Department of Physics
IIT Kanpur, Semester II, 2017-18

PHY103A: Physics II **Homework # 1** **Instructors: AKJ & SC**

Problem 1.1: The plot of the following function looks like a hill on the xy plane:

$$h(x, y) = \exp[(2xy - 3x^2 - 4y^2 - 18x + 28y - 5)/60].$$

(a) Where is the top of the hill located?

(b) How high is the hill?

(c) In what direction is the slope steepest at the point (1,1)?

(d) How steep is the slope of $h(x, y)$ at the point (1,1) in the direction $\mathbf{n} = (\hat{x}x + \hat{y}y)$?

Problem 1.2: The separation vector can be written as $\mathbf{R} = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$. If $R = |\mathbf{R}|$ is the magnitude of the separation vector, show that gradient $\boldsymbol{\nabla} R$ is a unit vector parallel to $\mathbf{R}$.

Problem 1.3: Find the scaler function $\phi(x, y, z)$ whose gradient is $\boldsymbol{\nabla}\phi = (2xy + z^3)\hat{\mathbf{x}} + x^2\hat{\mathbf{y}} + 3xz^2\hat{\mathbf{z}}$.

Problem 1.4: Evaluate the gradient of the following scalar functions: (i) $\phi = \ln|\mathbf{r}|$ and (ii) $\phi = 1/|\mathbf{r}|$.

Problem 1.5:

(a) Calculate the divergence $\boldsymbol{\nabla}\cdot\mathbf{E}$ of the vector $\mathbf{E} = \hat{\mathbf{r}}/r^n$, where n is an integer and $\hat{\mathbf{r}}$ is the unit vector corresponding to vector $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$.

(b) What is $\boldsymbol{\nabla}\cdot\mathbf{E}$ when $n = 2$?

(c) In what physical context the vector-functions of type $\mathbf{E} = \hat{\mathbf{r}}/r^2$ are encountered?

Problem 1.6: Suppose $\boldsymbol{\nabla}\cdot\mathbf{E} = 0$, $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$, $\boldsymbol{\nabla}\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$ and $\boldsymbol{\nabla}\times\mathbf{B} = \dfrac{\partial\mathbf{E}}{\partial t}$. Show that

$$\nabla^2\mathbf{E} = \frac{\partial^2\mathbf{E}}{\partial t^2} \qquad \text{and} \qquad \nabla^2\mathbf{B} = \frac{\partial^2\mathbf{B}}{\partial t^2}.$$

Problem 1.7 (Griffiths Prob 1.33): Verify Stoke's theorem for the function $\mathbf{v} = (xy)\hat{\mathbf{x}} + (2yz)\hat{\mathbf{y}} + (3zx)\hat{\mathbf{z}}$, using the shaded area shown in Fig. 1(a).

Problem 1.8 (Griffiths Prob 1.53): Verify divergence theorem for the function $\mathbf{v} = r^2\cos\theta\hat{\mathbf{r}} + r^2\cos\phi\hat{\theta} - r^2\cos\theta\sin\phi\hat{\phi}$, using the one octant of the sphere of radius R as the volume [see Fig. 1(b)].

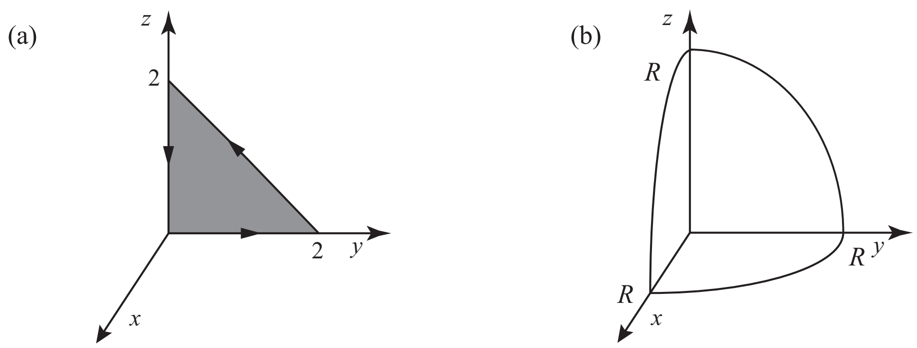

FIG. 1: