Department of Physics
IIT Kanpur, Semester II, 2017-18

PHY103A: Physics II | **Quiz # 1** | **Time: 15 Minutes** | **Max Marks: 10**

Name: .... **Roll No.:** ....

Problem 1: The separation vector can be written as $\mathbf{R} = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$. If $R = |\mathbf{R}|$ is the magnitude of the separation vector, calculate the gradient $\boldsymbol{\nabla} R$. (2.5 Marks)

Solution 1:
We have the magnitude of the separation vector given as

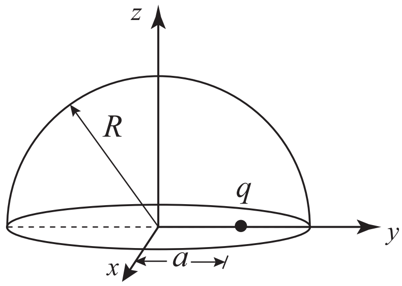
$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}.$$

Therefore the gradient is

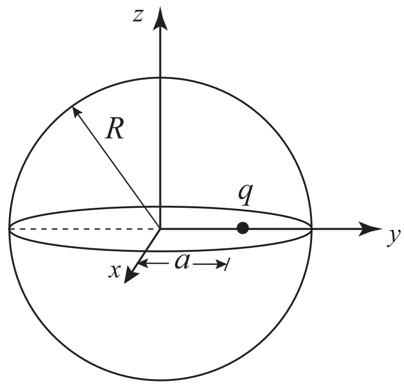
$$\begin{aligned}\boldsymbol{\nabla} R &= \left[\frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}\right]\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \\ &= \frac{(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = \frac{\mathbf{R}}{R}.\end{aligned}$$

Thus the gradient is a unit vector parallel to $\mathbf{R}$.

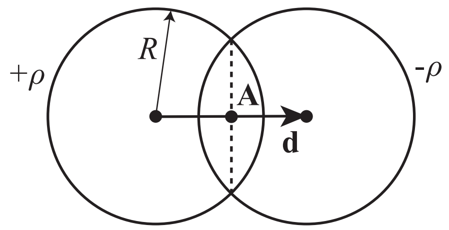
Problem 2: Consider a hemispherical bowl of radius R. A charge q is placed at a distance a from the center of the bowl as shown in the figure. Calculate the total electric flux through the hemispherical surface. (2.5 Marks)



Solution 2: Let us consider the whole spherical bowl as shown in the figure. The electric flux due to the charge through the entire spherical surface is equal to q/ϵ_0. We see that the electric fluxes through the upper hemispherical surface and the lower hemispherical surface are equal. Thus we have that the electric flux through the upper hemispherical surface is $q/2\epsilon_0$.

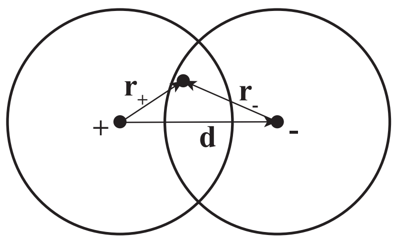


Problem 3: Two spheres, each of radius R and carrying charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the positive center to the negative center $\mathbf{d}$. Find the field at a point $\mathbf{A}$ midway between the centers in the overlap region. (2.5 Marks)



Solution 3: The field inside a uniformly charged sphere of charge density ρ is given by $\mathbf{E} = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho}{3\epsilon_0}\mathbf{r}$. Let us take $\mathbf{r}_+$ and $\mathbf{r}_-$ to be the radius vectors to a point in the overlap region from the centers of the positively and negatively charged sphere, respectively. The electric field in the overlap region is equal to the sum of the electric fields due to the positively and negatively charged spheres. Thus the electric field E_{overlap} at any point in the overlap region is

$$E_{\text{overlap}} = \frac{\rho}{3\epsilon_0}\mathbf{r}_+ - \frac{\rho}{3\epsilon_0}\mathbf{r}_- = \frac{\rho}{3\epsilon_0}(\mathbf{r}_+ - \mathbf{r}_+) = \frac{\rho}{3\epsilon_0}\mathbf{d}$$



Problem 4: Consider two concentric spherical shells, of inner and outer radii a and b, respectively. Suppose the inner one carries a charge q, and the outer one a charge -q (both of them uniformly distributed over the surface). Calculate the energy of this configuration. (2.5 Marks)

Solution 4: The electric field due to the two shells is given by $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$, for $(a < r < b)$, and $\mathbf{E}(\mathbf{r}) = 0$, otherwise. The energy of this configuration is:

$$W = \frac{\epsilon_0}{2}\int E^2 d\tau = \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2 \int_a^b \left(\frac{1}{r^2}\right)^2 4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$