Assignment-IV

1. A farmer owns 50 acre of land. Each acre of wheat yields a profit of ₹12 thousands and each acre of rice yields a profit of ₹18 thousands. The farmer has 100 workers and 120 tons of fertiliser. The labour and fertiliser per acre of wheat are 3 and 2 and the corresponding values for rice are 2 and 4. Find the number of acres for wheat and rice to maximise the profit.

2. Solve the following problems graphically

   (a) Minimize $x + y$
   subject to
   $x + y \geq 6$
   $3x - y \geq 9$
   $x \geq 0, y \geq 0.$

   (b) Maximize $10x + 35y$
   subject to
   $8x + 6y \leq 48$
   $4x + y \leq 20$
   $y \geq 5$
   $x \geq 0, y \geq 0.$

3. Convert the following problem to a dual problem and solve the dual problem graphically. From the solution of the dual problem, find the solution of the original problem.
   Maximize $z = 2x_1 + 4x_2 + 2x_3$
   subject to the constraints
   $2x_1 + x_2 + 2x_3 \leq 1$
   $x_1 + 4x_2 - x_3 \leq 2$
   $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

4. Use the simplex method to find the maximum value of
   $z = 2x_1 - x_2 + 2x_3$
   subject to the constraints
   $2x_1 + x_2 \leq 10$
   $x_1 + 2x_2 - 2x_3 \leq 20$
   $x_1 + 2x_3 \leq 5$
   $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

5. Discuss whether the population, whose Leslie matrix is given below, will survive in the long run

$$\begin{pmatrix} 0 & 2 & 3 \\ 0.5 & 0 & 0 \\ 0 & 0.25 & 0 \end{pmatrix}$$

6. A population has four life stages. The dominant eigenvalue is found to be 1.2 and a corresponding eigenvector is $[3\ 7\ 10\ 2]^T$. If, at time step 500, the population has reached the long-term growth rate with 4400 individuals, find the number of individuals in each life stage at time step 501.

7. The Leslie matrix for an insect population is

$$\begin{pmatrix} 0 & 20 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \end{pmatrix}$$

If $\boldsymbol{x}^{(0)} = [0\ 100\ 0]^T$ is the initial population, discuss the long-term behaviour of the population.

8. Consider the Leslie matrix given below for a population:

$$\begin{pmatrix} 0 & 0 & 50 & 30 \\ b_1 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{pmatrix}$$

If $b_1 = 0.03$, discuss whether harvesting only the youngest class is possible. If yes, then find the harvesting constant h and the distribution of the various age classes. Do the same analysis for $b_1 = 0.01$.

9. Consider the Leslie matrix given below for a population:

$$\begin{pmatrix} 0 & \alpha & 3 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

If $\alpha = 4$, discuss whether uniform harvesting is possible. If yes, then find the harvesting constant h and the distribution of the various age classes. Do the same analysis when α is changed to 1.

10. A person bets ₹1 in every bet, and he starts with ₹4. The probability of winning is 0.6, and losing is 0.4. Find the probability that the person attains ₹10 before becoming ruined. Also, find the probability of becoming infinitely rich.

11. A person bought a share of stock for ₹100, and it is believed that the stock price moves (day by day) up ₹1 with probability $p = 0.55$ and down ₹1 with probability $q = 0.45$. What is the probability that the stock reaches the high value of ₹115 before the low value of ₹90? What is the probability that the person will become infinitely rich?

12. A bird's feather moves down 5 meters per minute with probability $p = \frac{2}{3}$ and moves up (due to random wind) 5 meters per minute with probability $q = \frac{1}{3}$. Suppose the feather is at a distance of 25 metres from the top and 50 metres from the ground. Find the probability that the feather reaches the top before hitting the ground.

13. A walker takes a unit step to the right with probability $p = 0.55$ and a unit step to the left with probability $q = 0.45$. If the walker starts at $x = 4$, find the probability of reaching $x = 10$ before reaching $x = -5$.