{W}^n, \boldsymbol{V}^n, \boldsymbol{U}^n)$. In **Step 3.2**, for the given $\boldsymbol{W}^{n+1}$ and $\boldsymbol{U}^n$ the transmitter precoders $\{\boldsymbol{V}_k\}_{k=1}^K$ are computed to jointly reduce the transmit power of all the transmitter and relay nodes, Hence $\text{SINR}_{\min}$ remains unchanged. This holds because at the optimality, SINR of at least one stream must be equal to $\text{SINR}_{\min}$ otherwise we can further reduce the objective without violating the SINR constraints.

$$\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \boldsymbol{V}, \boldsymbol{U}^n) = \text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \boldsymbol{V}^n, \boldsymbol{U}^n)$$

In **Step 3.3**, the transmitter precoders $\{\boldsymbol{V}_k\}_{k=1}^K$ are upscaled to $\{\tilde{\boldsymbol{V}}_k\}_{k=1}^K$ by same constant $\left(\sqrt{\tilde{P}_s/\tilde{P}_v}\right)$ and $\tilde{P}_s \geq \tilde{P}_v$. Hence $\text{SINR}_{k,l}$; $l \in \mathcal{D}_k$, $k \in \mathcal{K}$ is increased as follows: (see equation below) Therefore $\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \boldsymbol{U}^n) \geq \text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \boldsymbol{V}, \boldsymbol{U}^n)$.

In **Step 4.2**, for the given $\boldsymbol{W}^{n+1}$ and $\tilde{\boldsymbol{V}}$ the relay precoding matrices $\{\boldsymbol{U}_m\}_{m=1}^M$ are computed to jointly reduce the transmit power of all relay nodes, Hence $\text{SINR}_{\min}$ remains unchanged as in **Step 3.2**.

$$\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \boldsymbol{U}) = \text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \boldsymbol{U}^n)$$

In **Step 4.3**, the relay precoding matrices $\{\boldsymbol{U}_m\}_{m=1}^M$ are upscaled to $\{\tilde{\boldsymbol{U}}_m\}_{m=1}^M$ by same constant $\left(\sqrt{\tilde{P}_s/\tilde{P}_u}\right)$ and $\tilde{P}_s \geq \tilde{P}_u$. Hence $\text{SINR}_{k,l}$; $l \in \mathcal{D}_k$, $k \in \mathcal{K}$ is increased similarly as in the case of upscaling the transmitter precoders (**Step 3.3**). Therefore

$$\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \tilde{\boldsymbol{U}}) \geq \text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \boldsymbol{U})$$

In **Step 5**, the transmitter precoders and relay precoding matrices are updated, therefore

$$\text{SINR}_{\min}(\boldsymbol{W}^{n+1}, \boldsymbol{V}^{n+1}, \boldsymbol{U}^{n+1}) \geq \text{SINR}_{\min}(\boldsymbol{W}^n, \boldsymbol{V}^n, \boldsymbol{U}^n)$$

Therefore in each iteration of the proposed max–min SINR algorithm, the objective function of problem (12) is non-decreasing.
Since $\|\boldsymbol{V}_k\|_F \leq P_{s,k}$, $\|\boldsymbol{W}_k\|_F \leq d_k$ and $\|\boldsymbol{U}_m\|_F \leq P_{\mathrm{r},m}$ for $k \in \mathcal{K}, m \in \mathcal{M}$, the $\|\boldsymbol{V}_k\|_F, \|\boldsymbol{W}_k\|_F, \|\boldsymbol{U}_m\|_F$ are bounded. Therefore, the sequences $\{\boldsymbol{V}^n\}$, $\{\boldsymbol{W}^n\}$ and $\{\boldsymbol{U}^n\}$ must have convergent subsequences. Let $\{\boldsymbol{V}^{n_t}\}_{t=1}^{\infty}$, $\{\boldsymbol{W}^{n_t}\}_{t=1}^{\infty}$ and $\{\boldsymbol{U}^{n_t}\}_{t=1}^{\infty}$ are the subsequences converging to $\bar{\boldsymbol{V}}$, $\bar{\boldsymbol{W}}$ and $\bar{\boldsymbol{U}}$, respectively. We have

$$\begin{aligned} &\lim_{t \to \infty} \text{SINR}_{\min}(\boldsymbol{W}^{n_t}, \boldsymbol{V}^{n_t}, \boldsymbol{U}^{n_t}) \\ &= \text{SINR}_{\min}(\lim_{t \to \infty} (\boldsymbol{W}^{n_t}, \boldsymbol{V}^{n_t}, \boldsymbol{U}^{n_t})) \\ &= \text{SINR}_{\min}(\bar{\boldsymbol{W}}, \bar{\boldsymbol{V}}, \bar{\boldsymbol{U}}) \\ &\triangleq \overline{\text{SINR}}_{\min} \end{aligned} \tag{53}$$

Here the objective function of problem (13) is continuous, we can take the limit inside the objective function. Since $\{\text{SINR}_{\min}(\boldsymbol{W}^n, \boldsymbol{V}^n, \boldsymbol{U}^n)\}_{n=1}^{\infty}$ is monotonically non-decreasing $\lim_{n \to \infty} \text{SINR}_{\min}(\boldsymbol{W}^n, \boldsymbol{V}^n, \boldsymbol{U}^n) = \overline{\text{SINR}}_{\min}$. □

Now, we analyse the worst-case complexity of Algorithm 1. The computational cost of each iteration of Algorithm 1 is primarily from solving two SOCPs, i.e. $\mathcal{Q}_v^{\mathrm{r}}$ (31) and $\mathcal{Q}_u^c$ (48). According to [25], an SOCP problem can be efficiently solved using primal-dual interior point method at worst-case complexity of the order of $\mathcal{O}(N_{\text{opt}}^2 D_{\text{soc}})$, where N_{opt} and D_{soc} is the dimension of the optimisation variable and total second-order-cone (SOC) constraints, respectively. Assuming $N_{s,k} = N_s \ \forall k \in \mathcal{K}$ and

$$\begin{aligned} \text{SINR}_{k,l}(\boldsymbol{W}^{n+1}, \tilde{\boldsymbol{V}}, \boldsymbol{U}^n) &= \frac{|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \tilde{\boldsymbol{v}}_k^{(l)}|^2}{\sum_{p=1, p \neq l}^{d_k} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \tilde{\boldsymbol{v}}_k^{(p)}|^2 + \sum_{j=1, j \neq k}^{K} \sum_{p=1}^{d_j} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \tilde{\boldsymbol{v}}_j^{(p)}|^2 + \sigma_{\mathrm{r}}^2 \|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}}\|^2 + \sigma_d^2 \|(\boldsymbol{w}_k^{(l)})\|^2} \\ &= \frac{(\tilde{P}_s/\tilde{P}_v) |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)}|^2}{\sum_{p=1, p \neq l}^{d_k} (\tilde{P}_s/\tilde{P}_v) |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(p)}|^2 + \sum_{j=1, j \neq k}^{K} \sum_{p=1}^{d_j} (\tilde{P}_s/\tilde{P}_v) |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)}|^2 + \sigma_{\mathrm{r}}^2 \|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}}\|^2 + \sigma_d^2 \|(\boldsymbol{w}_k^{(l)})\|^2} \\ &= \frac{|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)}|^2}{\sum_{p=1, p \neq l}^{d_k} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(p)}|^2 + \sum_{j=1, j \neq k}^{K} \sum_{p=1}^{d_j} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)}|^2 + \dfrac{\sigma_{\mathrm{r}}^2 \|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}}\|^2 + \sigma_d^2 \|(\boldsymbol{w}_k^{(l)})\|^2}{\tilde{P}_s/\tilde{P}_v}} \\ &\geq \frac{|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)}|^2}{\sum_{p=1, p \neq l}^{d_k} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(p)}|^2 + \sum_{j=1, j \neq k}^{K} \sum_{p=1}^{d_j} |(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)}|^2 + \sigma_{\mathrm{r}}^2 \|(\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}}\|^2 + \sigma_d^2 \|(\boldsymbol{w}_k^{(l)})\|^2} \\ &= \text{SINR}_{k,l}(\boldsymbol{W}^{n+1}, \boldsymbol{V}, \boldsymbol{U}^n) \end{aligned}$$

$N_{r,m} = N_r$, $\forall m \in \mathcal{M}$ and $d_k = d$, $\forall d \in \mathcal{K}$, the dimension of optimisation variable in (31) is $KN_s d$ and total dimension of all SOC constraints is $K^2d^2 + KN_s d + MKdN_r$. Therefore the worst-case complexity of solving SOCP based transmitter precoders design problem (31) is approximately $\mathcal{O}(K^4N_s^3d^3 + K^3MN_s^3N_r d^3)$. Similarly, the worst-case complexity of solving SOCP based relay precoding matrices design problem (48) is approximately $\mathcal{O}(M^4N_r^6 + KM^3N_r^6 d)$.

Algorithm 1: Proposed inexact BCU based max–min SINR algorithm

Inputs: $\{P_{s,k}\}_{k=1}^K$, $\{P_{r,m}\}_{m=1}^M$. Let $\tilde{P}_s = \min(P_{s,1}, \ldots, P_{s,K}, P_{r,1}, \ldots, P_{r,M})$ $P_{s,k} = \alpha_k \tilde{P}_s$ where $\alpha_k \geq 1$; $k \in \mathcal{K}$ and $P_{r,m} = \beta_m \tilde{P}_s$ where $\beta_m \geq 1$; $m \in \mathcal{M}$.

Step 1: Randomly initialise $\{V_k^0\}_{k=1}^K$, $\{U_m^0\}_{m=1}^M$. Set $n=0$ and define maximum number of iterations $N_{\max}$.

Step 2: Compute the receiver filters $\{W_k^{n+1}\}_{k=1}^K$ with fixed transmitter precoders $\{V_k^n\}_{k=1}^K$ and relay precoding matrices $\{U_m^n\}_{m=1}^M$ using (16).

Step 3.1: Compute $\gamma_{\min} = \min_{l \in \mathcal{D}_k, k \in \mathcal{K}} \mathrm{SINR}_{k,l}(\{V_k^n\}_{k=1}^K, \{U_m^n\}_{m=1}^M, \{W_k^{n+1}\}_{k=1}^K)$.

Step 3.2: Compute the transmitter precoder matrices $\{V_k\}_{k=1}^K$ and $\tilde{P}_v$ with fixed relay precoding matrices $\{U_m^n\}_{m=1}^M$ and receiver filters $\{W_k^{n+1}\}_{k=1}^K$ by solving the SOCP problem $\mathcal{Q}_v^r(\tilde{\gamma})$ (31) with $\tilde{\gamma} = \gamma_{\min}$.

Step 3.3: Since $\tilde{P}_v \leq \tilde{P}_s$, upscale the transmitter precoders $\{V_k\}_{k=1}^K$ $\tilde{V}_k = \sqrt{(\tilde{P}_s/\tilde{P}_v)}V_k$; $\forall k \in \mathcal{K}$. –

Step 4.1: Update minimum SINR $\gamma_{\min}$ as

$$\gamma_{\min} = \min_{\substack{l \in \mathcal{D}_k \\ k \in \mathcal{K}}} \mathrm{SINR}_{k,l}(\{\tilde{V}_k\}_{k=1}^K, \{U_m^n\}_{m=1}^M, \{W_k^{n+1}\}_{k=1}^K)$$

Step 4.2: Compute the relay precoding matrices $\{U_m\}_{m=1}^M$ and $\tilde{P}_u$ with fixed transmitter precoding matrices $\{\tilde{V}_k\}_{k=1}^K$ and receiver filters $\{W_k^{n+1}\}_{k=1}^K$) by solving the SOCP problem $\mathcal{Q}_u^c(\tilde{\gamma})$ with $\tilde{\gamma} = \gamma_{\min}$.

Step 4.3: Since $\tilde{P}_u \leq \tilde{P}_s$, upscale the relay precoding matrices $\{U_m\}_{m=1}^M$ and update minimum SINR $\gamma_{\min}$

$$\tilde{U}_m = \sqrt{\frac{\tilde{P}_s}{\tilde{P}_u}}U_m; \quad \forall m \in \mathcal{M}$$

$$\gamma_{\min} = \min_{\substack{l \in \mathcal{D}_k \\ k \in \mathcal{K}}} \mathrm{SINR}_{k,l}(\{\tilde{V}_k\}_{k=1}^K, \{\tilde{U}_m\}_{m=1}^M, \{W_k^{n+1}\}_{k=1}^K$$

Step 5: Update the transmitter precoders $V_k^{n+1} = \tilde{V}_k$; $k \in \mathcal{K}$ and relay precoding matrices $U_m^{n+1} = \tilde{U}_m$; $m \in \mathcal{M}$.

Step 6: Compute $J_n = \gamma_{\min}$, where J is the objective function in problem (12) and n is the iteration index. Test the stopping criterion $(J_n - J_{n-1})/(\max\{J_{n-1}, 1\}) \leq \epsilon$. If stopping criterion is satisfied or maximum number of iterations reached, terminate the algorithm, else $n = n + 1$ and go to **Step 2**.

5 Simulation results

In this section, simulation results are presented to demonstrate the performance of the algorithm developed for the joint transceiver and relay design for MIMO interference AF relay system. We consider a symmetric network described as $(N_d \times N_s, d)^K + N_r^M$ where $N_{s,k} = N_s$, $N_{d,k} = N_d$, $d_k = d$; $k \in \mathcal{K}$ and $N_{r,m} = N_r$; $m \in \mathcal{M}$. The entries of all the channel matrices $\{H_{m,k}\}$ and $\{G_{k,m}\}$ are independent and identically distributed (i.i.d) complex Gaussian random variables with mean zero and variance one. We assume that entries of all noise vectors are i.i.d complex Gaussian random variables with mean zero and variance one, i.e. $\sigma_r = \sigma_d = \sigma = 1$. So the SNR at a link is defined as $\mathrm{SNR} = P/\sigma^2 = P$ where P is the maximum power allowed at any node in the network, i.e. $P_{s,k} = P$ for $k \in \mathcal{K}$ and $P_{r,m} = P$ for $m \in \mathcal{M}$. Thus we assume the same SNR for all the transmitter–relay and relay–receiver links. We average all the results over randomly generated 200 channel matrices. The stopping criterion parameter ϵ is set as 10^{-3}. The maximum allowable iterations for terminating the algorithms is $N_{\max} = 300$. We use disciplined convex programming toolbox CVX for solving the convex optimisation problems [29].

We use the following benchmarks for comparing the performance of the proposed Algorithm 1.

1. Minimum per-stream SINR maximisation based solution (max–min SINR (exact)): This algorithm attempts to solve the max-min SINR based joint transceiver and relay design problem. However, the transmitter precoders (18) and relay matrices design sub-problems (47) at each iteration are solved either using bisection search and a sequence of SOCP feasibility problems or by iteratively solving their corresponding inverse problems (as explained in Tables 1 and 2).
2. Sum interference and noise leakage minimisation based solution (min leakage): This algorithm attempts to solve the joint transceiver and relay design problems to minimise the sum leakage of interference and noise signals in the desired signal space at each receiver subject to sum power constraints at the transmitter and relay nodes [19].
3. Maximum per-stream MSE minimisation based solution (min–max MSE): This algorithm attempts to solve the joint transceiver and relay design problems to minimise the maximum per-stream MSE subject to transmit power constraints at each transmitter and relay node [23].
4. Sum-MSE minimisation based solution (min sum-MSE): This algorithm attempts to solve the joint transceiver and relay design problems to minimise the system wide sum-MSE of all the users subject to transmit power constraints at each transmitter and relay node. This algorithm has been adapted from [22, 30].
5. Minimum per-user SINR maximisation based non-iterative solution (max–min SINR Noiter): This algorithm has been adapted from [31] which designs the relay matrices using the SDR technique and Gaussian randomisation to maximise the minimum per-user SINR in a cognitive relay network. Since we do not have any constraint on interference to primary user in our system model, for fair comparison, we use the channel matched beamformers, i.e. the transmitter and receiver beamforming vectors for kth user are properly scaled left and right singular vectors corresponding to the d_k dominant singular values of the equivalent channel matrix between kth source and kth destination $T_{k,k}$ in (6).

Fig. 2 plots the values of minimum SINR per stream among all the users achieved by the proposed algorithm for a random channel realisation of the $(2 \times 4, 1)^4 + 2^4$ system at SNR = 25 dB. We also plot the minimum SINR per stream achieved by the max–min SINR (exact) algorithm. It is observed that the minimum per-stream SINR values are, as expected, non-decreasing over the iterations. In convergence performance comparisons, we also observed that the convergence speed of the max–min SINR (exact) algorithm is quite slow, especially at moderate to high SNR, whereas in the proposed algorithm, most of the improvement in the SINR occurs in the first few iterations. This property makes the proposed algorithm attractive for practical implementation.

Figs. 3 and 4 illustrate the average end-to-end minimum user-rate versus SNR for the $(2 \times 4, 1)^4 + 2^4$ and $(4 \times 4, 2)^3 + 4^3$ systems,

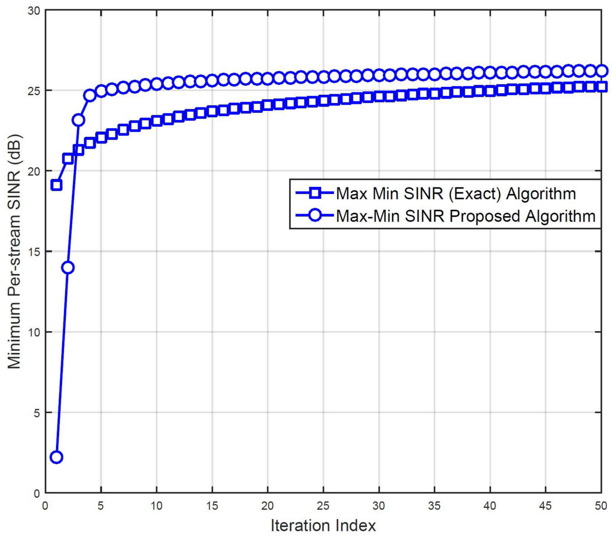

Fig. 2 *Convergence behaviour of Algorithm 1 for a random channel realisation of a* $(2 \times 4, 1)^4 + 2^4$ *system*

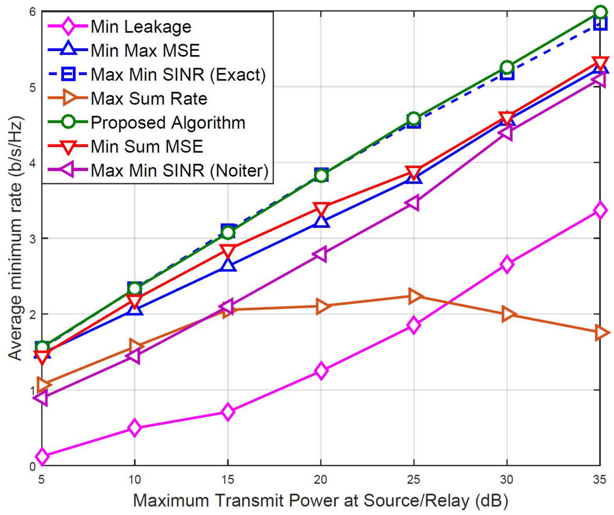

Fig. 3 *Average minimum user-rate versus SNR for a* $(2 \times 4, 1)^4 + 2^4$ *system*

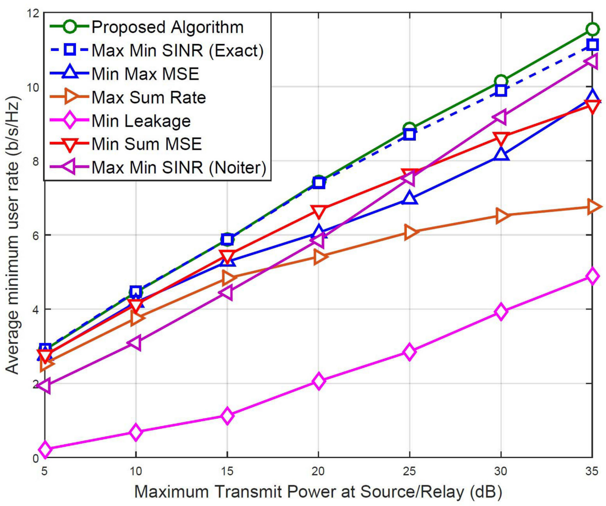

Fig. 4 *Average minimum user-rate versus SNR for a* $(4 \times 4, 2)^3 + 4^3$ *system*

respectively. We observe that the proposed max–min SINR algorithm and max–min SINR (exact) yield almost the same average performance in terms of minimum user-rate. Actually, the performance of the proposed algorithm in our simulations is slightly better than the max–min SINR (exact) algorithm for large

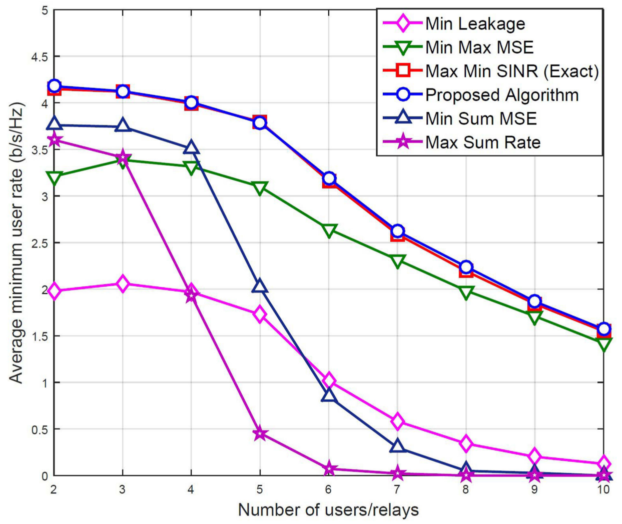

Fig. 5 *Average minimum user rate versus number of users/relays for a* $(2 \times 2, 1)^K + 2^K$ *system*

SNR values. This is because the convergence speed of the max–min SINR (exact) algorithm is quite slow, especially at high SNR and it may take a large number of iterations to converge. Moreover, the proposed Algorithm 1 outperforms all other benchmarks in terms of average minimum user-rate in all SNR regions for both the system configurations. This is because the proposed algorithm improves fairness among all the users' data-streams such that all the data-streams in the network achieve almost same received SINR.

Next, we consider an application scenario with a large number of users/relays where as the number of antennas at each node is small due to size constraints. Fig. 5 illustrates the average minimum user-rate for the $(2 \times 2, 1)^K + 2^K$ system over increasing values of K. We observe that the proposed Algorithm 1 achieves similar performance as that of the max–min SINR (exact) and outperform all other benchmarks in terms of average minimum user-rate for all values of K.

Next, we evaluate the performance of the proposed algorithm in a network where the number of users and relays are different. Fig. 6 shows the average minimum user-rate versus SNR for the $(2 \times 2, 1)^4 + 2^M$ system (i.e. $K = 4$) and different values of M. We observe that the average minimum user-rate of Algorithm 1 can be improved by increasing the number of relays in the system. Fig. 7 shows the average minimum user-rate versus SNR for the $(2 \times 2, 1)^K + 2^4$ system, i.e. ($M = 4$) with different values of K. It is observed that the average minimum user-rate of Algorithm 1 deteriorates as the number of users in the system increases with fixed number of relays. This is because the inter user interference in the system increases with the number of users in the system.

Fig. 8 compares the average CPU-time of the proposed max–min SINR algorithm and the max–min SINR (exact) algorithm. We can observe that the proposed algorithm considerably outperforms the max–min SINR (exact) algorithm. This is not surprising since in each iteration of the max–min SINR (exact) algorithm, a sequence of SOCPs need to be solved for transmitter precoder design sub-problem and relay matrices design sub-problem while only one SOCP is solved for updating transmitter precoders and relay matrices in each iteration of the proposed algorithm. Therefore, we see that the proposed algorithm yields similar performance at a substantially reduced computational cost than that of max–min SINR (exact) algorithm.

Next, we compare the proposed algorithm with MIMO interference DF relay system where each transmitter–receiver pair communicate with the help of a dedicated decode and forward (DF) relay node. We design the transceiver independently on each hop, i.e. transmitter–DF relay and DF relay–receiver according to the following MIMO interference channel solutions. (i) Total interference-leakage minimisation based design [32], (ii) max-SINR based design [32], (iii) maximum per-stream MSE

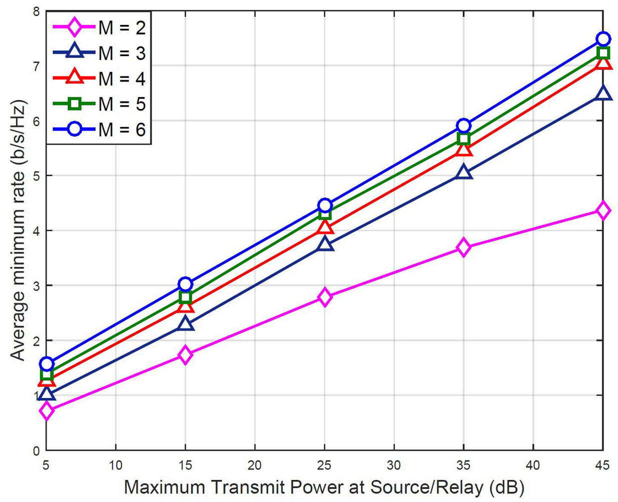

Fig. 6 *Average minimum user rate versus SNR for a* $(2 \times 2, 1)^4 + 2^M$ *system*

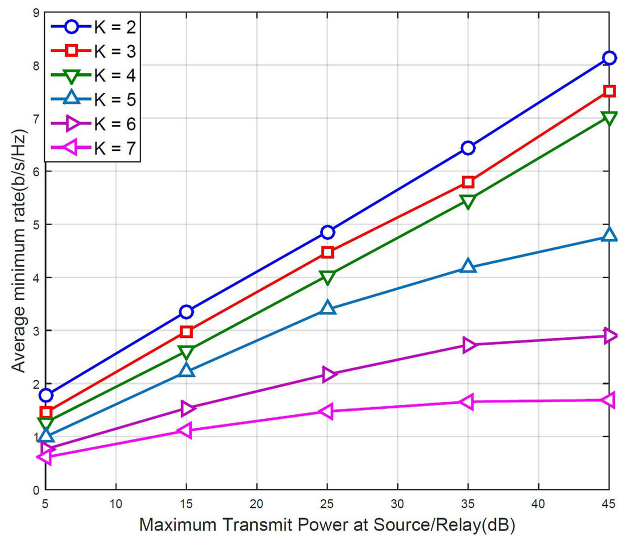

Fig. 7 *Average minimum user rate versus SNR for a* $(2 \times 2, 1)^K + 2^4$ *system*

minimisation based design [33] and (iv) minimum per-stream SINR maximisation based design [34]. The end-to-end rate of *k*th user in an interference DF relay system is computed as half of the minimum between rates of the *k*th transmitter – *k*th DF relay link and *k* DF relay – *k*th receiver link. We simulate the following three strategies for the interference AF relay system. (i) Time division multiple access based beamforming in which all the AF relays assist *k*th transmitter–receiver pair at time-slot *k*. The transmitter precoder, relay precoding matrices and receiver filter are jointly designed using MMSE and max-SINR criteria, respectively, (ii) maximum per-stream MSE minimisation and (iii) the proposed max–min SINR based solution. Fig. 9 illustrates the average end-to-end minimum user-rate versus SNR for the $(2 \times 2, 1)^4 + 2^4$ system. We observe that the proposed algorithm outperforms all the other strategies in all the SNR regions. For the DF relay case, interference-leakage minimisation and max-SINR algorithms perform worse because IA is infeasible for the considered systems on either hop [10]. Min–max MSE and max–min SINR algorithms attempt to provide certain QoS for each user. For the AF relay case, AF-TDMA distributed beamforming does not encounter co-channel interference but is inefficient in the proper utilisation of available resources.

Finally, Fig. 10 illustrates the average BER versus SNR for the $(2 \times 4, 1)^4 + 2^4$ system. For each channel realisation, 5000 independent quadrature phase shift keying symbols per user are transmitted to obtain the averaged BER performance. We observe that the proposed max–min SINR based algorithm has better BER

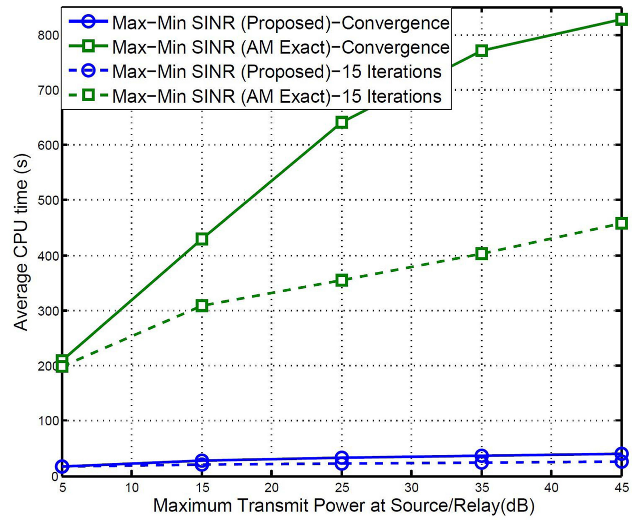

Fig. 8 *Average CPU time versus SNR for a* $(2 \times 2, 1)^4 + 2^4$ *system*

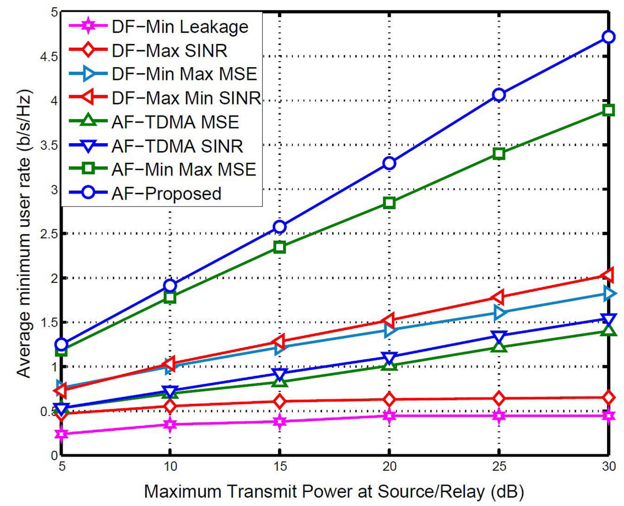

Fig. 9 *Average minimum user rate versus SNR for a* $(2 \times 2, 1)^4 + 2^4$ *system*

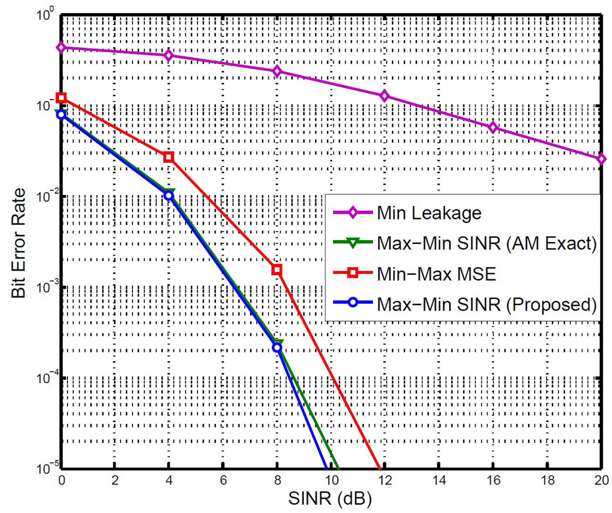

Fig. 10 *Average BER versus SNR for a* $(2 \times 4, 1)^4 + 2^4$ *system*

performance than MSE and leakage-minimisation based designs. In terms of BER, the min-leakage algorithm performs worst there is no diversity gain [35].

6 Conclusions

In this paper, we consider the max–min fairness based joint transceiver–relay design problem for the MIMO interference AF relay channel. We have presented an iterative algorithm based on inexact BCU method to jointly optimise the receiver filters, transmit precoders and relay precoding matrices for all the users and relays such that the minimum SINR per stream improves at each iteration. Simulation results show that the proposed algorithm has much lower computational complexity than the exact alternating minimisation based approach and achieves a higher average minimum rate than the benchmark strategies.

While relays have potential benefits in a MIMO interference channel, some limitations need to be tackled for their implementation in a realistic environment. Acquisition of global and perfect channel knowledge at a central processing unit would require large CSI acquisition overhead and is challenging in practice. Developing algorithms which reduces this overhead and are robust to CSI errors are topics for future work.

7 References

[1] Yang, Y., Hu, H., Xu, J., *et al.*: 'Relay technologies for WiMax and LTE-advanced mobile systems', *IEEE Commun. Mag.*, 2009, **47**, (10), pp. 100–105

[2] Sendonaris, A., Erkip, E., Aazhang, B.: 'User cooperation diversity – Part I: System description', *IEEE Trans. Commun.*, 2003, **51**, (11), pp. 1927–1938

[3] Tse, D., Viswanath, P.: '*Fundamentals of wireless communication*' (Cambridge University Press, Cambridge, 2005)

[4] Laneman, J.N., Tse, D.N.C., Wornell, G.W.: 'Cooperative diversity in wireless networks: efficient protocols and outage behavior', *IEEE Trans. Inf. Theory*, 2004, **50**, (12), pp. 3062–3080

[5] Gou, T., Jafar, S.A.: 'Degrees of freedom of the K user $M \times N$ MIMO interference channel', *IEEE Trans. Inf. Theory*, 2010, **56**, (12), pp. 6040–6057

[6] Cadambe, V.R., Jafar, S.A.: 'Interference alignment and degrees of freedom of the K-user interference channel', *IEEE Trans. Inf. Theory*, 2008, **54**, (8), pp. 3425–3441

[7] Cadambe, V.R., Jafar, S.A.: 'Degrees of freedom of wireless networks with relays, feedback, cooperation, and full duplex operation', *IEEE Trans. Inf. Theory*, 2009, **55**, (5), pp. 2334–2344

[8] Al-Shatri, H., Ganesan, R.S., Klein, A., *et al.*: 'Interference alignment using a MIMO relay and partially-adapted transmit/receive filters'. Proc. 2012 IEEE WCNC, pp. 459–464

[9] Chen, S., Cheng, R.S.: 'Achieve the degrees of freedom of K-User MIMO interference channel with a MIMO relay'. Proc. 2010 IEEE GLOBECOM, pp. 1–5

[10] Razaviyayn, M., Lyubeznik, G., Luo, Z.-Q.: 'On the degrees of freedom achievable through interference alignment in a MIMO interference channel', *IEEE Trans. Signal Process.*, 2012, **60**, (2), pp. 812–821

[11] Chen, X., Song, S.H., Letaief, K.B.: 'Interference alignment in dual-hop MIMO interference channel', *IEEE Trans. Wirel. Commun.*, 2014, **13**, (3), pp. 1274–1283

[12] Berger, S., Kuhn, M., Wittneben, A., *et al.*: 'Recent advances in amplify-and-forward two-hop relaying', *IEEE Commun. Mag.*, 2009, **47**, (7), pp. 50–56

[13] Oyman, O., Paulraj, A.J.: 'Design and analysis of linear distributed MIMO relaying algorithms', *IEE Proc. Commun.*, 2006, **153**, (4), pp. 565–572

[14] Ganesan, R.S., Al-Shatri, H., Weber, T., *et al.*: 'Cooperative zero forcing in multi-pair multi-relay networks'. Proc. IEEE PIMRC 2012, September 2012, pp. 1740–1745

[15] Fadel, M., El-Keyi, A., Sultan, A.: 'QOS-constrained multiuser peer-to-peer amplify-and-forward relay beamforming', *IEEE Trans. Signal Process.*, 2012, **60**, (3), pp. 1397–1408

[16] Chen, H., Gershman, A.B., Shahbazpanahi, S.: 'Distributed peer-to-peer beamforming for multiuser relay networks'. Proc. IEEE ICASSP 2009, Taipei, 2009, pp. 2265–2268

[17] Fazeli-Dehkordy, S., Shahbazpanahi, S., Gazor, S.: 'Multiple peer-to-peer communications using a network of relays', *IEEE Trans. Signal Process.*, 2009, **57**, (8), pp. 3053–3062

[18] Chalise, B.K., Vandendorpe, L.: 'Optimization of MIMO relays for multipoint-to-multipoint communications: nonrobust and robust designs', *IEEE Trans. Signal Process.*, 2010, **58**, (12), pp. 6355–6368

[19] Truong, K.T., Sartori, P., Heath, R.W.: 'Cooperative algorithms for MIMO amplify-and-forward relay networks', *IEEE Trans. Signal Process.*, 2013, **61**, (5), pp. 1272–1287

[20] Choi, H.-J., Song, C., Park, H., *et al.*: 'Transceiver designs for multipoint-to-multipoint MIMO amplify-and-forward relaying systems', *IEEE Trans. Wirel. Commun.*, 2014, **13**, (1), pp. 198–209

[21] Khandaker, M.R.A., Rong, Y.: 'Interference MIMO relay channel: joint power control and transceiver-relay beamforming', *IEEE Trans. Signal Process.*, 2012, **60**, (12), pp. 6509–6518

[22] Nguyen, K., Rong, Y., Nordholm, S.: 'MMSE-based transceiver design algorithms for interference MIMO relay systems', *IEEE Trans. Wirel. Commun.*, 2015, **14**, (11), pp. 6414–6424

[23] Singh, V.P., Chaturvedi, A.K.: 'Min–max mean squared error-based linear transceiver design for multiple-input–multiple-output interference relay channel', *IET Commun.*, 2015, **9**, (6), pp. 853–861

[24] Horn, R.A., Johnson, C.R.: '*Matrix analysis*' (Cambridge University Press, Cambridge UK, 1985)

[25] Lobo, M., Vandenberghe, L., Boyd, S., *et al.*: 'Applications of second-order cone programming', *Linear Algebr. Appl.*, 1998, **284**, (1), pp. 193–228

[26] Boyd, S.P., Vandenberghe, L.: '*Convex optimization*' (Cambridge University Press, Cambridge, UK, 2004)

[27] Wiesel, A., Eldar, Y.C., Shamai, S.: 'Linear precoding via conic optimization for fixed MIMO receivers', *IEEE Trans. Signal Process.*, 2006, **54**, (1), pp. 161–176

[28] Christopoulos, D., Chatzinotas, S., Ottersten, B.: 'Weighted fair multicast multigroup beamforming under per-antenna power constraints', *IEEE Trans. Signal Process.*, 2014, **62**, (19), pp. 5132–5142

[29] Grant, M., Boyd, S.: 'CVX: Matlab software for disciplined convex programming, version 2.0 beta'. http://cvxr.com/cvx, September 2013

[30] Mu, H., Tugnait, J.K.: 'MSE-based source and relay precoder design for cognitive multiuser multi-way relay systems', *IEEE Trans. Signal Process.*, 2013, **61**, (7), pp. 1770–1785

[31] Zhang, G., Li, Q., Zhang, Q., *et al.*: 'Signal-to-interference-plus-noise ratio-based multi-relay beamforming for multi-user multiple-input multiple-output cognitive relay networks with interference from primary network', *IET Commun.*, 2015, **9**, (2), pp. 227–238

[32] Gomadam, K., Cadambe, V.R., Jafar, S.A.: 'A distributed numerical approach to interference alignment and applications to wireless interference networks', *IEEE Trans. Inf. Theory*, 2011, **57**, (6), pp. 3309–3322

[33] Chen, C.-E., Chung, W.-H.: 'An iterative minmax per-stream mse transceiver design for MIMO interference channel', *IEEE Wirel. Commun. Lett.*, 2012, **1**, (3), pp. 229–232

[34] Liu, Y.-F., Dai, Y.-H., Luo, Z.-Q.: 'Max-min fairness linear transceiver design for a multi-user MIMO interference channel', *IEEE Trans. Signal Process.*, 2013, **61**, (9), pp. 2413–2423

[35] Ning, H., Ling, C., Leung, K.K.: 'Feasibility condition for interference alignment with diversity', *IEEE Trans. Inf. Theory*, 2011, **57**, (5), pp. 2902–2912