Department of Physics
IIT Kanpur, Semester II, 2016-17

PSO201A: Quantum Physics **Homework # 1**

Problem 1.1: Radiance and Energy density

The radiance R(ν) is defined as the energy radiated by a blackbody per unit area per unit time per unit frequency interval (dν). R(ν) is proportional to the energy density $\rho(\nu)$ in a cavity. $\rho(\nu)$ is defined as the energy per unit volume per unit frequency. Prove that R(ν) = (c/4)$\rho(\nu)$. (Hint: Show that the energy radiated out by an area dA of the blackbody surface is same as the energy incident on this area element due to a hemispherical volume of radius $R = cdt$, where c is the speed of light and dt is the time over which the area element dA radiates.)

Problem 1.2: Blackbody Radiation formula

As we derived in the class, the energy density $\rho(\nu)d\nu$ of standing waves inside a cavity with metallic walls is given by

$$\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\bar{\mathcal{E}}d\nu \tag{1}$$

where $\bar{\mathcal{E}}$ is the average energy of a standing wave.

(a) The classical theory assumes that a standing wave inside a cavity can have any value for the energy and therefore the classical expression for average energy is given by

$$\bar{\mathcal{E}} = \frac{\int_0^\infty \mathcal{E}P(\mathcal{E})d\mathcal{E}}{\int_0^\infty P(\mathcal{E})d\mathcal{E}} \tag{2}$$

Using the Boltzmann probability distribution for $P(\mathcal{E})$, that is, $P(\mathcal{E}) = \dfrac{e^{-\mathcal{E}/kT}}{kT}$, show that

$$\bar{\mathcal{E}} = kT \tag{3}$$

The above relation is also the statement of the law of equipartition of energy. Using the above relation show that

$$\rho(\nu)d\nu = \frac{8\pi\nu^2 kT}{c^3}d\nu. \tag{4}$$

The above relation is called the Rayleigh-Jeans formula for blackbody radiation. This formula is correct only in situations in which $h\nu \ll kT$.

(b) However, according to Planck's hypothesis (quantum theory) a standing wave inside a cavity can have only discrete energy values given by $\mathcal{E} = nh\nu$, where h is the Planck's constant, ν is the frequency of the standing wave and $n = 0, 1, 2...$ is an integer. The expression for average energy therefore now becomes

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^\infty \mathcal{E}P(\mathcal{E})}{\sum_{n=0}^\infty P(\mathcal{E})} \tag{5}$$

Prove that, with Planck's hypothesis, the average energy of the standing wave is $\bar{\mathcal{E}} = \dfrac{h\nu}{e^{h\nu/kT} - 1}$ and thus that the energy density is given by

$$\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT} - 1}d\nu \tag{6}$$

The above expression is known as Planck's formula for blackbody radiation and it turns out to be correct for blackbody radiation at all frequencies.

(c) Show that for small frequencies, that is, $h\nu \ll kT$, the Planck's radiation formula reduces to the Rayleigh-Jeans formula.

Problem 1.3: Stefan's law

Stefan's law seeks to find a relation between the temperature T of the blackbody and the total radiance R_T, which is the total energy radiated out by the blackbody per unit area per unit time. R_T is defined as: $R_T = \int_0^\infty R(\nu)d\nu$. In the class we had derived the Planck's law for blackbody spectrum, which quantifies the energy density $\rho(\nu)d\nu$ per unit frequency in the blackbody radiation and is given by: $\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}d\nu$.

(a) Using the relation $R(\nu) = \frac{c}{4}\rho(\nu)$, show that the total radiance depends on temperature T as $R_T = \sigma T^4$. This is called Stefan's law and the constant σ is called the Stefan-Boltzmann constant.

(b) Find the numerical value of the Stefan's constant.

Problem 1.4: Wien's Displacement law

(a) Starting from Planck's radiation formula, derive Wien's displacement law: $\lambda_{\text{max}}T = 2.898 \times 10^{-3}$ m-K by solving $d\rho(\lambda)/d\lambda = 0$. Here λ_{max} is the wavelength at which the radiance from the blackbody reaches its maximum and T is the temperature of the blackbody. (Hint: Set $hc/\lambda kT = x$ and show that the equation $d\rho(\lambda)/d\lambda = 0$ leads to $e^{-x} + x/5 - 1 = 0$. Then show that $x = 4.965$ is the solution.

(b) Suppose an object is heated to about 3000 K. Of what color would this source appear— Reddish, Greenish, Bluish? (Hint: Use Wien's displacement law).

Problem 1.5: Photoelectric effect

(a) The minimum electromagnetic energy that a human eye can detect is 1×10^{-18} J. How many photons of 600 nm wavelength does that correspond to?

(b) Show that it is impossible for a photon to give up all its energy and momentum to a free electron. This is the reason why the photoelectric effect can take place only when the photons strike bound electrons.

(c) Now, explain qualitatively how the energy and momentum conservations are simultaneously satisfied when photoelectric effect takes place with bound electrons.

(d) The work function of sodium, or the energy required to remove an electron from sodium is 2.3 eV. We have two sources of light. First is an intense, one watt HeNe laser at 633 nm and the second is the torch-light in a mobile phone. Which one of the two sources has a finite probability of ejecting an electron from Sodium and why?