Solutions: Homework # 2

Solution 2.1

We have

$$\Gamma(t_1, t_2) = \iint W(\omega_1, \omega_2) e^{i\omega_1 t_1} e^{-i\omega_2 t_2} d\omega_1 d\omega_2$$

(a) For $W(\omega_1, \omega_2) = S(\omega_1)\delta(\omega_1 - \omega_2)$, the cross-correlation function becomes

$$\Gamma(t_1, t_2) = \int S(\omega) e^{-i\omega(t_2 - t_1)} d\omega = \int S(\omega) e^{-i\omega\tau} d\omega = \int \frac{1}{\sqrt{2\pi}\Delta\omega} \exp\left[-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\right] e^{-i\omega\tau} d\omega = e^{-i\omega_0\tau} \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right]$$

Therefore the intensity is

$$I(t) = \Gamma(t, t) = \int S(\omega) d\omega = \int \frac{1}{\sqrt{2\pi}\Delta\omega} \exp\left[-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\right] d\omega = 1,$$

which is a constant as a function of time.

(b) We see that the cross-correlation function depends only on the time-difference $\tau = t_2 - t_1$ and is the Fourier transform of the spectral density $S(\omega)$. From the expression of the cross-correlation function derived above we find that the degree of coherence function is $\gamma(\tau) = \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right]$. Thus $1/\Delta\omega$, which is the standard deviation of $\gamma(\tau)$, can be taken as the coherence time of the field.

Solution 2.2

(a) We have

$$\tilde{v}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(t) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[a e^{i(\omega - \omega_a)t} + b e^{i(\omega - \omega_b)t}\right] d\omega = \frac{1}{2\pi} \left[a\delta(\omega - \omega_a) + b\delta(\omega - \omega_b)\right]$$

Therefore, the cross-spectral density function is given by

$$\begin{aligned} W(\omega_1, \omega_2) &= \langle \tilde{v}^*(\omega_1)\tilde{v}(\omega_2) \rangle = \tilde{v}^*(\omega_1)\tilde{v}(\omega_2) \\ &= \frac{1}{(2\pi)^2} \Big[a^2\delta(\omega_1 - \omega_a)\delta(\omega_2 - \omega_a) + b^2\delta(\omega_1 - \omega_b)\delta(\omega_2 - \omega_b) \\ &\qquad + ab\delta(\omega_1 - \omega_a)\delta(\omega_2 - \omega_b) + ab\delta(\omega_1 - \omega_b)\delta(\omega_2 - \omega_a) \Big] \end{aligned}$$

(b) The intensity is given by

$$I(t) = \langle V^*(t)V(t) \rangle = V^*(t)V(t) = a^2 + b^2 + 2ab\cos(\omega_a - \omega_b)t$$

(c) the temporal correlation function is given by

$$\begin{aligned} \Gamma(t_1, t_2) &= \langle V^*(t_1)V(t_2) \rangle = V^*(t_1)V(t_2) \\ &= (a e^{i\omega_a t_1} + b e^{i\omega_b t_1})(a e^{-i\omega_a t_2} + b e^{-i\omega_b t_2}) \\ &= a^2 e^{i\omega_a(t_1 - t_2)} + b^2 e^{i\omega_b(t_1 - t_2)} + ab e^{i(\omega_a t_1 - \omega_b t_2)} + ab e^{i(\omega_b t_1 - \omega_a t_2)} \end{aligned}$$

(d) Since the intensity and the temporal correlation function depend on the time origin, it is clear that the field is not stationary.

(e) The degree of temporal coherence is given by

$$\gamma(t_1, t_2) = \frac{|\Gamma(t_1, t_2)|}{\sqrt{\Gamma(t_1, t_1)\Gamma(t_2, t_2)}} = \frac{|V^*(t_1)V(t_2)|}{\sqrt{V^*(t_1)V(t_1)V^*(t_2)V(t_2)}} = 1$$

Solution 2.3

(a) We have

$$\Gamma(t_1, t_2) = \iint W(\omega_1, \omega_2) e^{i\omega_1 t_1} e^{-i\omega_2 t_2} d\omega_1 d\omega_2$$

For $W(\omega_1, \omega_2) = [S_+[\omega_1 - (\omega_0 + \Omega)] + S_-[\omega_1 - (\omega_0 - \Omega)]]\delta(\omega_1 - \omega_2)$, the cross-correlation function becomes

$$\Gamma(t_1, t_2) = \iint [S_+[\omega_1 - (\omega_0 + \Omega)] + S_-[\omega_1 - (\omega_0 - \Omega)]]\delta(\omega_1 - \omega_2) e^{i\omega_1 t_1} e^{-i\omega_2 t_2} d\omega_1 d\omega_2$$

or,

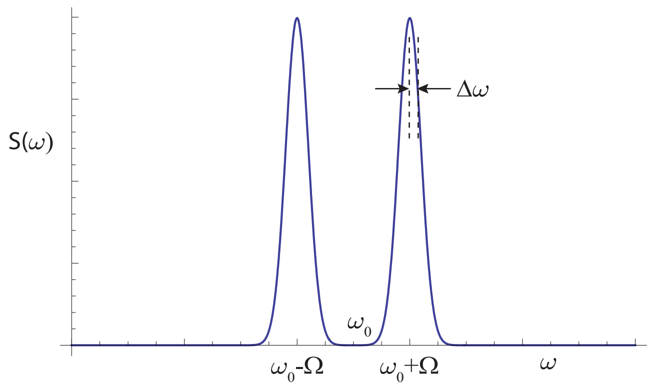
$$\Gamma(\tau) = \int S_+[\omega_1 - (\omega_0 + \Omega)] e^{-i\omega\tau} d\omega + \int S_-[\omega_1 - (\omega_0 - \Omega)] e^{-i\omega\tau} d\omega$$

Therefore the intensity is

$$I(t) = \Gamma(t, t) = \text{const}$$

We find that the cross-correlation function depends only on the time difference and the intensity is independent of time. Therefore, the field is wide sense stationary.

(b) See the plot below



(c) Suppose that the input field is $V(t)$. Therefore, the field at an output port of the Mach-Zehnder interferometer is given by $V(t) = kV(t - t_1) + kV(t - t_2)$. The intensity at the output is therefore given by

$$\begin{aligned} I_0 = \langle V^*(t)V(t)\rangle &= |k^2| \left[\langle V(t - t_1)|^2 + |V(t - t_2)|^2 + \langle V^*(t - t_1)V(t - t_2)\rangle + \text{c.c.}\right] \\ &= |k^2| \left[\Gamma(0) + \Gamma(0) + \Gamma(\tau) + \Gamma^*(\tau)\right] \end{aligned}$$

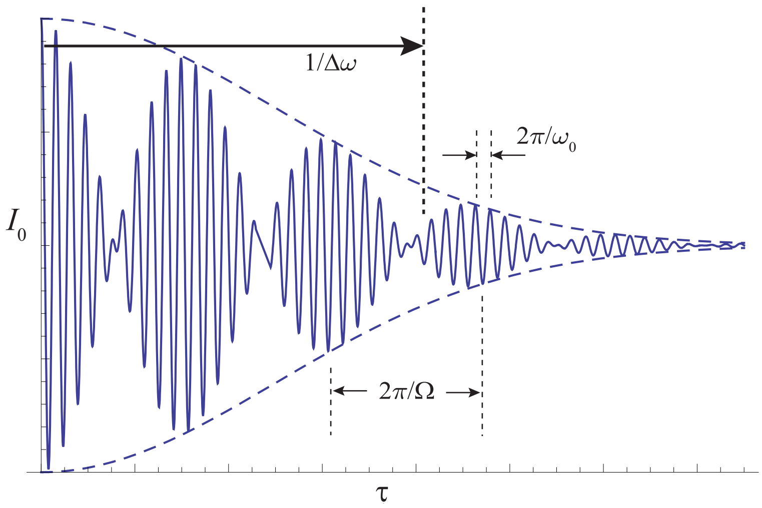
We have

$$\Gamma(\tau) = \int S_+[\omega_1 - (\omega_0 + \Omega)]e^{-i\omega\tau}d\omega + \int S_-[\omega_1 - (\omega_0 - \Omega)]e^{-i\omega\tau}d\omega$$
$$= e^{-i(\omega_0+\Omega)\tau} \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right] + e^{-i(\omega_0-\Omega)\tau} \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right]$$
$$= 2e^{-i\omega_0\tau} \cos(\Omega\tau) \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right]$$

From the above expression, we get that $\Gamma(0) = 2$. Therefore, the intensity I_0 can be written as

$$I_0 = |k^2| \left[2 + 2 + 2\exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right] \cos(\Omega\tau)\cos(\omega_o\tau)\right]$$
$$= 4|k^2| \left[1 + \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right] \cos(\Omega\tau)\cos(\omega_o\tau)\right]$$

(d) See the plot below

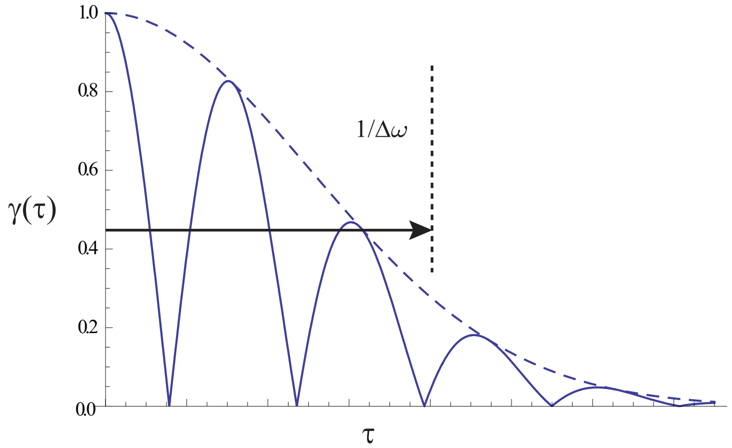


(e) The degree of coherence function can be calculated as

$$\gamma(\tau) = \frac{|\Gamma(\tau)|}{\sqrt{\Gamma(0)\Gamma(0)}} = \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right] \cos(\Omega\tau)$$

We find that the degree of coherence function is a product of two functions, one is decaying exponential and the other one is an oscillating function. As a result the degree of coherence goes down to zero for some τ and then starts increasing again. So, one cannot take the standard deviation of the degree of coherence function as the coherence time in this case. However, the standard deviation of the exponential function can still be taken as the coherence time since that overall decides the time scale after which the coherence function does not increase up to a very large value. So, $1/\Delta\omega$ can be taken as the coherence time. Please note that $\Delta\omega$ is the frequency bandwidth of the individual peaks in the spectral density and is not the frequency bandwidth of the total spectral density.

(f) See the plot below



Solution 2.4

(a)

$$W(\mathbf{r_1},\mathbf{r_2},\omega) = -\left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}\int_{\mathcal{A}} S(\mathbf{r}',\omega)e^{-ik(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}'}d^2\mathbf{r}', \tag{1}$$

Since the source is a square aperture, we'll work in the cartesian coordinate system. Also, since the source is spatially incoherent, we have over the area of interest, $S(\mathbf{r}',\omega) \to S(x',y',0;\omega) = S_0$, where K is a constat. Therefore, we can write the above equation as

$$W(x_1,y_1,z;x_2,y_2,z;\omega) = -S_0\left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}\int_{\mathcal{A}} e^{-ik(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}'}dx'dy', \tag{2}$$

Since we are are working within the far-field approximation we have

$$\mathbf{s_1} = \left(\frac{x_1}{r_1},\frac{y_1}{r_1},\frac{z}{r_1}\right) \approx \left(\frac{x_1}{z},\frac{y_1}{z},1\right)$$
$$\mathbf{s_2} = \left(\frac{x_2}{r_2},\frac{y_2}{r_2},\frac{z}{r_2}\right) \approx \left(\frac{x_2}{z},\frac{y_2}{z},1\right)$$
$$(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}' = \left[\frac{(x_2-x_1)x'}{z}+\frac{(y_2-y_1)y'}{z}\right]$$

The expression for the cross-spectral density now becomes

$$W(x_1,y_1,z;x_2,y_2,z;\omega) = \left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}K\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} e^{-i\frac{k}{z}[(x_2-x_1)x'+(y_2-y_1)y']}dx'dy'$$
$$W(x_1,y_1,z;x_2,y_2,z;\omega) = \left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}K\int_{-a/2}^{a/2} e^{-i\frac{k(x_2-x_1)x'}{z}}dx'\int_{-a/2}^{a/2} e^{-i\frac{k(y_2-y_1)}{z}y'}dy'$$
$$W(x_1,y_1,z;x_2,y_2,z;\omega) = \left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}Ka^2\text{Sinc}\left(\frac{k(x_2-x_1)a}{2z}\right)\text{Sinc}\left(\frac{k(y_2-y_1)a}{2z}\right)$$

So, we got the two Sinc functions as expected.

(b) The first zero of the Sinc function occurs at $x_2-x_1 = 2\pi z/(ka)$ and therefore $[4\pi z/(ka)]^2$ can be taken to be the coherence area of the source. As we can see, the coherence area increases as a function of z. This is because as z increases the sum total of the fields coming from the source to any two arbitrary points in the far-field become more and more indistinguishable.

(c)

$$W(\mathbf{r_1},\mathbf{r_2},\omega) = -\left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}\int_{\mathcal{A}} S(\mathbf{r}',\omega)e^{-ik(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}'}d^2\mathbf{r}', \tag{3}$$

We'll work in the cartesian coordinate system. Since the source is spatially incoherent, we have over the area of interest, $S(\mathbf{r}',\omega) \to S(x',y',0;\omega) = S_0$. Therefore, we can write the above equation as

$$W(x_1,y_1,z;x_2,y_2,z;\omega) = -S_0\left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2}\int_{\mathcal{A}} e^{-ik(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}'}dx'dy', \tag{4}$$

Since we are are working within the far-field approximation we have

$$\mathbf{s_1} = \left(\frac{x_1}{r_1},\frac{y_1}{r_1},\frac{z}{r_1}\right) \approx \left(\frac{x_1}{z},\frac{y_1}{z},1\right)$$

$$\mathbf{s_2} = \left(\frac{x_2}{r_2},\frac{y_2}{r_2},\frac{z}{r_2}\right) \approx \left(\frac{x_2}{z},\frac{y_2}{z},1\right)$$

$$(\mathbf{s_2}-\mathbf{s_1})\cdot\mathbf{r}' = \left[\frac{(x_2-x_1)x'}{z} + \frac{(y_2-y_1)y'}{z}\right]$$

The expression for the cross-spectral density now becomes

$$W(x_1,y_1,z;x_2,y_2,z;\omega) = \left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2} S_0 \int_{-b/2}^{b/2} e^{-i\frac{k(x_2-x_1)x'}{z}}dx'$$

$$\times\left[\int_{-d-a/2}^{-d+a/2} e^{-i\frac{k(y_2-y_1)}{z}y'}dy' + \int_{d-a/2}^{d+a/2} e^{-i\frac{k(y_2-y_1)}{z}y'}dy'\right]$$

$$W(x_1,y_1,z;x_2,y_2,z;\omega) = \left(\frac{k}{2\pi}\right)^2 \frac{\exp[ik(r_2-r_1]}{r_1 r_2} S_0 a^2 \text{Sinc}\left(\frac{k(x_2-x_1)b}{2z}\right)\text{Sinc}\left(\frac{k(y_2-y_1)a}{2z}\right)2a\cos\left(\frac{k(y_2-y_1)d}{z}\right).$$

Solution 2.5

(a) The cross-spectral density function for this beam is given by:

$$W(x_1,y_1,z;x_2,y_2,z) = U^*(x_1,y_1,0)U(x_2,y_2,0)$$

The degree of coherence of such a beam can now be written as

$$\mu(x_1,y_1,z;x_2,y_2,z) = \frac{W(x_1,y_1,z;x_2,y_2,z)}{\sqrt{U^*(x_1,y_1,0)U(x_1,y_1,0)}\sqrt{U^*(x_2,y_2,0)U(x_2,y_2,0)}} = 1$$

Therefore this beam is fully coherent and the coherence area of this beam is infinite.

(b) The spectral amplitude $a(q_x,q_y)$ is the amplitude of the plane-wave components of the field. The spectral amplitude is related with the transverse field amplitude through the following formula:

$$a(q_x,q_y) = \left(\frac{1}{2\pi}\right)^2 \iint_{\infty}^{\infty} U(x,y,0)e^{-i(q_x x+q_y y)}dxdy$$

For the Gaussian field amplitude of this problem, the spectral amplitude is given by

$$a(q_x,q_y) = A\left(\frac{1}{2\pi}\right)^2 \iint_{\infty}^{\infty} \exp\left(-\frac{x^2+y^2}{w_0^2}\right)e^{-i(q_x x+q_y y)}dxdy$$

We already derived Gaussian integrals in Homework 1. Here we would simple apply them. Making use of the

Gaussian integral

$$\int_{\infty}^{\infty} e^{-\alpha x^2+\beta x} dx = e^{\beta^2/4\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2},$$

we get

$$\begin{aligned} a(q_x, q_y) &= A\left(\frac{1}{2\pi}\right)^2 \int_{\infty}^{\infty} \exp\left(-\frac{x^2}{w_0^2}\right) e^{-iq_x x} dx \int_{\infty}^{\infty} \exp\left(-\frac{y^2}{w_0^2}\right) e^{-iq_y y} dy \\ &= A\left(\frac{1}{2\pi}\right)^2 \left(\frac{\pi}{1/w_0^2}\right)^{1/2} e^{-w_0^2 q_x^2/4} \left(\frac{\pi}{1/w_0^2}\right)^{1/2} e^{-w_0^2 q_y^2/4} \\ &= A\frac{w_0^2}{4\pi} e^{-w_0^2(q_x^2+q_y^2)/4} \end{aligned}$$

We see that the width of the spectral amplitude is inversely proportional to the width of the filed amplitude, i.e., the width of the beam. This means that a narrower beam has much broader spectral content.

(c) Using the diffraction integral and the field amplitude for the Gaussian beam, we can write the field amplitude at $z = z$ as

$$U(x, y, z) = -\frac{ik}{2\pi} \iint A \exp\left[-\frac{x'^2 + y'^2}{w_0^2}\right] \frac{e^{ikr}}{r} dx' dy'$$

Within the approximation, $z >> (x - x')^2 + (y - y')^2$, we have

$$\begin{aligned} r = [(x - x')^2 + (y - y')^2 + z^2]^{1/2} &= z\left[z + \frac{(x - x')^2 + (y - y')^2}{z^2}\right]^{1/2} \\ \approx z\left[z + \frac{(x - x')^2 + (y - y')^2}{2z^2}\right] &= z + \frac{(x - x')^2 + (y - y')^2}{2z} \end{aligned}$$

Substituting the above approximation for r in the diffraction integral we get,

$$U(x, y, z) = -\frac{iAk}{2\pi z} e^{ikz} e^{ik(x^2+y^2)/2z} \iint \exp\left[-\frac{x'^2 + y'^2}{w_0^2}\right] e^{-ik(xx'+yy')/z} e^{ik_0(x'^2+y'^2)/2z} dx' dy'$$

The above equation can be written as a product of two separate integrals

$$\begin{aligned} U(x, y, z) = -\frac{iAk}{2\pi z} e^{ikz} e^{ik(x^2+y^2)/2z} &\int_{\infty}^{\infty} \exp\left[-\frac{x'^2}{w_0^2}\right] e^{-ikxx'/z} e^{ikx'^2/2z} dx' \\ \times &\int_{\infty}^{\infty} \exp\left[-\frac{y'^2}{w_0^2}\right] e^{-ikyy'/z} e^{iky'^2/2z} dy' \end{aligned}$$

Rearranging, we get

$$\begin{aligned} U(x, y, z) = -\frac{iAk_0}{2\pi z} e^{ikz} e^{ik(x^2+y^2)/2z} &\int_{\infty}^{\infty} \exp\left[-x'^2\left(\frac{1}{w_0^2} - \frac{ik}{2z}\right)\right] e^{-ikxx'/z} dx' \\ &\int_{\infty}^{\infty} \exp\left[-y'^2\left(\frac{1}{w_0^2} - \frac{ik}{2z}\right)\right] e^{-ikyy'/z} dy' \end{aligned}$$

Using the Gaussian integral

$$\int_{\infty}^{\infty} e^{-\alpha x^2+\beta x} dx = e^{\beta^2/4\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2},$$

we evaluate the above integrals to get

$$U(x,y,z) = -\frac{iAk}{2\pi z} e^{ikz} e^{ik(x^2+y^2)/2z} \exp\left[\frac{(-ikx/z)^2}{4\left(\frac{1}{w_0^2} - \frac{ik_0}{2z}\right)}\right] \left[\frac{\pi}{\left(\frac{1}{w_0^2} - \frac{ik_0}{2z}\right)}\right]^{1/2}$$
$$\times \exp\left[\frac{(-iky/z)^2}{4\left(\frac{1}{w_0^2} - \frac{ik}{2z}\right)}\right] \left[\frac{\pi}{\left(\frac{1}{w_0^2} - \frac{ik}{2z}\right)}\right]^{1/2}$$

Rearranging we get,

$$U(x,y,z) = -\frac{Aw_0}{w(z)} e^{ikz} e^{i[kz - \tan^{-1}(z/z_R)]} e^{ik(x^2+y^2)/2R(z)} e^{-(x^2+y^2)/w^2(z)},$$

where

$$z_R = \frac{kw_0^2}{2}; \qquad w(z) = w_0\sqrt{1 + \frac{z^2}{z_R^2}}; \qquad R(z) = z + \frac{z_R^2}{z}$$

$w_0(z)$ is the size of the beam after it has propagated a distance z. We see that the size of the beam increases as it propagates. z_R is called the Rayleigh range. It is the z-distance over which the beam size increases by a factor of $\sqrt{2}$. More intuitively, it is the z-range over which the beam size can be said to have not changed too much. $R(z)$ is the radius of curvature of the beam at z. We see that at $z = 0$, the radius of curvature is ∞. This is due to the fact that the phase-front of a Gaussian beam is completely flat at $z = 0$

(d) Yes, the beam is still spatially fully coherent. This means that a spatially completely coherent beam remains completely coherent upon propagation.

Solution 2.6

The field $V(x,t)$ at location x on the screen at time t can be written in terms of the fields at the two slits as

$$V(x,t) = k_1 V_1(d, t - t_1) + k_2 V_2(-d, t - t_2),$$

where $V_1(d, t - t_1)$ and $V_2(-d, t - t_2)$ are the field amplitudes at the $z = 0$ plane at $(d, t - t_1)$ and $(-d, t - t_2)$, respectively, and where k_1 and k_2 are the attenuation constants. Now, the intensity $I(x)$ at the screen is then given by

$$\begin{aligned} I(x) = \langle V(x,t)^* V(x,t)\rangle &= |k_1|^2 \langle V_1(d, t-t_1)^* V_1(d, t-t_1)\rangle + |k_2|^2 \langle V_2(-d, t-t_2)^* V_2(-d, t-t_2)\rangle \\ &+ k_1^* k_2 \langle V_1(d, t-t_1)^* V_2(-d, t-t_2)\rangle + k_2^* k_1 \langle V_2(-d, t-t_2)^* V_1(d, t-t_1)\rangle \\ &= |k_1|^2 I(d, t-t_1) + |k_2|^2 I(-d, t-t_2) + k_1^* k_2 \Gamma(d, t_1; -d, t_2) + \text{c.c.}, \end{aligned}$$

where $I(d, t - t_1)$ is the intensity at $(d, t - t_1)$, and $\Gamma(d, t_1; -d, t_2)$ is the cross-correlation function, etc. Now, since it is a constant-amplitude plane wave field, the field amplitude of the incoming field $z = 0$ is independent of the spatial location. So, we can write the above intensity as

$$I(x) = |k_1|^2 I_0(t - t_1) + |k_2|^2 I_0(t - t_2) + k_1^* k_2 \Gamma(t_1, t_2) + \text{c.c.},$$

We know that the field is stationary in time. Therefore, we have

$$\Gamma(t_1, t_2) = \Gamma(\tau) = \int S(\omega) e^{-i\omega\tau} d\omega = \int \frac{1}{\sqrt{2\pi}\Delta\omega} \exp\left[-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\right] e^{-i\omega\tau} d\omega = e^{-i\omega_0\tau} \exp\left[-\frac{\Delta\omega^2\tau^2}{2}\right],$$

where $\tau = t_2 - t_1$ and it can be shown to be

$$\tau = t_2 - t_1 \approx \frac{1}{c}\left[R + \frac{(x+d)^2}{2R}\right] - \frac{1}{c}\left[R + \frac{(x-d)^2}{2R}\right] = \frac{2xd}{cR}.$$

Also, we have $I_0(t - t_1) = I_0(t - t_2) = \Gamma(0) = 1$. Therefore, the intensity can now be written as

$$\begin{aligned} I(x) &= |k_1|^2 + |k_2|^2 + 2|k_1 k_2| \exp\left[-\frac{\Delta\omega^2 \tau^2}{2}\right] \cos(\omega_0 \tau + \phi_k) \\ &= |k_1|^2 + |k_2|^2 + 2|k_1 k_2| \exp\left[-\frac{2x^2 d^2 \Delta\omega^2}{c^2 R^2}\right] \cos\left(\frac{\omega_0 2xd}{cR} + \phi_k\right), \end{aligned}$$

where $k_1^* k_2 = |k_1||k_2|e^{-i\phi_k}$. We find that the intensity $I(x)$ as a function of x varies in a sinusoidal manner. The visibility of these interference fringes decreases as a functions x. This visibility function is essentially the degree of coherence function $\gamma(\tau) = \exp\left[-\frac{\Delta\omega^2 \tau^2}{2}\right] = \exp\left[-\frac{2x^2 d^2 \Delta\omega^2}{c^2 R^2}\right]$. The standard deviation of this envelope function is $\frac{cR}{2d\Delta\omega}$. So by measuring the width of this envelope one can measure $\Delta\omega$.