ODE: Assignment-5

1. Verify that $y = x^2 \sin x$ and $y = 0$ both are solutions of the initial value problem
$$x^2y'' - 4xy' + (x^2+6)y = 0, \quad y(0) = y'(0) = 0.$$
Does it contradict the uniqueness?

2. Find general solution of the following differential equations given a known solution y_1:
(i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0 \qquad y_1 = 1/x$
(ii) $(1-x^2)y'' - 2xy' + 2y = 0 \qquad y_1 = x$

3. Verify that $\sin x/\sqrt{x}$ is a solution of $x^2y'' + xy' + (x^2 - 1/4)y = 0$ over any interval on the positive x-axis and hence find its general solution.

4. Solve the following differential equations:
(i) $y'' - 4y' + 3y = 0 \qquad$ (ii) $y'' + 2y' + (\omega^2+1)y = 0, \quad \omega$ is real.

5. Solve the following initial value problems:
(i) $y'' + 4y' + 4y = 0 \qquad y(0) = 1,\ y'(0) = -1$
(ii) $y'' - 2y' - 3y = 0 \qquad y(0) = 1,\ y'(0) = 3$

6. The equation
$$x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$
where a, b are constants, is called the Euler-Cauchy equation. Show that under the transformation $x = e^t$ (when $x > 0$) for the independent variable, the above reduces to
$$\frac{d^2u}{dt^2} + (a-1)\frac{du}{dt} + bu = 0,$$
which is an equation with constant coefficients. In the above $y(x) = y(e^t) = u(t)$
Hence solve: (i) $x^2y''+2xy'-12y = 0$ (ii) $x^2y''+5xy'+13y = 0$ (iii) $x^2y''-xy'+y = 0$

7. Find a particular solution of each of the following equations by the method of undetermined coefficients and hence find its general solution:
(i) $y'' + 4y = 2\cos^2 x + 10e^x \qquad$ (ii) $y'' + y = \sin x + (1+x^2)e^x$
(iii) $y'' - y = e^{-x}(\sin x + \cos x) \qquad$ (iv) $y''' - 3y'' - y' + 3y = x^2e^x$

8. By using the method of variation of parameters, find the general solution of:
(i) $y'' + 4y = 2\cos^2 x + 10e^x \qquad$ (ii) $y'' + y = x\sin x$
(iii) $y'' + y = \cot^2 x \qquad$ (iv) $x^2y'' - x(x+2)y' + (x+2)y = x^3, \quad x > 0.$
[Hint. $y = x$ is a solution of the homogeneous part]