Analysis and Prediction of Vertical Cycloidal Rotor Wind Turbine with Variable Amplitude Pitching

Palash Jain
Research Associate
paljain.iitk@gmail.com

Abhishek
Assistant Professor
abhish@iitk.ac.in

Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Kanpur, India

ABSTRACT

This paper predicts and analyzes the aerodynamic performance of a Vertical Axis Wind Turbine (VAWT) with variable amplitude dynamic blade pitching. This study contributes to the physics based understanding of the dependence of power extracted by the turbine on various design parameters. An aerodynamic model based on double multiple streamtube theory coupled to an airfoil table lookup based blade element theory analysis and attached unsteady aerodynamics is used for performance prediction. Present analysis gives excellent correlation with experimental results for a lab scale straight bladed prototype with sinusoidally varying blade pitch angle. It is concluded that the amplitude of sinusoidal blade pitching must be varied with wind speed and tip speed ratio to maximize the power extracted from the turbine for wide range of wind speeds and tip speed ratios. High (about 35°) pitch amplitudes work best for tip speed ratios below 0.5 and the amplitude should be lowered upto 10° beyond tip speed ratio of 2. This analysis can be used to optimize the design of VAWT of different size and power rating.

INTRODUCTION

World energy demand stand at around 156 PWh (petawatt-hour) aggregated over the year 2012 (Ref. 1). Around 80% of this energy demand is fulfilled by fossil fuels including petroleum, coal and natural gas. With the shift in focus towards renewable sources, wind energy which currently constitutes 4% of total electricity generation of 19 PWh, is slated to play an increasingly important role. One of the major limiting factors in the universal switch over to wind energy is highly intermittent and sometimes inadequate supply of wind in the areas with peak demand. Further, at present the Horizontal Axis Wind Turbines (HAWTs), the traditional choice for harnessing wind energy, work efficiently only in the areas where fast moving laminar wind is available. Most of the wind power generation farms of several Mega-Watt capacity consist of large horizontal axis wind turbines of several 100 kWs capacity which feed electricity directly to the grid. These wind turbine farms are usually located away from inhabited areas and rely on the horizontal non-turbulent wind profiles. New transmission lines are required to bring generated wind power from its point of generation to markets. This makes the integration of large-scale wind power plants into existing grid system a challenging task. Therefore, a more decentralized approach is needed wherein the points of power generation and consumption are collocated (Ref. 2).

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This calls for an investigation of small wind turbines which can be deployed in inhabited areas without encroaching into useful real estate. Such wind turbines may generate DC power and can be integrated with existing photovoltaic equipment to augment power generation. The wind profile in densely populated areas tend to be more turbulent due to presence of man-made structures causing obstructions in the path of the wind. (Ref. 3) looked into the details of the early developments that led to the design of modern horizontal and vertical axis wind turbines. For long VAWTs were considered a less viable alternative for deployment than their horizontal axis counterpart and less efforts were put in to the research and development of such turbines. (Ref. 4) reported that the traditional workhorse, horizontal axis turbines, perform poorly in wind conditions atypical of inhabited areas such as rooftops and are often noisier than VAWTs due to higher tip speeds. They also have large footprint to allow for yawing. Vertical axis turbines on the contrary perform better in severe / non-ideal wind climates, such as rooftops, found in populated urban / rural settings. In addition VAWTs have smaller footprint, omni-directional and can operate in very low wind speed. The VAWTs being used commercially are of fixed pitch type (either Darrius type or H-rotor type) and hence suffer from low efficiency, especially at low tip speed ratio (tip speed ratio = blade speed / wind speed) and are self-starting for certain wind directions and relatively higher wind speeds (Refs. 2, 5–7). The solution lies in variable pitch VAWTs, these are known to have improved efficiencies (Refs. 8–11) at lower tip speed ratios (close to 1.5), can self-start at very low wind speeds and arbitrary wind directions.

Vertical Axis Wind Turbines, on the basis of wind interaction, can be classified into the drag based Savonius type or the lift based Darrieus type. The drag based VAWTs despite having better starting performance, capture little of the net available wind energy. The lift based VAWTs have better performance and were therefore pursued for further research. Extensive experimental studies on design and optimization of Darrieus-type wind turbines were carried out by Sandia National Laboratories. An overview of the science and economics of these studies are provided in (Ref. 12). Many of these designs being troposkien or eggbeater shaped had fixed pitch configuration which is one of the reasons behind their under-performance. (Ref. 2) presents a compelling case and some challenges associated with implementing variable pitch in H-Darrieus designs. However, some studies have shown that variable pitch can help overcome the challenge associated with poor starting performance of the VAWTs. Various active and passive blade pitch change mechanisms such as sinusoidal forcing through cam or gears, mass-stabilized pitch control, passive pitching under action of aerodynamic loads etc are discussed in (Ref. 13). Optimization of blade pitch angle using genetic algorithm is discussed in (Ref. 8) to get the best performance out of a VAWT of a given size. Despite of significant performance benefits, the lack of commercial success for variable pitch vertical axis wind turbines can be attributed to the mechanical complexity of these blade pitch change mechanisms.

An active reliable pitch changing mechanism based on the classical four-bar linkage has been studied experimentally in (Refs. 14–16). Apart from experimental measurement of performance of model scale turbines at various blade pitch amplitudes, 2D Computational Fluid Dynamic (CFD) predictions were also carried out by Refs. (Refs. 15, 16) to understand the behavior of these turbines. These model scale turbines have been reported to be self starting at wind speeds as low as 1.5 m/s to 2 m/s making them ideal for standalone operation in urban settings where the average annual wind speed may be low, as they exhibit higher efficiency at lower wind speeds when compared to HAWT and fixed pitch VAWT designs. Lower footprint in terms of space required for operation also make them ideal for use in high density wind energy farms such as one described in (Ref. 17). However, the physical mechanisms involved in power generation from the variable blade pitch VAWT is not fully understood. Therefore, this study focuses on investigating the aerodynamics of variable amplitude variable pitch vertical axis wind turbine.

A number of physics based theories as well as computational methods have been previously employed to predict the performance with a varying degree of accuracy. Reference (Ref. 18) gives a detailed account of several of these methods for a small sized VAWT. Momentum based models are the simplest of these and comprise of Single Stream Tube theory, Multiple Stream Tube theory and Double Multiple Stream Tube theory (Refs. 19, 20). The function of stream tubes is to determine the velocity induced by the turbine blades. Subsequently Blade Element analysis is carried out to determine individual blade contribution to net performance. Instead of computing inflow by momentum conservation, vortex models such as that of (Refs. 21, 22) may also be used to determine induced inflow for VAWT performance prediction by accounting for 3D effects due to trailed vortices. Irrespective of the method for inflow calculation, the shed vorticity due to unsteadiness in the flow may have to be modelled using attached unsteady aerodynamic models such as Theodorsen's theory or Wagner's function (Ref. 23). If the turbine is operating under deep stall condition unsteady aerodynamic models for separated flow such Leishman-Beddoes (Ref. 24) dynamic stall model would have to be incorporated for better performance prediction. Numerical methods like 2D and 3D CFD are computationally expensive but provide a detailed picture of the flow field across the turbine. When compared with the experimental results, 2D CFD methodology shows considerable deviation while 3D CFD provides an excellent correlation (Ref. 25). This deviation is more pronounced for smaller turbines operating at lower speeds.

The VAWT with capability of changing the pitch amplitude by variation of length of one of the linkages of four bar mechanism presents the possibility of extracting the best possible performance for wide range of tip speed ratios and configurations. The present study focuses on establishing an analytical formulation which is first systematically validated using the experimental data available in literature. Then the effect of turbine design parameters is studied and their impact on the performance is understood. Fundamental understanding of the key mechanisms governing the performance of the turbine is also carried out. The case for varying the amplitude of sinusoidal blade pitching is established.

METHODOLOGY FOR AERODYNAMIC ANALYSIS

The goal of the present analysis is to predict the aerodynamic performance of vertical axis wind turbine with dynamic blade pitching and establish the physics governing it. The influence of various design parameters on the performance is also studied. For this, an aerodynamic analysis based on Double Multiple Stream Tube (DMST) model coupled to Blade Element Theory (BET) is established. The DMST model is based on the one discussed in (Ref. 26). The sectional aerodynamic properties are estimated using airfoil lookup table which uses local angle of attack and Reynolds number to determine lift and drag. This analysis is systematically validated with the experimental and computational results presented in (Ref. 15) for a small scale VAWT.

Double Multiple Stream Tube (DMST) Theory

Fig. 1 shows the interaction of incoming wind with the VAWT. The rotor is assumed to be a disk which is encountered twice by the air flowing through the turbine, hence the theory is referred to as Double Multiple Streamtube Theory. In the upstream half, wind is slowed once it encounters the turbine and

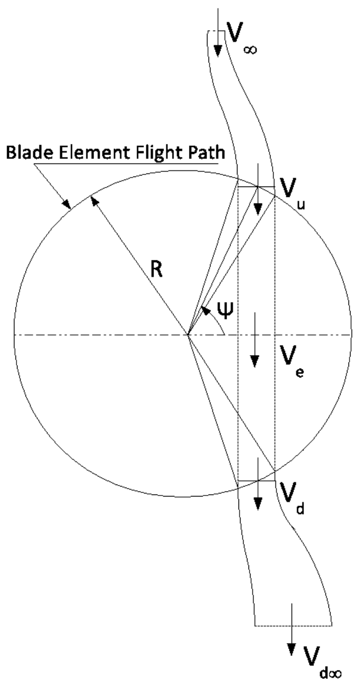


Fig. 1. Interaction of wind flowing through the turbine along a streamtube

this results in a net drag in the opposite direction. The velocity of the incoming wind, v_0 is reduced by a factor a_{u_i} such that after passing the first actuator disk it becomes

$$v_{u_i} = v_0 a_{u_i} \tag{1}$$

and the transition velocity becomes

$$v_{e_i} = v_0(2a_{u_i} - 1) \tag{2}$$

based on the analogy with helicopter momentum theory. The inflow factor a_{u_i} is unknown at this point and subsequent analysis is performed parametrically.

Applying conservation of momentum for a streamtube across disk D_{u_i} of area ΔA gives the drag force

$$F_{u_i} = \rho.v_{u_i}(v_0 - v_{e_i}).\Delta A \tag{3}$$
$$= \rho.v_0^2.2a_{u_i}.(1 - a_{u_i}).hR\sin\psi\Delta\psi \tag{4}$$

where ρ is the density of air, h is the length of the finite height element and ψ is the azimuthal location of disk.

Similarly, in the downstream half the wind flowing with velocity v_{e_i} inside the stream tube i encounters the second actuator disk D_{d_i} in the downstream half. The inflow factor for the air crossing D_{d_i} is assumed a_{d_i}. The wind velocity experienced by the disk is given by

$$v_{d_i} = v_{e_i} a_{d_i} \tag{5}$$
$$= v_0(2a_{u_i} - 1)a_{d_i} \tag{6}$$

and the velocity far downstream becomes

$$v_\infty = v_{e_i}(2a_{d_i} - 1) \tag{7}$$
$$= v_0(2a_{u_i} - 1)(2a_{d_i} - 1) \tag{8}$$

The inflow factor a_{d_i} is allowed to take any non-zero value, resulting in either deceleration or acceleration of the flow. The force acting opposite to the direction of the wind is given by

$$F_{d_i} = \rho.v_{d_i}(v_{e_i} - v_\infty).\Delta A \tag{9}$$
$$= -\rho.v_0^2.2a_{d_i}.(1 - a_{d_i})(2a_{u_i} - 1)^2.hR\sin\psi\Delta\psi \tag{10}$$

Blade Element Theory

The air flow over the blade airfoil generate lift and drag forces which are the driving forces behind lift-based Darrieus VAWT. The flow around the blade airfoil in a VAWT is highly directional and the inflow angle can vary anywhere between -180° to +180° depending on the azimuthal location, local incident velocity and tip speed ratio as:

$$\phi = \tan^{-1}\frac{v_i \sin\psi}{v_i \cos\psi + \lambda v_0} \tag{11}$$

where ϕ is the inflow angle and v_i assumes values v_{u_i} or v_{d_i} depending on the half in consideration. Fig. 2 depicts the direction of lift and drag forces in the upstream and downstream halves respectively. The pitch angle, θ of the blade being prescribed as input, is known at all times. Then the angle of attack for the airfoil can be found as

$$\alpha = \theta - \phi \tag{12}$$

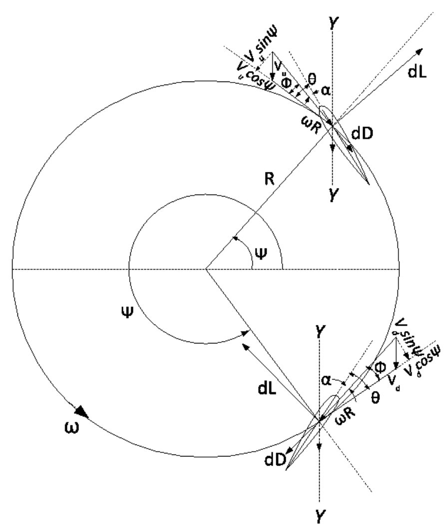


Fig. 2. Forces acting on VAWT blades in accordance with blade element theory

Based on the angle of attack defined above and local Reynolds number, the sectional lift and drag coefficients are calculated using table lookup. For the purpose of current analysis, NACA0015 airfoil tables for angles 0° - 180° given in (Ref. 27) are used.

The resultant force coefficient, C_Y in the direction opposite to the incoming wind is given by

$$C_Y = C_L sin(\psi - \phi) - C_D cos(\psi - \phi) \tag{13}$$

For the upstream half, this drag force translates to

$$F_{u_i} = \frac{1}{2}\rho v_{u_i}^2 C_Y \frac{c.n_b}{2\pi}\Delta\psi \tag{14}$$

$$= \frac{1}{2}\rho\left(\frac{v_{u_i} sin\psi}{sin\phi}\right)^2 \sigma.R.C_Y\Delta\psi \tag{15}$$

$$= \frac{1}{2}\rho\sigma R\left(\frac{sin\psi}{sin\phi}\right)^2 v_0^2(1-a_{u_i})^2.C_Y\Delta\psi \tag{16}$$

where the term σ denotes rotor solidity of the VAWT which is a function of turbine radius, R, blade chord, c and number, n_b; and given by

$$\sigma = \frac{n_b c}{2\pi R} \tag{17}$$

Similarly for the downstream half,

$$F_{d_i} = \frac{1}{2}\rho v_{d_i}^2 C_Y \frac{c.n_b}{2\pi}\Delta\psi \tag{18}$$

$$= \frac{1}{2}\rho\left(\frac{v_{d_i} sin\psi}{sin\phi}\right)^2 \sigma.R.C_Y\Delta\psi \tag{19}$$

$$= \frac{1}{2}\rho\sigma R\left(\frac{sin\psi}{sin\phi}\right)^2 v_0^2(1-a_{d_i})^2(2a_{u_i}-1).C_Y\Delta\psi \tag{20}$$

Equating the resistant force obtained from momentum and blade element theories in Eq. 4 with Eq. 16, an implicit expression in terms of inflow factor a_{u_i} is obtained as

$$\frac{a_{u_i}}{1-a_{u_i}} = F \tag{21}$$

$$\text{where} \quad F = \frac{1}{4}\sigma.\frac{sin\psi}{sin^2\phi}[C_L sin(\psi-\phi) - C_D cos(\psi-\phi)] \tag{22}$$

Note that both sides of Eq. 22 depend on a_{u_i}; hence it is solved for numerically by subsequently updating as

$$a_{u_i} = \frac{F}{1+F} \tag{23}$$

The iteration is terminated once convergence is obtained between initial and updated values of a_{u_i} within set permissible tolerance. The tangential component of the force coefficient per blade, which is same as the torque coefficient, C_{Q_u} that drives the turbine is given by

$$C_{T_u} = -(C_L \sin\phi + C_D \cos\phi) \tag{24}$$

The corresponding torque about the central shaft considering one stream tube in the upstream half is given by

$$Q_{u_i} = \frac{n_b\Delta\theta}{2\pi} \times \left(\frac{1}{2}\rho hcRv_{u_i}^2 C_{T_u}\right) \tag{25}$$

where n_b is the number of blades, $\Delta\theta$ is the angular span of each streamtube and h is the height of the blade element along the span of the turbine.

For the complete upstream half the power coefficient C_{P_u} is summed up as

$$C_{P_u} = \frac{\sum Q_{u_i}}{\frac{1}{2}\rho sRv_0^2} \times \lambda \tag{26}$$

The downstream half has exactly similar expression of power coefficient,

$$C_{P_d} = \frac{\sum Q_{d_i}}{\frac{1}{2}\rho sRv_0^2} \times \lambda \tag{27}$$

and the total power coefficient is the sum of upstream and downstream components

$$C_P = C_{P_u} + C_{P_d} \tag{28}$$

The power coefficient is the net resultant of contribution from all the blades throughout a complete revolution. There are several variations of this theory taking into account various improvements for better estimation of the inflow in the turbine. Reference 28 attempts to study the effect of wind that is not completely transverse to the axis of the turbine. This can be used to model the turbines with curved blades such as the troposkein design of Sandia National Laboratories. A different inflow model is discussed in (Ref. 29) for a cycloidal rotor along with CFD validation. Apart from inflow calculation, two key modeling aspects that affect the performance prediction of a VAWT with dynamic blade pitching are unsteady aerodynamics and virtual camber.

VALIDATION OF BASELINE MODEL

The present analysis can be used to predict the performance of a Darrieus VAWT of any size and shape. Prerequisites being lift based blades and uniform rotational speed. Validation of the present formulation is carried out using the experimental and computational (CFD) results given in 15 for a small scale wind turbine. The dimensional parameters of the small scale VAWT used by (Ref. 15) is included in Table 1. Fig. 3 shows the schematic of the turbine with a 4-bladed variable pitch rotor. The pitch change is achieved by 4-bar linkage mechanism in crank-rocker configuration whose ground link is located in the central hub of the assembly as shown in Fig. 4. The length of this link can be changed to modify the amplitude of pitch angle. The input link is the rigid radial link which connects the hub with the blade near its leading edge. Similarly the trailing edge of the blade is connected to the other end of the ground link completing the 4-bar, with the blade itself acting as the floating fourth link. The ground link can be aligned to the oncoming wind direction with the aid of an active/passive mechanism, e.g. a vertical floating wing/vane or motor driven yawing mechanism. The pitch variation achieved is nearly sinusoidal as shown in Fig. 5.

Table 1. Dimensional parameters of the baseline model of VAWT used in the present study

Length of the blades, h	0.254 m
Radius of turbine, R	0.127 m
Blade chord, c	0.0635
Number of blades, n_b	4
Wind velocity, V_0	10 m/s

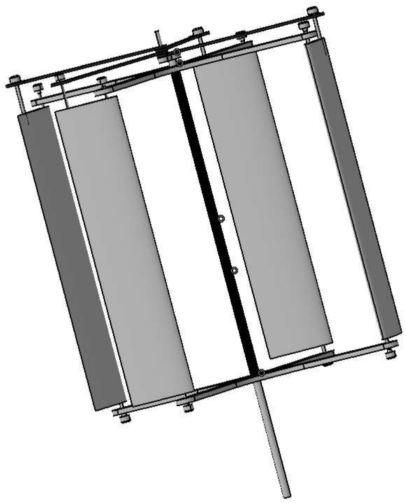


Fig. 3. Computer generated graphic of the VAWT used in the present study

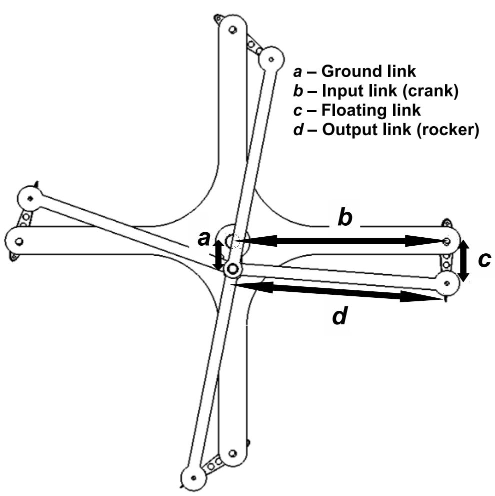


Fig. 4. Schematic of four-bar mechanism showing various links

Fig. 6 compares the predicted power coefficient at various tip speed ratios from present model with the experimental measurements and CFD predictions (available for only two pitch amplitudes) from (Ref. 15) for a variable pitch turbine operating at different sinusoidal pitch amplitudes. The predic-

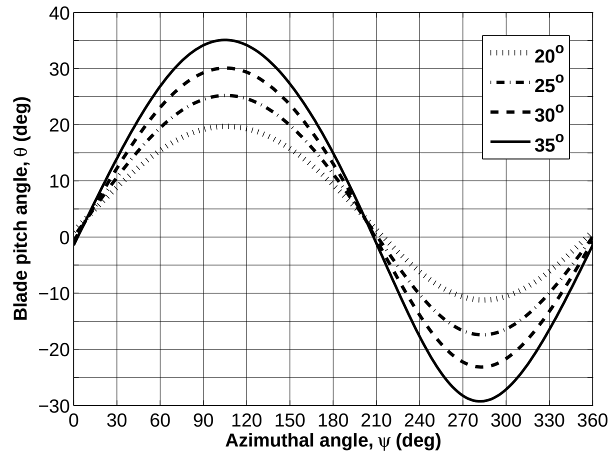
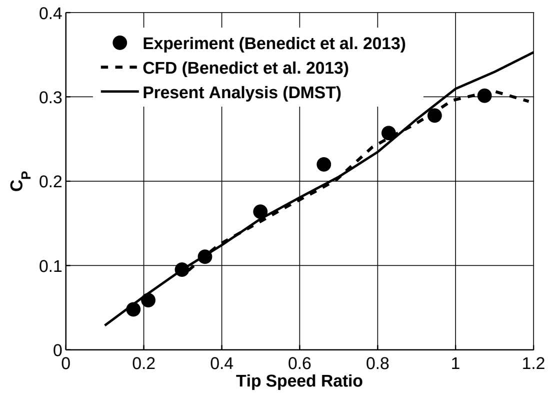


Fig. 5. Azimuthal angle of attack variation assumed for variable pitch VAWT

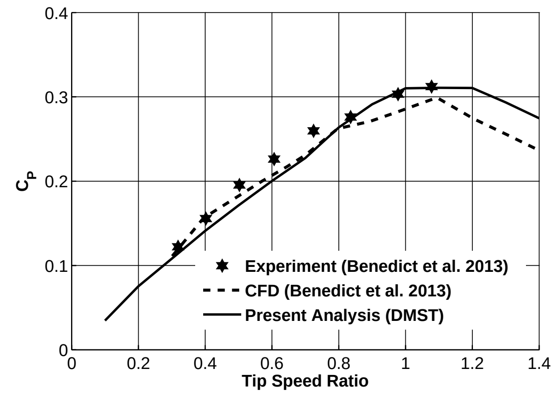
tions show very good correlation with the test data for all pitch amplitudes. Note that these results only indicate the aerodynamic power extracted by the turbine and the actual usable power transferred to the generator may be less due to friction, vibrations and inertial effects. The behaviour of power coefficient vs. tip speed ratio (TSR) indicates that the variable pitch VAWT of this type delivers peak performance in a narrow band of tip speed ratios. At higher TSRs, the blade tends to stall resulting in decrease in power output and at lower TSRs the torque output is not sustainable for profitable turbine operation. This narrow bandwidth of optimal power output can be improved by use of different blade pitch amplitudes at different tip speed ratios as discussed later in text.

The mechanism of power output for $TSR < 1.0$ is the same as for $TSR > 1.0$. Most of the contribution to torque comes from lift acting on the blades. At lower tip speed ratios, the drag coefficient becomes significant which acts counter to the direction of turbine rotation. However at very low TSRs some of the retreating half experiences reverse flow, where drag contribution is favorable. This can be understood by analyzing the blade pitch angle and angle of attack, shown in Fig. 7 and 8, for the case with VAWT operating at 25° pitch amplitude and 0.6 and 1.4 tip speed ratios Both these cases have roughly the same power output. For $TSR = 0.6$, the blade airfoil operates beyond its static stall limit as seen in Fig. 7. Therefore, the drag acting on the blades as seen in Fig. 9a becomes predominant. This phenomenon is similar to a Savonius-type wind turbine which have anemometer-like or flat-plate surface to create differential drag on the advancing and retreating sides. On the contrary at higher tip speed ratios (angle of attack variation shown in 8), the lift component in the upstream half is the major contributor to the total torque as shown in Fig. 9b.

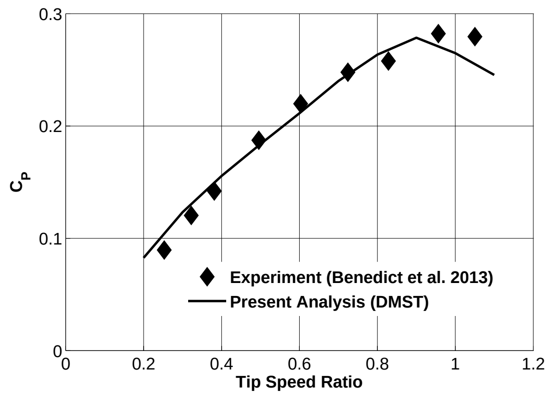
To understand the physics of power generation the lift and drag forces acting on the turbine at four azimuthal locations, representative of each quadrant, are shown in Fig. 10. This leads to identifying the regions of favorable power extraction and otherwise. The sudden dip in performance of the turbine



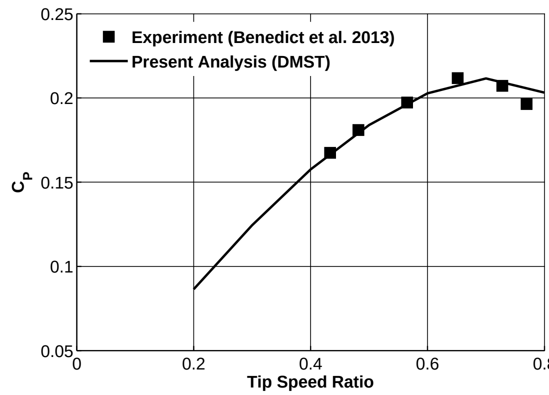
(a) Pitch amplitude = 20°



(b) Pitch amplitude = 25°



(c) Pitch amplitude = 30°



(d) Pitch amplitude = 35°

Fig. 6. Predicted and measured C_P vs. Tip Speed Ratio (TSR) 15

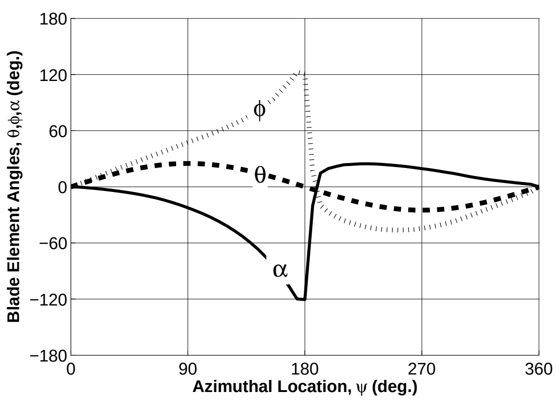


Fig. 7. Azimuthal variation of blade pitch angle (θ), inflow angle (ϕ) and angle of attack (α) at tip speed ratio (λ) = 0.6

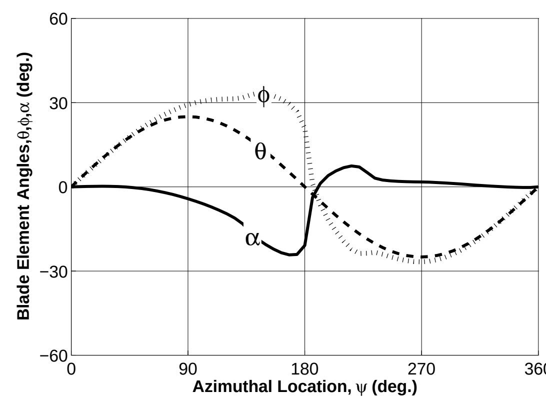
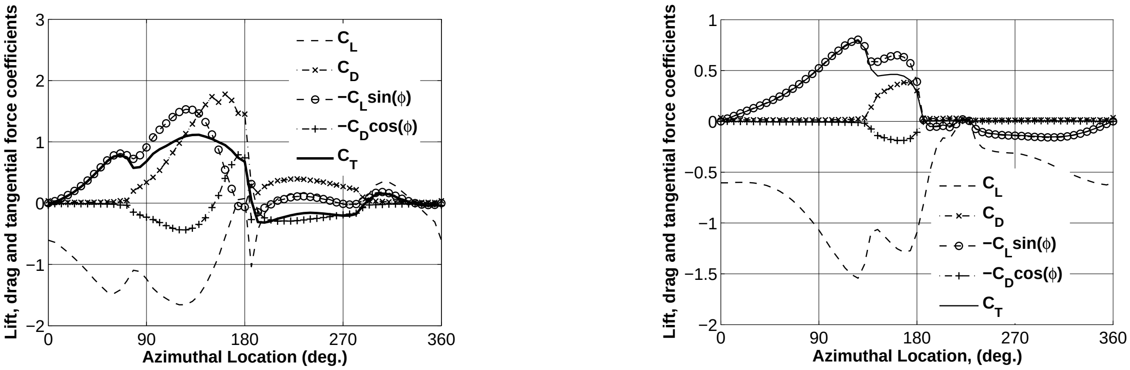


Fig. 8. Azimuthal variation of blade pitch angle (θ), inflow angle (ϕ) and angle of attack (α) at tip speed ratio (λ) = 1.4

after TSR $= 1.2$ can be explained by loss in lift advantage at the advancing side of the upstream half which is the major torque producing region. However, this can be overcome by reducing the pitch amplitude to push the lift back in the driving direction. The peculiar property of this kind of turbine is that the magnitude and direction of lift can be controlled by setting the pitch amplitude right and thereby making angle of attack favorable for turbine acceleration. However, the drag contribution is predominantly dependent on the direction of inflow experienced by the blades which is mostly dependent on the tip speed ratio of operation. The effect of various parameters on the VAWT performance as seen in later section will be explained based on this figure.



(a) Tip Speed Ratio, λ = 0.6 (b) Tip Speed Ratio, λ = 1.4

Fig. 9. Lift and drag contribution to tangential force as a function of azimuthal location.

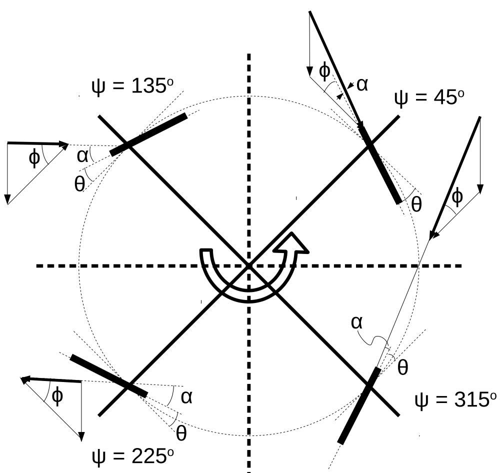


Fig. 10. Approximate direction and magnitude of turbine inflow at different azimuthal location for pitch amplitude = 25° and λ = 1.4.

EFFECT OF DESIGN PARAMETERS ON THE PERFORMANCE OF VAWT

This section investigates the effect of various design parameters on the performance of a variable pitch VAWT under given conditions. The use of blade element theory in the current analysis allows for identification of the physics behind sub-optimal performance of VAWT under off-design conditions. The product of rotor diameter and blade span which determines the area of wind interacting with the turbine and is used for determination of dimensional power output from the turbine is preserved during the parametric study. This analysis also lays down the general guidelines for optimal design of a VAWT with variable amplitude dynamic blade pitching.

Rotor Solidity

Rotor solidity is the ratio of planform area of all the blades combined, to the rotor swept area. As evident from Eq. 17, solidity of a VAWT rotor is directly proportional to the number of blades and chord and inversely proportional to radius. Solidity affects the turbine inflow as seen in Eq. 22, which changes both the angle of attack and the dynamic pressure experienced by the blades. The effect of changing solidity by varying each of the components individually is discussed below.

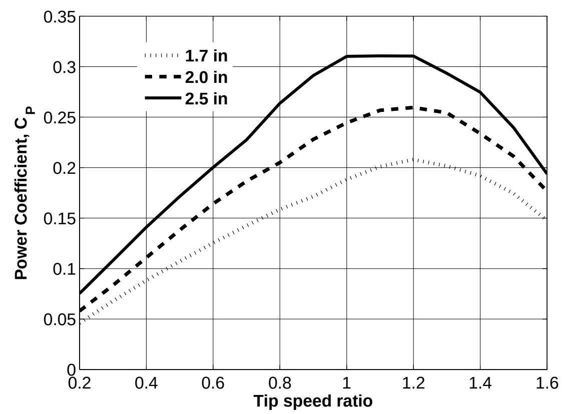


Fig. 11. Dependence of VAWT performance on blade solidity affected by blade chord at 25° pitch amplitude

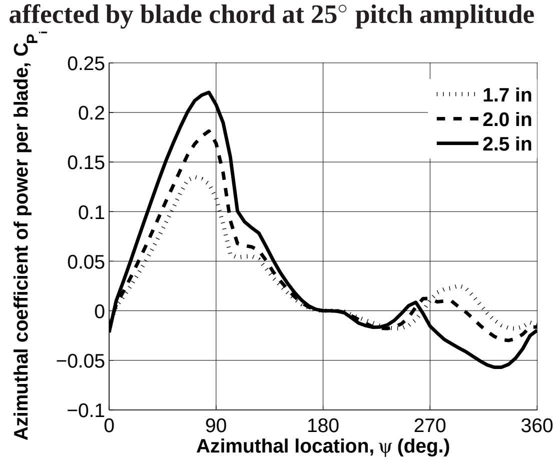


Fig. 12. Effect of blade chord on azimuthal variation of power coefficient C_{P_i} at $\lambda = 1.0$

Blade chord Fig. 11 shows the variation of power coefficient with chord length, power coefficient seems to be increasing with increasing chord length. The total power extracted from the turbine is the integrated effect of the tangential force coefficient (C_T) as defined in Eq. 24. Therefore, to understand the change in total power coefficient reported above, the variation of C_P vs. azimuth angle, shown in Fig. 12, is studied for $\lambda = 1$. In the analysis power coefficient increases due to increase in inflow resulting from increased solidity (see Eq. 22) with increase in chord length. There would also be increase in lift due to increased Reynolds number, but since the change in chord is relatively small, this effect would be less significant. Increased inflow results in greater C_L which in turn increases the sectional tangential force coefficient (C_T) on the upstream half of the turbine allowing for greater extraction of power in this region as shown in Fig. 12. However, this also results in slight increase in the power lost on the downstream half. It appears that the increase in resultant power extracted on the upstream half is greater than that lost on the downstream half, thereby allowing for greater total power coefficient. Since, the current analysis does not include 3D tip loss effects, therefore, the predicted quantitative improvement in performance may be an overestimate.

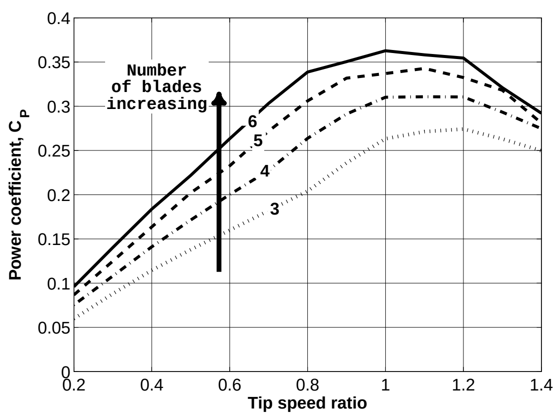


Fig. 13. Dependence of VAWT performance on blade solidity affected by number of blades at 25° pitch amplitude

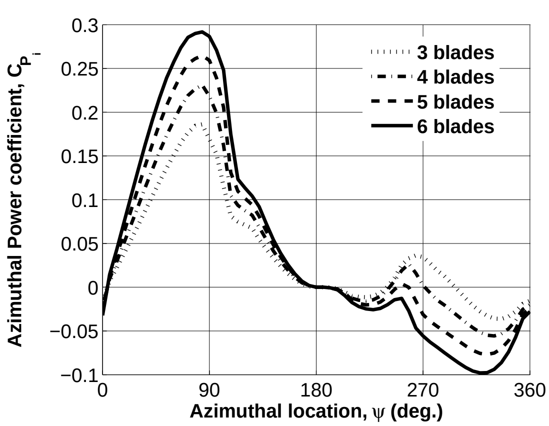


Fig. 14. Effect of number of blades on azimuthal power coefficient at $\lambda = 1.0$

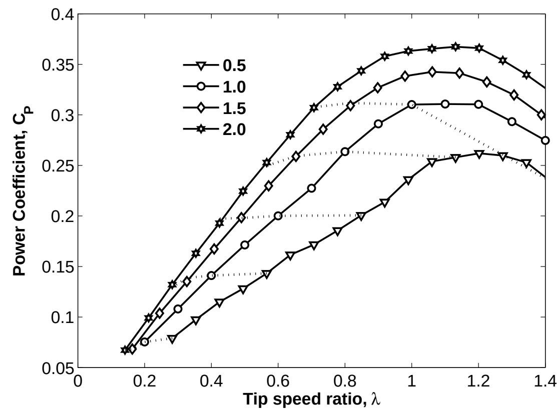


Fig. 15. Effect of turbine aspect ratio Æ on VAWT performance at 25° pitch amplitude

Number of blades Rotor solidity when varied by number of blades produces similar pattern in power coefficient. A 4-bladed rotor produces more power than a 3-bladed one and so on as can be seen from Fig. 13. However the difference in C_P between subsequent blade numbers keeps on decreasing. This is because more the number of blades in the turbine, the more it is capable of manipulating the inflow. This implies more power extraction in the upstream half but at the same time, more power loss in the downstream as seen in Fig 14. The major difference is, therefore caused by aggregating the effect of multiple blades showing, net increment. Advanced theories taking into account the inter-blade interference can put the upper limit to this trend.

Aspect Ratio Aspect ratio, Æ as defined here is the ratio of the VAWT blade span to its radius. Just like aircraft wings, it is a measure of the slenderness of the VAWT. Thus a high density wind farm will have adjacent VAWTs of high aspect ratio. However, in standalone applications such as urban environment, lower values are preferred due to limit on the height of turbine structure. The power rating of such turbines is dependent on the swept area, S of its rotor which is twice the product of radius and span. Thus aspect ratio plays a key role in the early design stages where sizing of the VAWT is performed. Using the current methodology ideal trends in the VAWT performance with respect to its aspect ratio are studied. Eq. 29 describes the variation of turbine radius, R with Æ.

$$R = \frac{1}{2}\sqrt{\frac{S}{Æ}} \tag{29}$$

The radius of VAWT affects not only its solidity but also the tip speed ratio at which it operates. When this tip speed ratio is maintained, the results are as shown in Fig. 15, which shows an increase in C_P with increasing Æ from 0.5 to 2.0. This is predictable given the solidity of a VAWT is inversely proportional to its radius. A slender turbine is able to extract power more efficiently from the area within which it is operating. Clearly, the present theory, as any other 2-D theory can not quantify the effect of turbine height on its C_P. The effect of variation of $\mathcal{AR}$ on turbine performance, when the rotational speed, Ω is kept constant, can be compared by looking at the points connected by dotted lines in Fig. 15.

Pitch amplitude

Based on the blade element theory the lift and drag experienced by the blades is dependent not only on the angle of attack at which the airfoil is operating but also on the local Reynolds number. This makes the optimum pitch amplitude for turbine operation sensitive to the magnitude of inflow velocity experienced locally by the blades. Thus, a mechanism enabling the variation of pitch amplitude will effectively allow the wind turbine to deliver peak performance at a wider range of wind speeds and tip speed ratios. This will also help starting the wind turbine at lower wind speeds.

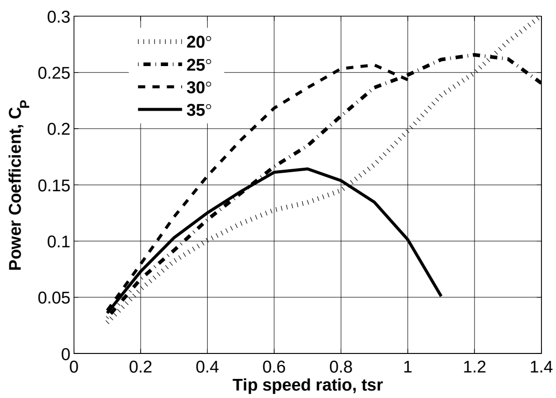


Fig. 16. Dependence of VAWT performance on pitch amplitude at 7 m/s wind speed

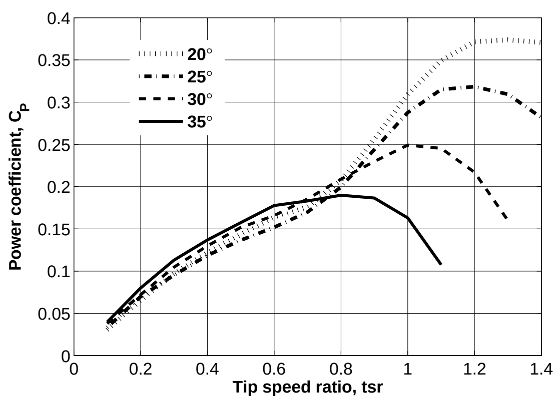


Fig. 17. Dependence of VAWT performance on pitch amplitude at 16 m/s wind speed

The performance of VAWT at different pitch amplitudes is compared at velocities of 7 and 16 m/s and the results are shown in figures 16 and 17. At 7 m/s the C_P is conspicuously dependent on the tip speed ratio of operation. At lower tip speed ratios 30° pitch amplitude has the airfoils performing at best angles. However, further at 35° pitch amplitude performance has shown drastic reduction. At higher operating tip speed ratios lower pitch amplitude are shown to deliver better performance. At $\lambda = 0.4$ there is an improvement of 25% in C_P with each 5° increment in pitch angle from 20° to 30°. This gap increases until the peak is achieved by the higher pitch amplitude curve.

At higher velocities this dependence is quite subtle. Fig. 6 at wind speed of 10 m/s has little dependence of performance on pitch amplitude up to stall limit. Similarly, the 16 m/s results predict 20° to be the pitch amplitude delivering best performance. Up to $\lambda = 0.8$, the difference in C_P between adjacent curves is negligible. However, beyond $\lambda = 1.1$, lower pitch amplitude curve separates from higher ones by a difference of more than 0.5 in C_P. This dependence calls for variable pitch amplitude mechanisms for turbines that operate in irregular wind environment or rotating at multiple tip speed ratios. This could also be the solution to the starting problem which left VAWTs far behind in the race of leader in the burgeoning wind turbine market.

Case for Variable Pitch Amplitude After establishing the dependence of VAWT performance on pitch amplitude, the application of this dependence is studied. A major problem with lift based Darrieus design is lack of sufficient torque at low tip speed ratios to self start the turbine. Variable pitch partially addresses this problem. However, the pitch changing arrangement that best facilitates the starting of turbine has slight disadvantage at higher tip speed ratios where the turbine is designed to operate. In this section performance of an improved design that allows changing the pitch amplitude is studied.

Fig. 18 shows the maximum proportion of available power that can be extracted at different tip speed ratios at 10 m/s wind, if the pitch amplitude is allowed to vary. This variation in pitch amplitude may be achieved by an active control system depending on the wind speed and RPM. These results show a sustained performance at a larger operating range of tip speed ratios, much larger than that with a fixed pitch amplitude. At lower tip speed ratio of 0.5, best performance is achieved by pitching the blades at 36° which is better than 8° output by more than 70%. As the tip speed ratio is increased the higher amplitude configurations stall, requiring to lower amplitude for sustaining the level of performance. Beyond λ = 2 higher pitch amplitudes are no longer viable and configuration as low as 10° amplitude are able to extract maximum power from the flow. This trend can be understood from the fact that the lower tip speed ratios prefer drag driven turbine as stated in Section . Thus higher angles of attack are favorable to generate more drag at the retreating side. On the contrary, lower angles of attack at the advancing side facilitates more lift in case of higher tip speed ratios. This effect is almost similar for all wind speeds as the power coefficient depends on non-dimensional tip speed ratio, apart from the slight impact of change in Reynolds number.

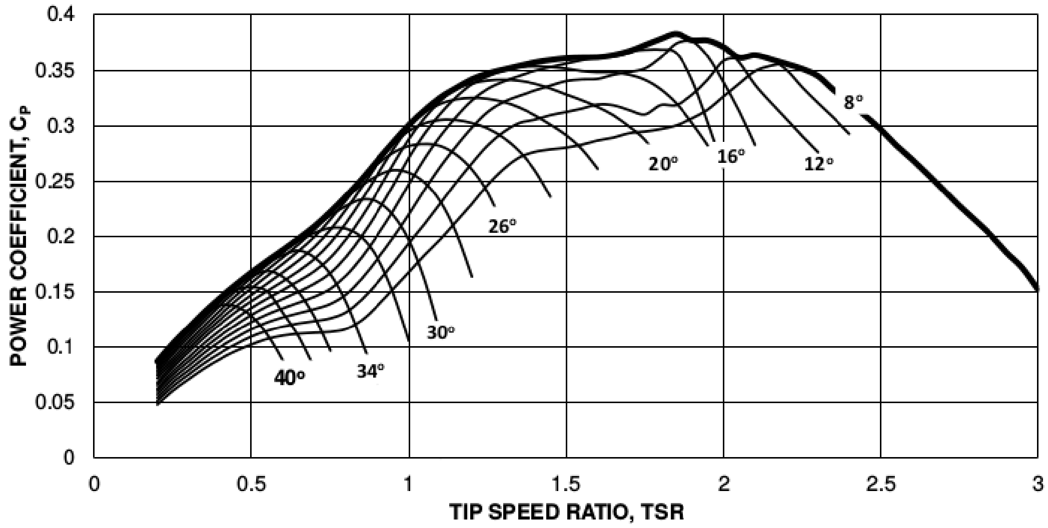


Fig. 18. Maximum possible power extracted by a variable pitch turbine

Directionality of flow

VAWTs are comparatively advantageous over the HAWTs because they do not require a yawing mechanism to align to the oncoming wind direction to extract energy. This leads to a less degree of mechanical complexity and continuity of operation in case of fluctuation in wind direction or turbulence. This advantage is, however, lost when a variable pitch design such as that of Fig 3 is used. Therefore, a systematic study is required to understand the impact of wind incoming at off-design angles. The analysis is performed by offsetting the pitch angle variation of Fig. 5 by incoming wind direction. The results as shown in Fig 19 depicts a significant drop in performance if the wind approaches from advancing side. However, VAWT is more tolerant for port-side wind upto 15° or more depending on tip speed ratio. Thus to tap the benefit of variable pitch design an active aligning mechanism is required. An ideal default alignment of offset would be such that on an average wind enters somewhere between 0° – 15°. This will take care of small disturbances prevalent in actual field.

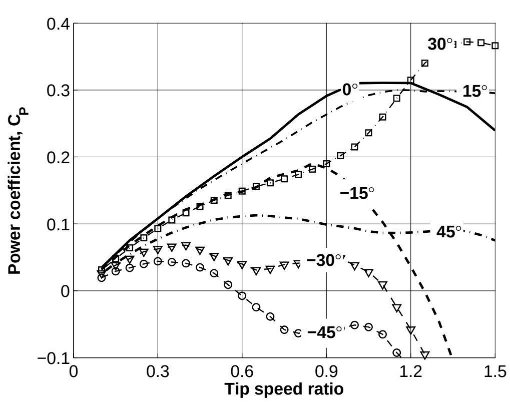


Fig. 19. Effect of flow direction on VAWT performance

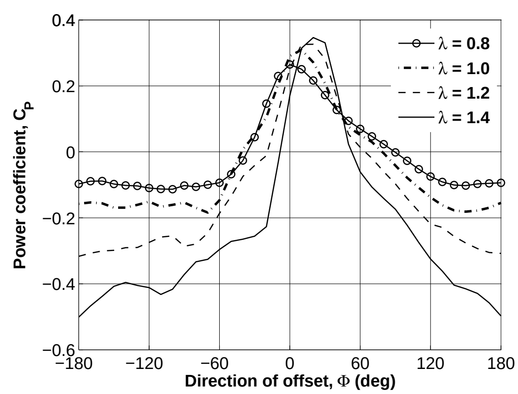


Fig. 20. Aerodynamic braking of VAWT by adjusting offset link

Commercial wind turbines regulations sometimes require redundant braking systems for turbine safety under high wind conditions and maintenance. Aerodynamic braking in HAWTs use spoiler flaps, pitching or yawing mechanisms. The dip in performance of VAWT at off-design angles of attack can be functionally used to brake the turbine. This can be achieved by adjusting the offset link phase angle such that net torque output is zero or negative. Offset controlled aerodynamic braking has an advantage over other methods because this capability is built into the design of the turbine and can be used as an storm protection device by manupulating the lift and drag forces on the blade section. This would help prevent the turbine from accelerating to very high rotational speeds to safeguard the structure and the generator from possible damage. Figure 20 shows the direction of the offset at which this braking can be achieved for various tip-speed ratios.

SUMMARY AND CONCLUSION

This paper discusses the modeling, performance prediction and fundamental understanding of the aerodynamic performance of a Vertical Axis Wind Turbine (VAWT) with variable amplitude blade pitching. An aerodynamic model based on double multiple stream tube theory coupled to lookup table based non-linear aerodynamics is developed. Based on this study for the turbine of Fig. 3 the following key conclusions can be made:

1. The variation in the amplitude of sinusoidal blade pitching allows for the maximum power extraction for wide range of tip speed ratios. Use of constant amplitude blade pitching results in quick reduction in C_P beyond the tip speed ratio corresponding to maximum value. High (about 35°) pitch amplitudes work best for tip speed ratios below 0.5 and the amplitude should be lowered upto 10° beyond tip speed ratio of 2. Maximum C_P of 0.38 is obtainable from a 10 m/s wind at $\lambda = 1.8$ and 18° pitch amplitude. The equilibrium tip speed ratio is dependent on a number of factors including freestream wind velocity, inertia of blades, and resistence at the bearings and electrodynamic brakes.

2. Parametric studies have shown an improvement of performance of VAWT with increasing solidity. The solidity is a function of number and chord of blades, and turbine aspect ratio. Increasing blade chord increases solidity and in turn improves performance at all tip speed ratios. This trend is, however, limited by 3-dimensional effects for calculating aerodynamic lift and drag which are ignored in the present study. Solidity is directly proportional to number of blades in the VAWT, which is also reflected in the increasing C_P of the turbine. But there is a decrease in performance per blade, as evident from the torque curves. This explains the difference between adjacent power curves to be getting smaller as blade number becomes high. When the aspect ratio was varied subjected to the condition that the blade swept area remains constant, the preformance curves show similar variation. This analysis confirms that solidity is the appropriate non-dimensional parameter to study turbine of any scale provided swept area is maintained.

3. A variable pitch VAWT is quite sensitive to the fluctuations in the wind direction. It can tolerate wind approaching from the advancing side by up to 15° while maintaining the same level of performance. However, beyond this $06\circ - 15^\circ$ range, performance mostly decreases, which requires the pitch alignment to be adjusted by an active control system depending on the wind direction. This attribute can be employed for aerodynamic braking of the turbine for the purpose of storm protection and maintenance. By adjusting the direction of offset link exactly opposite to default value, that is 180°, negative torque can be generated which is sufficient to brake the turbine at any tip speed ratio up to 1.4.

4. The current model which uses a combination of DMST and BEMT is a robust tool to predict the performance of lift-based VAWTs. It can be used to understand the physics behind the working, which enables preliminary design and sizing of the turbine. The time dependent forces and moments acting on the blades can be used for stress analysis and fatigue which are essential for designing turbine blades.

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