ODE: Assignment-I

1. Classify each of the following differential equations as linear, nonlinear and specify the order
   (i) $y'' + (\cos x)\, y = 0$ (ii) $y'' + x \sin y = 0$ (iii) $y' = \sqrt{1+y}$
   (iv) $y'' + (y')^2 + y = x$ (v) $y'' + xy' = \sin y$ (vi) $(x\sqrt{1+x^2}\, y')' = e^x\, y$

2. Find the differential equation of each of the following families of plane curves:
   (i) $xy^2 - 1 = cy$ (ii) $cy = c^2x + 5$ (iii) $y = ax^2 + be^{2x}$ (iv) $y = ax + b + c$
   (v) Circles touching the x-axis with centres on y-axis
   (vi) $y = a \sin x + b \cos x + b$,
   where a, b and c are arbitrary constants.

3. Verify that the given function in the left is a general solution to the corresponding differential equation in the right.

(i)	$x^3 + y^3 = 3cxy$	$x(2y^3 - x^3)y' = y(y^3 - 2x^3)$
(ii)	$y = ce^{-x} + x^2 - 2x + 4$	$y' + y = x^2 + 2$
(iii)	$y = cx - c^2$	$y'^2 - xy' + y = 0$

4. Verify that $y = 1/(x + c)$ is general solution of $y' = -y^2$. Find particular solutions such that (i) $y(0) = 5$, and (ii) $y(2) = -1/5$. In both the cases, find the largest interval I on which y is defined.

5. Find the solution of the initial value problem

$$xy' = y + \frac{2x^4}{y}\cos(x^2), \quad y(\sqrt{\pi/2}) = \sqrt{\pi}.$$

6. Reduce the differential equation

$$y' = f\left(\frac{ax + by + m}{cx + dy + n}\right),\ ad - bc \neq 0$$

   to a separable form. Also discuss the case of $ad = bc$.

7. For each of the following differential equations, draw several isoclines with appropriate lineal elements and hence sketch some solution curves

$$(i) \quad y' = x \qquad (ii) \quad y' = -x/y$$