PHY690G: Coherence and Quantum Entanglement **Semester I, 2018-19; IIT Kanpur**

End-Semester Examination

Saturday, Nov 24th, 2018; **Time: 9:00 am -12:00 pm;** **Maximum Marks: 100**
(Answer All Questions)

Q1: Consider a field $V(t)$ given by $V(t) = ae^{-i\omega_a t} + be^{-i\omega_b t}$, where ω_a and ω_b are two frequencies and a and b are two real numbers.

(a) Find out the cross-spectral density function $W(\omega_1, \omega_2)$ of the field. **(4 marks)**

(b) Find out the intensity $I(t)$ of the field. **(2 marks)**

(c) Find out the cross-correlation function $\Gamma(t_1, t_2)$ of the field. **(2 marks)**

(d) Find out the degree of coherence function of the field. **(2 marks)**

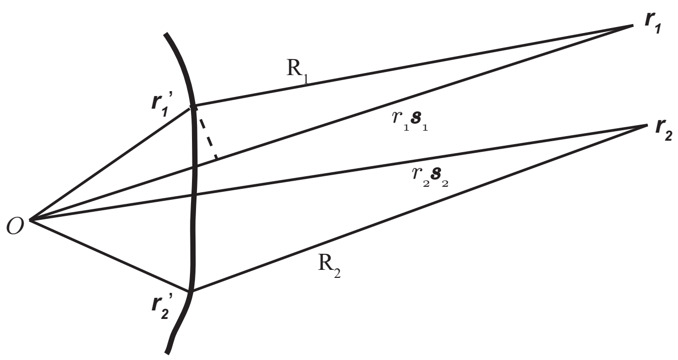
Q2: If $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is a cross-spectral density function, what are the conditions that $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ must satisfy in order for it to have a coherent mode representation? (state the conditions in words and also write them down in mathematical terms.) **(6 marks)**

Q3: The field amplitude $U(\boldsymbol{\rho}; z)$ is written in the paraxial approximation as $U(\boldsymbol{\rho}; z) = e^{ikz} \iint a(\boldsymbol{q}) e^{i\boldsymbol{q}\cdot\boldsymbol{\rho}} e^{-iq^2 z/(2k)} d\boldsymbol{q}$, where $a(\boldsymbol{q})$ is the angular spectrum, k is the wave-vector of the field, and $q = |\boldsymbol{q}|$.

(a) Derive an expression for the cross-spectral density function $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; z)$ in terms of the angular correlation function $\mathcal{A}(\boldsymbol{q}_1, \boldsymbol{q}_2)$. **(2 marks)**

(b) If $\mathcal{A}(\boldsymbol{q}_1, \boldsymbol{q}_2) = I(\boldsymbol{q}_1)\delta(\boldsymbol{q}_1 - \boldsymbol{q}_2)$, write down the expression for $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; z)$ in terms of $I(\boldsymbol{q})$. **(4 marks)**

Q4: Consider a source in the form of a surface as shown below. The cross-spectral density function at the source is given by $W(\boldsymbol{r}'_1, \boldsymbol{r}'_2) = S(\boldsymbol{r}'_1)\delta(\boldsymbol{r}'_1 - \boldsymbol{r}'_2)$. Taking k to be the wave-vector of the field, derive an expression for the cross-spectral density function $W(\boldsymbol{r}_1, \boldsymbol{r}_2)$ at far-away points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ in terms of the source spectral density $S(\boldsymbol{r}'_1)$. Here, $R_1 = |\boldsymbol{r}_1 - \boldsymbol{r}'_1|$, $R_2 = |\boldsymbol{r}_2 - \boldsymbol{r}'_2|$, $r_1 = |\boldsymbol{r}_1|$, $r_2 = |\boldsymbol{r}_2|$, $\boldsymbol{r}_1 = r_1\boldsymbol{s}_1$, and $\boldsymbol{r}_2 = r_2\boldsymbol{s}_2$. **(12 marks)**



Q5: Suppose that the state of a photon is given as an incoherent mixture with equal proportion of states representing a horizontally polarized photon and a 45^0-polarized photon. What is the probability that a photon in such a state gets detected as a 45^0-polarized photon? **(10 marks)**

Q6: Consider three independent uncorrelated fields. The first one is horizontally polarized, the second one is vertically polarized, and the third one is polarized along 45^0. If the three fields are incoherently mixed in equal proportion, what is the degree of polarization of the resultant field. **(6 marks)**

Q7: The polarization matrix J of a photon is given as $J = \frac{1}{8}\begin{pmatrix} 5 & \sqrt{3} \\ \sqrt{3} & 3 \end{pmatrix}$. Write down J as a sum of two matrices, one of which is completely polarized and the other one completely unpolarized. **(10 marks)**

Q8: The density matrix representing the state of a photon in the polarization basis is given by

$$\rho = \begin{pmatrix} 0.64 & \alpha \\ \beta & 0.36 \end{pmatrix}.$$

(a) What is the relationship between α and β? **(1 marks)**

(b) What are the ranges of values that α can take? **(4 marks)**

Q9: In a BB84 protocol, we are given the following information as below. Assuming no eavesdropping and channel errors, what is the secret key that will get shared between Alice and Bob at the end of the protocol? **(5 marks)**

Alice's Bases	DA	HV	DA	HV	HV	HV	DA	DA	HV	HV	DA	DA	HV	DA	HV	HV	HV	DA
Alice's random bits	1	0	0	1	0	1	1	0	0	1	0	1	0	0	1	0	1	1

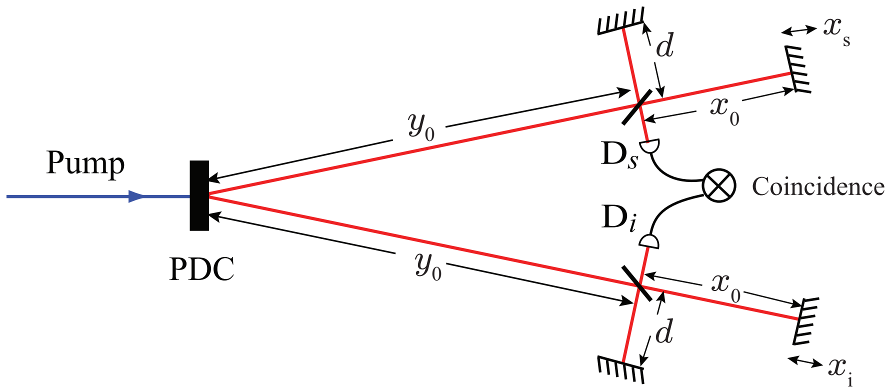
Bob's Bases	DA	DA	DA	HV	HV	DA	DA	HV	DA	HV	HV	HV	DA	DA	DA	HV	HV	HV
Bob's random bits	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Q10: Consider the interferometer shown below, in which y_0 and d are fixed distances. x_0 is the fixed distance of the signal and idler mirrors in the balanced position. From the balanced position, the displacements of the signal and idler mirrors are represented by x_s and x_i, respectively. Assume that the pump field is monochromatic with wavelength λ_0, that $x_0 \gg d \gg l_{\text{coh}}$, where l_{coh} is the coherence length of the signal/idler field, and that the coincidence time-window is much smaller compared to the time $(x_0 - d)/c$, where c is the speed of light. Finally, assume no loss, perfect mirrors, 50:50 beam splitters, and degenerate PDC.

(a) Show through explicit calculations, how should x_s and x_i be changed in order to see a HOM-like dip profile in the coincidence count rate. **(10 marks)**

(b) Show through explicit calculations, how should x_s and x_i be changed in order to see a HOM-like hump profile in the coincidence count rate. **(8 marks)**

(c) Does the HOM-like dip in this interferometer have a bunching interpretation? **(2 marks)**



Q11: Suppose Alice and Bob share two entangled particles in state: $|\Psi^-\rangle_{\text{AB}} = \frac{1}{\sqrt{2}}[|H\rangle_{\text{A}}|V\rangle_{\text{B}} - |V\rangle_{\text{A}}|H\rangle_{\text{B}}]$. Now suppose that Alice has another particle with an unknown state $|\phi\rangle_{\text{C}} = \alpha|H\rangle_{\text{C}} + \beta|V\rangle_{\text{C}}$, with $\alpha^2 + \beta^2 = 1$, that she wants to teleport to Bob. Describe mathematically the teleportation protocol in detail. **(10 marks)**