ODE: Assignment-2

1. Find general solution of the following differential equations:

$$(i)\ (x+2y+1)-(2x+y-1)y'=0 \quad (ii)\ y'=(8x-2y+1)^2/(4x-y-1)^2$$

2. Show that the following equations are exact and hence find their general solution:

$$(i)\ (\cos x\cos y-\cot x)=(\sin x\ \sin y)y' \quad (ii)\ y'=2x(ye^{-x^2}-y-3x)/(x^2+3y^2+e^{-x^2})$$

3. Show that if the differential equation is of the form

$$x^a y^b(my\,dx+nx\,dy)+x^c y^d(py\,dx+qx\,dy)=0,$$

where a,b,c,d,m,n,p,q $(mq\neq np)$ are constants, then $x^h y^k$ is an integrating factor. Hence find a general solution of $(x^{1/2}y-xy^2)+(x^{3/2}+x^2y)y'=0$.

4. Show that the equation $(3y^2-x)+2y(y^2-3x)y'=0$ admits an integrating factor which is a function of $(x+y^2)$. Hence solve the differential equation.

5. Given that $2\sin(y^2)+xy\cos(y^2)y'=0$ admits an integrating factor which is a function of x only. Hence solve the differential equation.

6. Given that $xy-(x^2+y^4)y'=0$ admits an integrating factor which is a function of y only. Hence solve the differential equation.

7. Reduce the following differential equations into linear form and solve:
(i) $y^2y'+y^3/x=x^{-2}\sin x$ (ii) $y'\sin y+x\cos y=x$ (iii) $y'=y(xy^3-1)$

8. Find the orthogonal trajectories of the following families of curves:
(i) $e^x\sin y=c$ (ii) $y^2=cx^3$

9. Find the family of oblique trajectories which intersect the family of straight lines $y=cx$ at an angle of 45^o.

10. Show that the following families of curves are self-orthogonal:
(i) $y^2=4c(x+c)$ (ii) $x^2/c^2+y^2/(c^2-1)=1$