Semester II, 2017-18
Department of Physics, IIT Kanpur

PHY103A: Lecture # 16

(Text Book: Intro to Electrodynamics by Griffiths, 3rd Ed.)

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Notes

- Quiz-1 is tomorrow (11:00 – 11:15 am).

Summary of Lecture # 15:

- Electric displacement and Dielectric constant $\mathbf{D} = \epsilon \mathbf{E}$

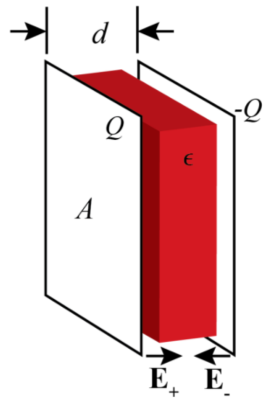
  Define: $\epsilon \equiv \epsilon_0 (1 + \chi_e)$ Define: $\epsilon_r \equiv \frac{\epsilon}{\epsilon_0} = (1 + \chi_e)$

- Clausius-Mossotti formula

  $$\mathbf{p} = \alpha\, \mathbf{E}_{\text{else}} \qquad \mathbf{P} = \epsilon_0 \chi_e\, \mathbf{E} \qquad \alpha = \frac{3\epsilon_0}{N} \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)}$$

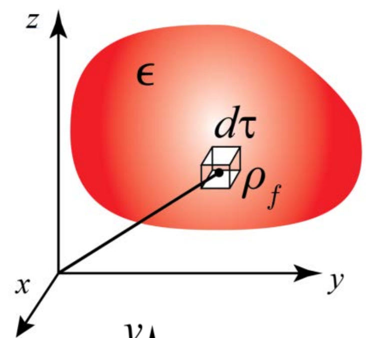
- Capacitance with dielectric filling

  $$C = \frac{Q}{V} = C_{\text{vac}} \epsilon_r$$



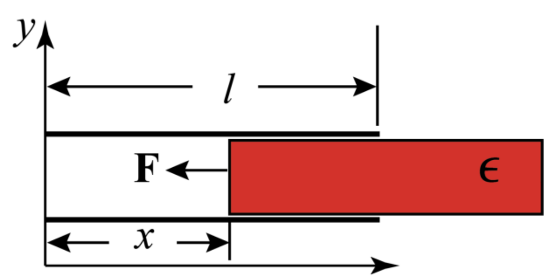
- Energy in dielectric systems

  $$W = \frac{1}{2} \int_{all\ space} \mathbf{D} \cdot \mathbf{E}\, d\tau$$



- Force in dielectric system

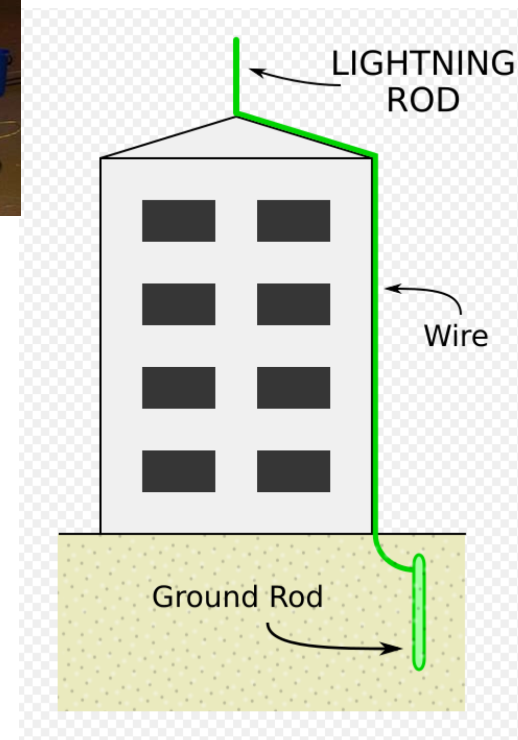
  $$F = \frac{1}{2} V^2 \frac{dC}{dx}$$



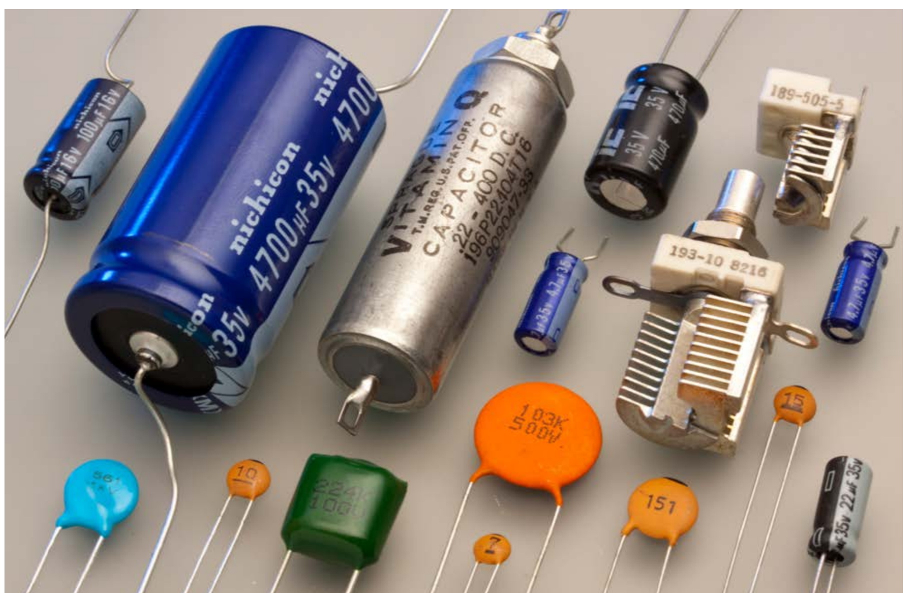
Electrostatics in action

- Faraday Cage

- Lightning rods



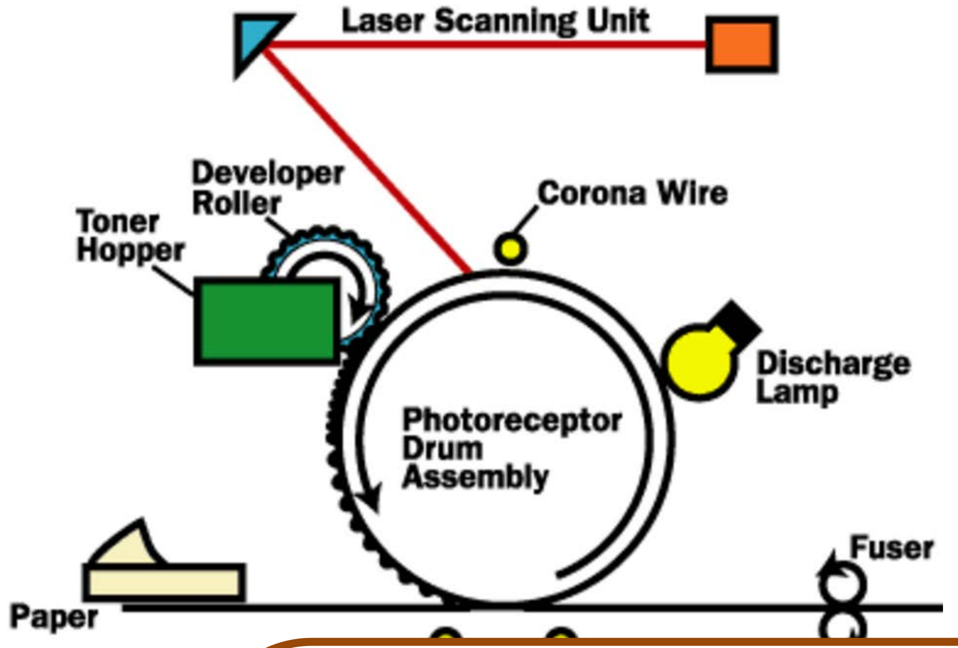
- Capacitor

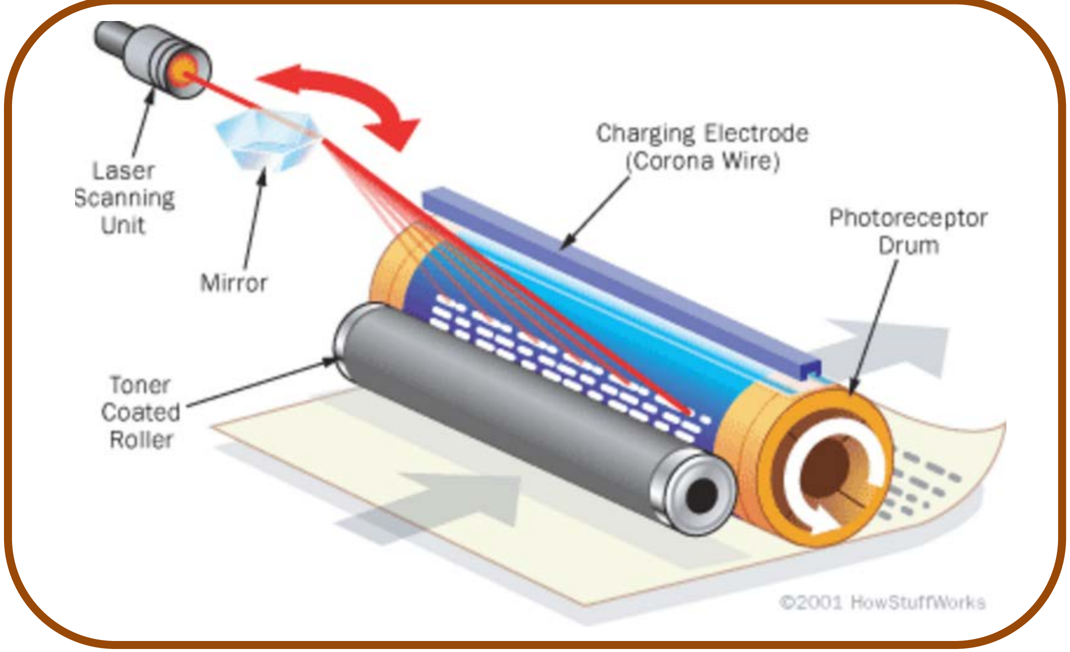


(From Wikipedia and Google Images)

Electrostatics in action

- Laser Printer





- Some fun stuff https://www.youtube.com/watch?v=jZEFuCxD7BE

(From Wikipedia and Google Images)

Magnetostatics

Stationary charges	→	Constant Electric field	→	Electrostatics
Steady currents	→	Constant Magnetic field	→	Magnetostatics
Non-stationary charges and/or Non-steady currents	→	Changing Electric and Magnetic field	→	Electrodynamics

Force on a point charge *Q*:

Electric Force: $\mathbf{F}_{\mathrm{elec}} = \mathrm{Q}\mathbf{E}$

Magnetic Force: $\mathbf{F}_{\mathrm{mag}} = \mathrm{Q}(\mathbf{v} \times \mathbf{B})$ **Lorentz Force Law**

Total Force:

$$\mathbf{F} = \mathbf{F}_{\mathrm{elec}} + \mathbf{F}_{\mathrm{mag}} = \mathrm{Q}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Work done by magnetic forces

The work done by a magnetic force is zero !

Why?

$$\mathbf{W}_{\text{mag}} = \int \mathbf{F}_{\text{mag}} \cdot d\mathbf{l} = \int Q(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$= \int Q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\, dt$$

$$= 0$$

- Magnetic forces do no work
- Magnetic forces can change the direction in which a particle moves.
- Magnetic forces do not change the speed with which a particle moves.

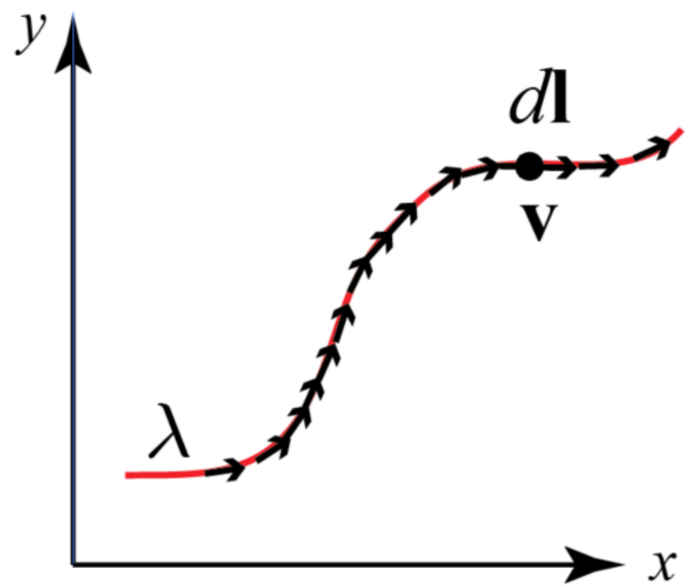
Currents

- Current is charge flow per unit time $I = \frac{dq}{dt}$
- It is measured in Coulombs per second, or Amperes (A).

Charge flowing in a wire is described by **Current**

$$\mathbf{I} = \frac{dq}{dt} = \frac{\lambda\, d\mathbf{l}}{dt} = \lambda \mathbf{v}$$

- The direction of current is in the direction of charge-flow.
- Conventionally, this is the direction opposite to the flow of electrons.



Magnetic force on a current carrying wire:

$$\mathbf{F}_{\text{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\, dq = \int (\mathbf{v} \times \mathbf{B})\, \lambda dl = \int (\mathbf{I} \times \mathbf{B})\, dl$$

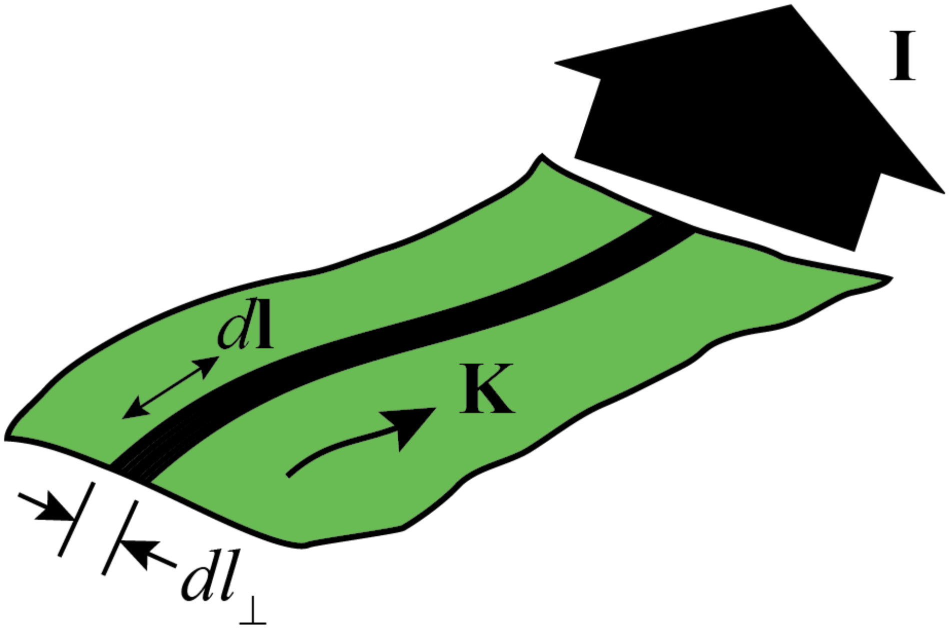
$$\boxed{\mathbf{F}_{\text{mag}} = \int (\mathbf{I} \times \mathbf{B})\, dl = I \int (d\mathbf{l} \times \mathbf{B})}$$

Currents

- Current is charge flow per unit time $I = \frac{dq}{dt}$
- It is measured in Coulombs per second, or Amperes (A).

Charge flowing on a surface is described by **surface current density**

$$\mathbf{K} = \frac{d\mathbf{I}}{dl_{\perp}} = \sigma \mathbf{v}$$



- Current density is a vector quantity.

Magnetic force on the surface current:

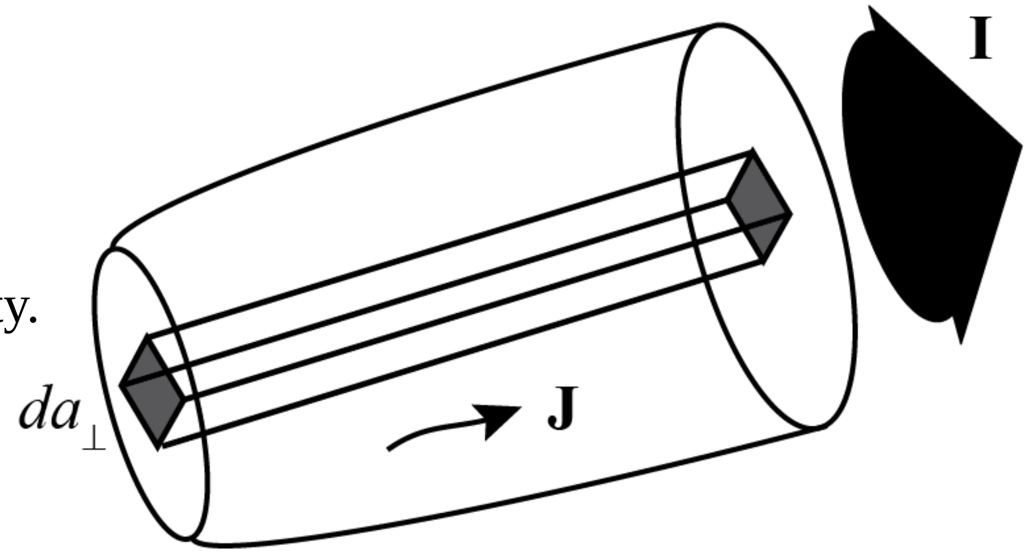
$$\mathbf{F}_{\text{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\, \sigma da = \int (\mathbf{K} \times \mathbf{B})\, da$$

Currents

- Current is charge flow per unit time $I = \frac{dq}{dt}$
- It is measured in Coulombs per second, or Amperes (A).

Charge flowing in a volume is described by **volume current density**

$$\mathbf{J} = \frac{d\mathbf{I}}{da_{\perp}} = \rho\mathbf{v}$$

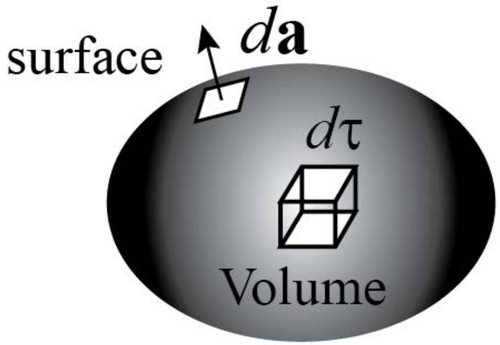


- Current density is a vector quantity.

Magnetic force on the volume current:

$$\mathbf{F}_{\text{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\,\rho d\tau = \int (\mathbf{J} \times \mathbf{B})\,d\tau$$

The Continuity Equation (Conservation of Charge)



$$\mathbf{J} = \frac{d\mathbf{I}}{da_\perp} \quad \Rightarrow \quad \mathrm{I} = \int_S \mathrm{J}\, da_\perp$$

$$\Rightarrow \quad \mathrm{I} = \int_S \mathbf{J} \cdot d\mathbf{a}$$

- Current crossing a surface
- Total charge per unit time crossing a surface

<u>For a closed surface</u>

$$\mathrm{I} = \oint_S \mathbf{J} \cdot d\mathbf{a}$$ Total charge per unit time crossing a closed surface

$$\oint_S \mathbf{J} \cdot d\mathbf{a} = \int_V (\nabla \cdot \mathbf{J}) d\tau$$ Total charge per unit time leaving the volume V.

But, total charge per unit time leaving the volume V is $-\frac{d}{dt}\left(\int_V \rho d\tau\right) = -\int_V \frac{d\rho}{dt}\, d\tau$

So, $\int_V (\nabla \cdot \mathbf{J}) d\tau = -\int_V \frac{d\rho}{dt}\, d\tau \quad \Rightarrow \quad \boxed{\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt}}$ **The Continuity Equation**