Semester II, 2017-18
Department of Physics, IIT Kanpur

PHY103A: Lecture # 14

(Text Book: Intro to Electrodynamics by Griffiths, 3rd Ed.)

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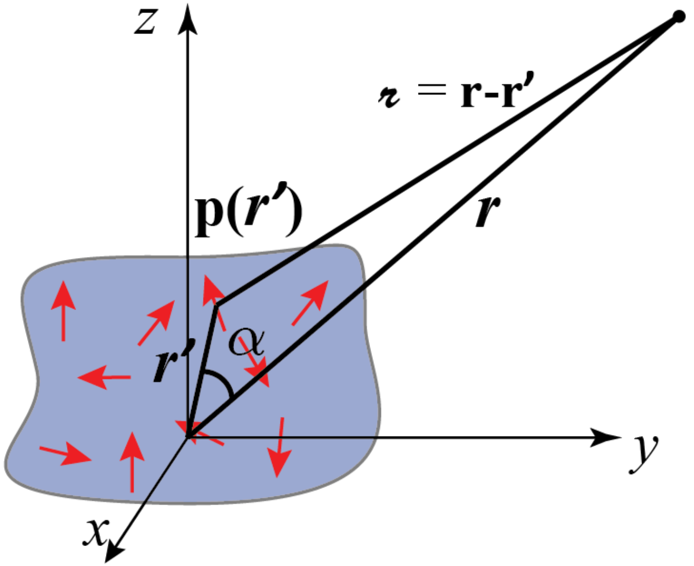
02-Feb-2018

Notes

- HW#5 and solutions to HW#4 have been posted on the course webpage.
- Quiz next Tuesday (06-Feb-2018), during tutorial
- Quiz timing: 11:00 – 11:15 am (15 Min).
- Course coverage (up to HW # 4, Griffiths Chap 3)

Summary of Lecture # 13:

- The field of a polarized object



$$V_{\text{dip}}(\mathbf{r}) = \int_{vol} \frac{1}{4\pi\epsilon_0} \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

$$= \frac{1}{4\pi\epsilon_0} \oint_{surf} \frac{\sigma_b}{\mathscr{r}} \, da' + \frac{1}{4\pi\epsilon_0} \int_{vol} \frac{\rho_b}{\mathscr{r}} d\tau'$$

$$\sigma_b = \mathbf{P}(\mathbf{r}') \cdot \widehat{\mathbf{n}}' \qquad \rho_b = -\boldsymbol{\nabla}' \cdot \mathbf{P}(\mathbf{r}')$$

Surface charge density   Volume charge density

- These bound charges are not just mathematical constructs. They are real.

- Total bound charge is zero.

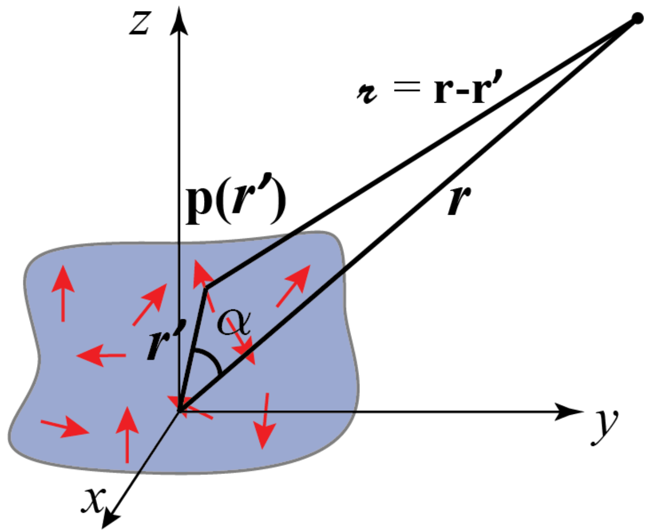
- Uniform Polarization $\boxed{\sigma_b \neq 0 \qquad \rho_b = 0}$

- Non-Uniform Polarization $\boxed{\sigma_b \neq 0 \qquad \rho_b \neq 0}$

Questions 1:

$$V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \oint_{surf} \frac{\sigma_b}{\mathscr{r}}\, da' + \frac{1}{4\pi\epsilon_0} \int_{vol} \frac{\rho_b}{\mathscr{r}} d\tau'$$

$$\sigma_b = \mathbf{P}(\mathbf{r}') \cdot \hat{\mathbf{n}}' \qquad \rho_b = -\boldsymbol{\nabla}' \cdot \mathbf{P}(\mathbf{r}')$$



Q: Is the decomposition unique?

Ans: Yes, it is. Because there is only one way in which an object can be divided into its surface and volume.

Questions 2:

Q: What happens to the Gauss's law when we have polarized objects?

Ans: We are going to answer this today.

Gauss's law in the presence of Dielectrics

Bound charges in a dielectric $\sigma_b = \mathbf{P}(\mathbf{r}) \cdot \hat{\mathbf{n}}$ $\rho_b = -\boldsymbol{\nabla} \cdot \mathbf{P}(\mathbf{r})$

In addition, there can be some free charge ρ_f in the dielectric as well

Total charge inside a dielectric is $\rho = \rho_b + \rho_f$

Gauss's law for the electric field is therefore

$$\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho = \rho_b + \rho_f = -\boldsymbol{\nabla} \cdot \mathbf{P}(\mathbf{r}) + \rho_f$$

Note: **E** is the total field, not just what is generated by the polarization

$$\boldsymbol{\nabla} \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = \rho_f$$

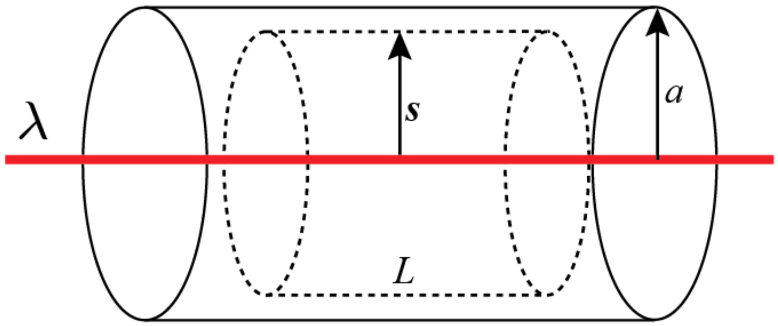
Define: $\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$ **Electric displacement**

$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_f$ Differential form of Gauss's law in presence of a dielectric

$\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f_{enc}}$ Integral form of Gauss's law in presence of a dielectric

Gauss's law in the presence of Dielectrics

Ex. 4.4 (Griffiths, 3rd Ed.): Find the electric displacement.



$$\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f_{enc}}$$

$$\oint D\hat{\mathbf{s}} \cdot da\,\hat{\mathbf{s}} = Q_{f_{enc}}$$

$$D(2\pi s L) = \lambda L$$

$$D = \frac{\lambda}{2\pi s} \quad \Rightarrow \quad \mathbf{D} = \frac{\lambda}{2\pi s}\hat{\mathbf{s}}$$

What is the electric field outside the dielectric $(s > a)$??

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}} \qquad (\text{since } \mathbf{D} \equiv \epsilon_0\mathbf{E} + \mathbf{P} \text{ and } \mathbf{P} = 0)$$

What is the electric field inside the dielectric $(s < a)$??

Since $\mathbf{P}$ inside is not known, electric field inside cannot be found.

Gauss's law in the presence of Dielectrics

Why did we not include the surface charge σ_b in the derivation of the Gauss's law?

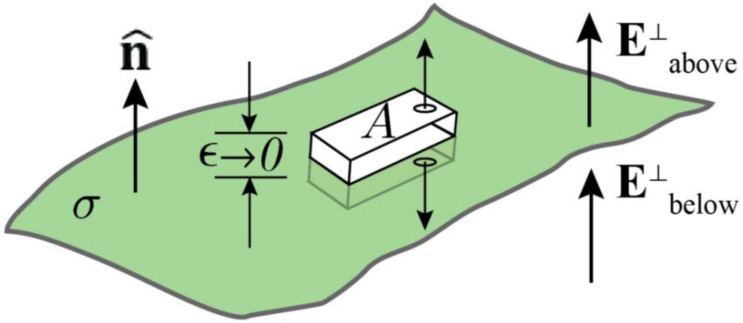
We don't need to. Volume charge takes care of everything.

When the Gaussian surface is drawn in the bulk of the material,
the surface charge does not come into picture.

On the surface, the Gauss's law anyway cannot be applied.

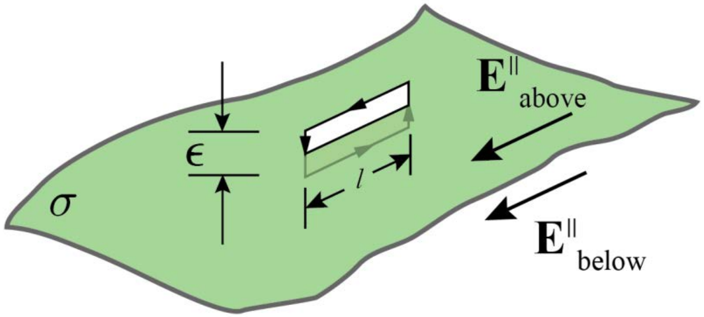
Boundary conditions in Dielectrics

Boundary Conditions on electric field



$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \longleftrightarrow \oint_{surf} \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\Rightarrow \boxed{E^{\perp}{}_{\text{above}} - E^{\perp}{}_{\text{below}} = \frac{\sigma}{\epsilon_0}}$$



$$\nabla \times \mathbf{E} = 0 \longleftrightarrow \oint_{path} \mathbf{E} \cdot d\mathbf{l} = \mathbf{0}$$

$$\Rightarrow \boxed{E^{||}{}_{\text{above}} - E^{||}{}_{\text{below}} = 0}$$

Boundary Conditions on electric displacement

$$\nabla \cdot \mathbf{D} = \rho_f \longleftrightarrow \oint \mathbf{D} \cdot d\mathbf{a} = Q_{fenc} \Rightarrow \boxed{D^{\perp}{}_{\text{above}} - D^{\perp}{}_{\text{below}} = \sigma_f}$$

$$\nabla \times \mathbf{D} = \epsilon_0(\nabla \times \mathbf{E}) + \nabla \times \mathbf{P} = \nabla \times \mathbf{P} \Rightarrow \boxed{D^{||}{}_{\text{above}} - D^{||}{}_{\text{below}} = P^{||}{}_{\text{above}} - P^{||}{}_{\text{below}}}$$

Electric Field Vs Electric Displacement

Similarity:

- Divergence and Gauss's Law: $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0} \iff \nabla \cdot \mathbf{D} = \rho_f$

- Boundary condition: $E^{\perp}{}_{\text{above}} - E^{\perp}{}_{\text{below}} = \dfrac{\sigma}{\epsilon_0} \iff D^{\perp}{}_{\text{above}} - D^{\perp}{}_{\text{below}} = \sigma_f$

Difference:

- Curl is not zero: $\nabla \times \mathbf{E} = 0$ But $\nabla \times \mathbf{D} = \epsilon_0(\nabla \times \mathbf{E}) + \nabla \times \mathbf{P} = \nabla \times \mathbf{P} \neq 0$

- $\mathbf{E} = -\nabla V$ But $\mathbf{D} \neq -\nabla f$

- $E^{||}{}_{\text{above}} - E^{||}{}_{\text{below}} = 0$ But $D^{||}{}_{\text{above}} - D^{||}{}_{\text{below}} = P^{||}{}_{\text{above}} - P^{||}{}_{\text{below}}$