Department of Physics
IIT Kanpur, Semester II, 2017-18

PHY103A: Physics II **Homework # 2** **Instructors: AKJ & SC**

Problem 2.1: Divergence theorem in cylindrical coordinates (Griffiths 3rd ed. Prob 1.42)

Consider the following vector function $\mathbf{V} = s(2 + \sin^2 \phi)\hat{\mathbf{s}} + s \sin \phi \cos \phi \hat{\boldsymbol{\phi}} + 3z\hat{\mathbf{z}}$

(a) Find the divergence of $\mathbf{V}$.

(b) Verify the divergence theorem for $\mathbf{V}$, using the quarter cylinder shown in Fig. **??**.

(c) Find the curl of $\mathbf{V}$.

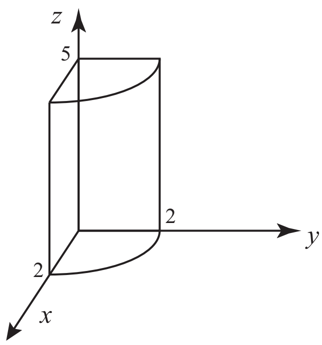

FIG. 1:

Problem 2.2: Applications of the Dirac delta function (Griffiths 3rd ed. Prob 1.46)

(a) What is the electric charge density $\rho(\mathbf{r})$ of a point charge q at $\mathbf{r}'$?

(b) What is the electric charge density of an electric dipole, that consists of a point charge $-q$ at the origin and a point charge $+q$ at $\mathbf{a}$?

(c) What is the electric charge density of a uniform, infinitesimally thin spherical shell of radius R and total charge Q, centered at the origin?

Problem 2.3: Calculating charge density given an Electric field (Griffiths 3rd ed. Prob 2.42)

What is the charge density corresponding to electric field $\mathbf{E}(\mathbf{r}) = \frac{A}{r}\,\hat{\mathbf{r}} + \frac{B \sin\theta \cos\phi}{r}\,\hat{\phi}$, where A and B are constants.

Problem 2.4: Physical Electrostatic field (Griffiths 3rd ed. Prob 2.20)

Which of these two can be a physical electrostatic field?

(a) $\mathbf{E} = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}$

(b) $\mathbf{E} = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$

Problem 2.5: Calculating electric field for a given charge distribution

(a) What is the electric field inside a uniformly charged sphere of charge density $\rho = k$ and radius R [See Fig. **??**(a)]? (Griffiths 3rd ed. Prob 2.12)

(b) The charge density of a spherical shell is $\rho = \dfrac{k}{r^2}$ in the region $a \leq r \leq b$ [See Fig. **??**(b)]. Find the electric field in the three regions: (i) $r \leq a$, (ii) $a \leq r \leq b$, (iii) $r \geq b$. (Griffiths 3rd ed. Prob 2.15)

(c) Two spheres, each of radius R and carrying charge densities $+\rho$ and ρ, respectively, are placed so that they partially overlap[Fig. **??**(c)]. Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value (Griffiths 3rd ed. Prob 2.18)

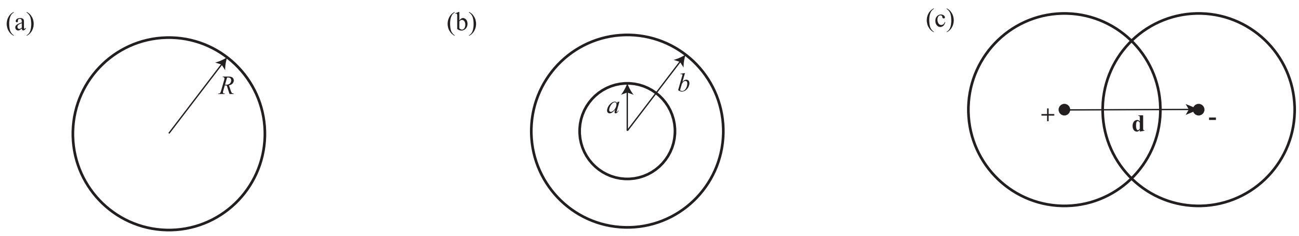

FIG. 2:

Problem 2.6: Scalar and Vector Potentials (Griffiths 3rd ed. Prob 1.52)

Consider the following vector functions:

(1) $\mathbf{v_1} = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} - 2xz\hat{\mathbf{z}}$.

(2) $\mathbf{v_2} = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3zx\hat{\mathbf{z}}$.

(3) $\mathbf{v_3} = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$.

(a) Which is of following vectors can be expressed as the gradient of a scalar?

(b) Which is of following vectors can be expressed as the curl of a vector? Find that vector.

Problem 2.7: Electric Potential (Griffiths 3rd ed. Prob 2.44)

An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the "north" pole and the center.

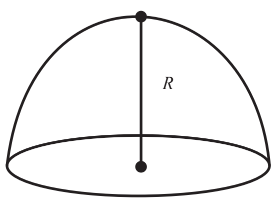

FIG. 3: