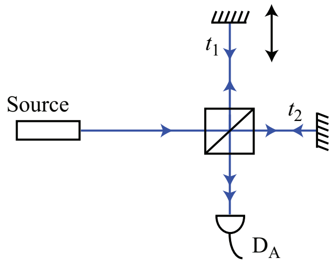
Homework # 4

Due in class on Thursday, September 27th, 2018

Problem 4.1: One-Photon Interference (2+2+4+2+2+2+2+2+2=20 marks)



(a) Consider the one-photon Michelson interferometer as shown above. The state of the photon at the source is $|\psi\rangle = \int_0^\infty V(\omega)|\omega\rangle d\omega$, where $|\omega\rangle$ represents one-photon at frequency ω and $V(\omega)$ is the field amplitude of a stationary random field at frequency ω. Show that the probability of detecting a photon at D_A is given by

$$P_A = |k_1|^2\langle\langle\psi|\hat{E}^{(-)}(t-t_1)\hat{E}^{(+)}(t-t_1)|\psi\rangle\rangle_e + |k_2|^2\langle\langle\psi|\hat{E}^{(-)}(t-t_2)\hat{E}^{(+)}(t-t_2)|\psi\rangle\rangle_e + 2\mathrm{Re}k_1^*k_2\langle\langle\psi|\hat{E}^{(-)}(t-t_1)\hat{E}^{(+)}(t-t_2)|\psi\rangle\rangle_e,$$

where k_1 and k_2 are overall constants and t_1 and t_2 are the travel-times of a photon in the two alternative paths.

(b) Use the representation $\hat{E}^{(+)}(t) = \int_0^\infty \hat{a}(\omega)e^{-i\omega t}d\omega$ and show that $\hat{E}^{(-)}(t) = \int_0^\infty \hat{a}^\dagger(\omega)e^{i\omega t}d\omega$.

(c) Since the field is stationary we can write $\langle V^*(\omega)V(\omega')\rangle_e = S(w)\delta(\omega-\omega')$, where $S(\omega)$ is the spectral density of the field. Using the above relation show that

$$\langle\langle\psi|\hat{E}^{(-)}(t-t_1)\hat{E}^{(+)}(t-t_2)|\psi\rangle\rangle_e = \int_0^\infty S(\omega)e^{-i\omega\tau}d\omega$$

(d) Next, assuming that the field is quasi-monochromatic with mean frequency at ω_0, show that

$$\langle\langle\psi|\hat{E}^{(-)}(t-t_1)\hat{E}^{(+)}(t-t_2)|\psi\rangle\rangle_e = e^{-i\omega_0\tau}\int_{-\infty}^\infty S_0(\omega)e^{-i\omega\tau}d\omega$$

where $S_0(\omega) = S(\omega+\omega_0)$.

(e) Finally, derive the one-photon interference law

$$P_A = |k_1|^2 I + |k_1|^2 I + 2|k_1||k_2|I|\gamma(\tau)|\cos(\omega_0\tau+\phi)$$

where $I = \int_{-\infty}^\infty S_0(\omega)d\omega$; $\gamma(\tau) = \int_{-\infty}^\infty S_0(\omega)e^{-i\omega\tau}d\omega/\int_{-\infty}^\infty S_0(\omega)d\omega$; and $\phi = \arg[k_1^*k_2] + \arg[\gamma(\tau)]$.

(f) The above expression is the same as that we get in the classical treatment. Discuss what are differences in the two treatments. Describe the situations in which the classical treatment would be inadequate.

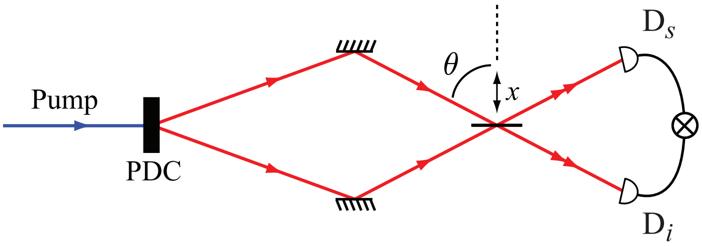
(g) Take $S(\omega) = 1/(\sqrt{2\pi}\Delta\omega)\exp[-(\omega-\omega_0)^2/(2\Delta\omega^2)]$. Here $\Delta\omega$ is the standard deviation of $S(\omega)$, that is, the frequency-width of the field. Calculate $\gamma(\tau)$ and show that the coherence time, i.e., the standard deviation of $\gamma(\tau)$ is $1/\Delta\omega$.

(h) Describe, both classically and quantum mechanically, what happens to the interference when $\tau \gg \tau_c$.

(i) Assume that the central wavelength of the source is λ and it has a wavelength-bandwidth $\Delta\lambda =$. Calculate the frequency bandwidth $\Delta\omega$ in terms of λ and $\Delta\lambda$. Calculate τ_c and plot $|\gamma(\tau)|$ for $\lambda = 700$nm and $\Delta\lambda =$ (i) 1nm , (ii) 10 nm and (iii) 100 nm.

(j) Explain why the coherence time depends on frequency-bandwidth? Also, can there be a situation in which the frequency-bandwidth has no affect on the coherence time of the field?

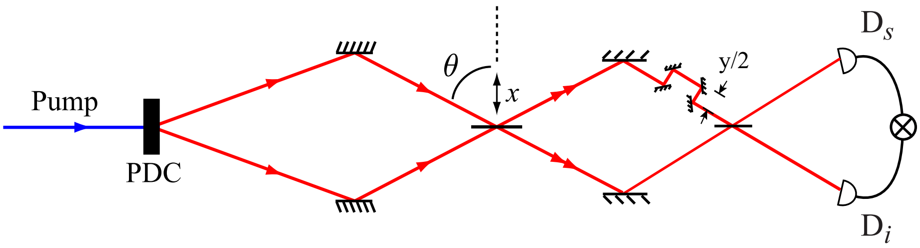
Problem 4.2: Hong-Ou-Mandel Effect (5+5+10=20 marks)



(a) Consider the Hong-Ou-Mandel experiment. Assuming $\theta \approx 0$ and $k_d = 0$, find the coincidence count rate of the two detectors.

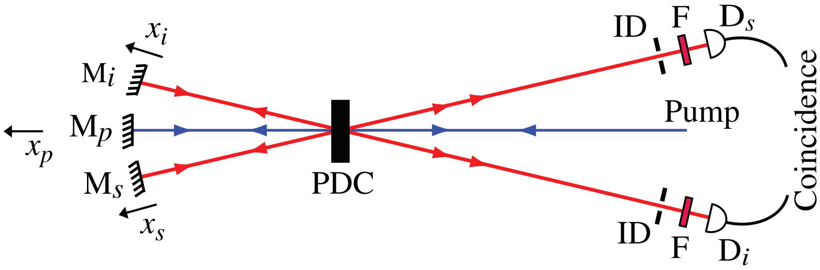
(b) Find the one-photon count rate at detectors D_s and D_i.

(c) Refer to the figure below. Assuming $x = 0$, calculate the coincidence count rate R_{si} as a function of the displacement y. If the central wavelength of each of the signal and idler field is λ_{s0}, find the fringe period of the coincidence interference pattern. (The above setup is the idea of quantum lithography. For more details refer to "A.N.Boto, et al. Phys. Rev. Lett. 85, 2733 (2000)."



Problem 4.3: Double-pass experiment (10+5+5+10+5=35 marks)

(a) Consider the situation shown in the figure. This is the double-pass setup that we discussed in the class. Here x_s, x_i and x_p are the displacements of the signal (M_s), idler (M_i) and the pump (M_p) mirrors, respectively, from the balanced position at which all the mirrors are equidistant from the PDC crystal. Calculate the coincidence count rate R_{si} and the one-photon count rates R_s and R_i as a function of the displacement of the three mirrors.



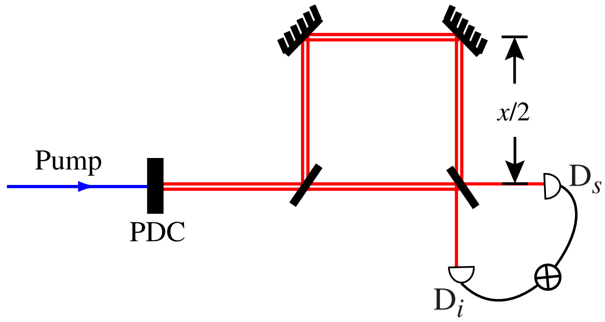
(b) Take $x_p = 0$. Now, find a way of moving the remaining two mirrors so that we only see the effect of the pump coherence function $\gamma(\Delta L)$.

(c) Take $x_p = 0$. And find a way of moving the remaining two mirrors so that we only see the effect of the signal-idler coherence function $\gamma'(\Delta L')$.

(d) What should be the condition for seeing a Hong-Ou-Mandel type profile, that is, a dip in the coincidence count rate R_{si}. What should be the condition for observing a hump in the coincidence count rate. Can this dip/hump be explained in terms of photon-bunching effect? Plot R_{si} as a function of the mirror displacements to show the dip/hump profile; plot both the curves on the same plot.

(e) Usually in a one-photon interference experiment, there are more than one ports for the photons to go out. This means that whenever we are observing a minimum in the intensity at one port, the intensity at the other port is at its maximum such that the total intensity is conserved. However, in the above experiment the two photons don't have any ports other than D_s and D_i. So, how do you explain the photon-number conservation when we are at a minimum. Where do the photons go? For more details and explanations, you could refer to "Phys. Rev. Lett. 72, 629632 (1994)."

Problem 4.4: Two-photon Coherence function (10+5+10=25 marks)



(a) Assuming that the coincidence detection system is much faster that the delay time x/c, calculate the coincidence count rate R_{si} and the one-photon count rate R_s and R_i in terms of x.

(b) Describe where do the entangled photons go when minima are observed in the coincidence count rate. Note that this setup is an alternative way of doing quantum lithography as described in problem 4.2?

(c) Design another experimental setup that would keep $\gamma'(\Delta L') = 1$ and that could be used for quantum lithography.