ODE: Assignment-8

1. Let $F(s)$ be the Laplace transform of $f(t)$. Find the Laplace transform of $f(at) \quad (a > 0)$.

2. Find the Laplace transforms:

   (a) $[t]$ (greatest integer function), (b) $t^m \cosh bt \quad (m \in$ non-negative integers),

   (c) $e^t \sin at$, (d) $\dfrac{e^t \sin at}{t}$, (e) $\dfrac{\sin t \cosh t}{t}$, (f) $f(t) = \begin{cases} \sin 3t, & 0 < t < \pi, \\ 0, & t > \pi, \end{cases}$

3. Find the Laplace transforms (Hint: use second shifting theorem):

   (a) $f(t) = \begin{cases} 1, & 0 < t < \pi, \\ 0, & \pi < t < 2\pi, \\ \cos t, & t > 2\pi, \end{cases}$ (b) $f(t) = \begin{cases} 0, & 0 < t < 1, \\ \cos(\pi t), & 1 < t < 2, \\ 0, & t > 2 \end{cases}$

4. Find the inverse Laplace transforms of

   (a) $\tan^{-1}(a/s)$, (b) $\ln \dfrac{s^2+1}{(s+1)^2}$, (c) $\dfrac{s+2}{(s^2+4s-5)^2}$, (d) $\dfrac{se^{-\pi s}}{s^2+4}$, (e) $\dfrac{(1-e^{-2s})(1-3e^{-2s})}{s^2}$.

5. Using convolution, find the inverse Laplace transforms:

   (a) $\dfrac{1}{s^2-5s+6}$, (b) $\dfrac{2}{s^2-1}$, (c) $\dfrac{1}{s^2(s^2+4)}$, (d) $\dfrac{1}{(s-1)^2}$.

6. Use Laplace transform to solve the initial value problems:

   (a) $y'' + 4y = \cos 2t, \qquad y(0) = 0,\ y'(0) = 1.$

   (b) $y'' + 3y' + 2y = \begin{cases} 4t & \text{if } 0 < t < 1 \\ 8 & \text{if } t > 1 \end{cases} \qquad y(0) = y'(0) = 0$

   (c) $y'' + 9y = \begin{cases} 8 \sin t & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases} \qquad y(0) = 0, y'(0) = 4$

   (d) $y_1' + 2y_1 + 6 \displaystyle\int_0^t y_2(\tau)\, d\tau = 2u(t), \quad y_1' + y_2' = -y_2, \qquad y_1(0) = -5, y_2(0) = 6$

7. Solve the integral equations:

   (a) $y(t) + \displaystyle\int_0^t y(\tau)\, d\tau = u(t-a) + u(t-b)$

   (b) $e^{-t} = y(t) + 2 \displaystyle\int_0^t \cos(t-\tau) y(\tau)\, d\tau$

   (c) $3 \sin 2t = y(t) + \displaystyle\int_0^t (t-\tau) y(\tau)\, d\tau$

8. Sketch the following functions and find their Laplace transforms:

   (a) $f(t) = \begin{cases} u(t) - 2u(t-1), & 0 \le t < 2, \\ f(t-2), & t > 2, \end{cases}$ (b) $f(t) = \begin{cases} t\Big(u(t) - u(t-1)\Big), & 0 \le t < 2, \\ f(t-2), & t > 2, \end{cases}$

   (c) $f(t) = \begin{cases} tu(t) - 2(t-1)u(t-1), & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$