Assignment-II

1. Solve the initial value problem
$$\frac{dx}{dt} = x(2-x), \quad x(0) = x_0.$$
What will happen if $t \to \infty$?

2. Solve the constant harvesting model
$$\frac{dx}{dt} = x(1-x) - h, \quad x(0) = x_0 > 0,$$
where $h > 0$. What will happen if $t \to \infty$?

3. Solve the constant effort harvesting model
$$\frac{dx}{dt} = x(1-x) - ex, \quad x(0) = x_0 > 0,$$
where $e > 0$. What will happen if $t \to \infty$?

4. Consider two possible sequences of end-of-year returns (in thousands):
$$\{20, 20, 20, 15, 10, 5\} \quad \text{and} \quad \{10, 10, 15, 20, 20, 20\}.$$
Which sequence is preferable if the interest rate, compounded annually, is (i) 3%, (ii) 5%, (iii) 10%

5. A bank account gives an interest rate of 6% compounded monthly. Suppose you initially invest ₹1000 at the beginning of January. Next, you deposit ₹10 at the end of every odd month and withdraw ₹20 at the end of every even month. How much money do you have after 5 years?

6. A pharmaceutical company ABC has a 4-year project with an initial cost ₹2,40,000 borrowed at an annual interest rate r. The projected cashflow at the end of 1st, 2nd, 3rd and 4th years are ₹25,000, ₹75,000, ₹1,50,000, and ₹1,50,000 respectively. Conclude whether the company project manager should accept or reject the project if the interest rate r is (i) 10%, (ii) 20%.

7. A stock will pay a dividend of ₹100, ₹150, ₹100, and ₹50 quarterly for this year. Another stock will pay a dividend of ₹200 each semi-annually. Which one is preferable, assuming 5% interest rate compounded continuously?

8. Find the general solution of the following difference equations:

   (a) $x_{n+1} - 5x_n + 6x_{n-1} = 2^n(5n+1)$

   (b) $x_{n+1} - 6x_n + 9x_{n-1} = 2^n(5n+1)$

   (c) $x_{n+1} - 6x_n + 9x_{n-1} = 3^n(5n+1)$

9. Find the solution of the following difference equations:

   (a) $x_{n+1} = 5x_n + n^2, \quad x_0 = 1$

   (b) $2x_{n+1} = x_n + \frac{n}{2^{n-1}}, \quad x_0 = 1$

   (c) $2x_n = x_{n-1} + 6^n, \quad x_0 = 1$

   (d) $x_{n+2} + 3x_{n+1} + x_n = 0, \quad x_0 = -1, x_1 = 0$

   (e) $4x_{n+1} + 4\sqrt{3}x_n + 3x_{n-1} = 0, \quad x_0 = -1, x_1 = 0$

10. A discrete population model of insects is given by

$$x_{n+1} = \begin{cases} 4x_n, & 0 \leq x_n \leq 1 \\ 0.5x_n + 3.5, & x_n > 1 \end{cases}$$

Find the equilibrium points and discuss their stability.

11. Consider the nonlinear equation for population growth

$$x_{n+1} = \frac{\lambda x_n}{1 + x_n}, \qquad \lambda > 0.$$

Find the equilibrium points and discuss their stability and any bifurcation that takes place. Solve the equation by substituting $y_n = 1/x_n$ and verify your results.

12. A drug of amount D is administered every T hours. Let D_n be the amount of drug at the beginning of the n-th interval $[nT, (n+1)T)$ where $n = 0, 1, 2, \cdots$. The blood removes a certain fraction f of the drug in each time interval. Find D_n and $\lim_{n\to\infty} D_n$.

13. The tent map T is defined as

$$T(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} < x \leq 1. \end{cases}$$

Consider the recurrence relation $x_{n+1} = T(x_n)$, $n \geq 0$. Find all the fixed points and the 2-periodic cycles, and discuss their stability.