Department of Physics
IIT Kanpur, Semester II, 2016-17

PSO201A: Quantum Physics

Homework # 2

Problem 2.1: Radiation manifesting as particles

(a) The Compton scattering experiment proved that a radiation field behaves as particles. In this experiment, it was found that when photons of wavelength λ_0 are incident on free electrons, photons at shifted wavelengths are generated. Assuming photons and electrons as point particles, write the momentum and energy conservation equations for the collision of a photon with an electrom and thereby show that the new wavelengths are generated around λ_1 with $\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_c(1 - \cos\theta)$. Here θ is the angle at which the photons scatter away with respect to the incident direction and $\lambda_c = \dfrac{h}{m_0 c}$ is the Compton wavelength, where m_0 is the rest mass of the electron.

(b) In terms of the incident and the scattered wavelengths of the photon, derive an expression for the kinetic energy with which the electron reoils.

(c) What is the Compton shift $\Delta\lambda$ at $\theta = \pi/2$ with (i) $\lambda_0 = 1.88 \times 10^{-2}$ Å(γ ray photons); (ii) $\lambda_0 = 1$ Å(X-ray photons), and (iii) $\lambda_0 = 5000$ Å(visible photons).

(d) Comment on whether the Compton shift can actually be observed experimentally with the above three photons. If not, discuss what are the limitations?

Problem 2.2: Electrons (material particle) manifesting as waves

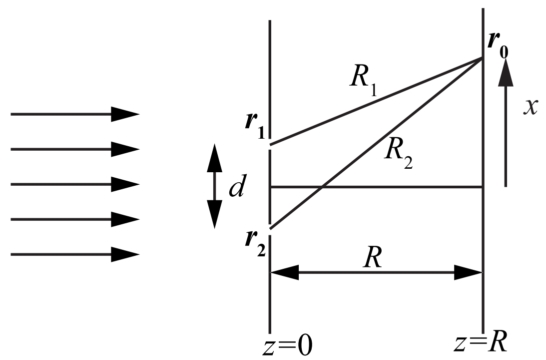
(a) In Davission-Germer experiment, the electrons behave as waves, since they are found to satisfy the Bragg reflection condition for atomic planes in crystal. Considering electrons to have a de-Broglie wavelength λ and assuming the distance between the atomic planes to be d, show that the electrons interfere constructively in certain direction given by $2d\sin\phi = n\lambda$, where ϕ is the angle between the incident direction of electrons and the crystal plane.

(b) Assuming the kinetic energy of the electrons to be 60 eV, find its de-Broglie wavelength.

(c) Taking the distance between atomic planes d to be 0.91 Å and using the de-Broglie wavelength of the electron from above, find the angle at which the first diffraction maximum occurs?

(d) In order for the third diffraction maximum to occur at the angle calculated in problem 2.2 (c), what should be the kinetic energy of the incoming electrons. Use $d = 0.91$ Å.

Problem 2.3: Young's double-slit interference with particles



(a) In this problem, we analyze Young's double-slit experiment with particles. Assume that the field amplitude of the wave coming to the double-slit plane at $z = 0$ is given by $E(\vec{r}, t) = Ae^{i(kz-\omega t)}$, where $k = \dfrac{2\pi}{\lambda}$ is the wave-vector and $\omega = 2\pi\nu$ is the angular frequency of the incoming wave. Find the expression for the fringe period of the interference pattern seen on a screen at $z = R$.

(b) Assume $R = 1$ m, $d = 1$ mm. Find the fringe period when the incident particles on the double-slit are photons with wavelength $\lambda = 5000$ Å.

(c) Assume $R = 1$ m, $d = 1$ mm. Find the fringe period when the incident particles on the double-slit are bullets of mass $m = 60$ g and speed $v = 200$ m/s.

(d) Can one observe bullet-fringes on the screen? If not, explain what restricts the observation?

Problem 2.4: Miscellaneous Conceptual Questions

(a) In the photoelectric effect, the maximum kinetic energy of a photoelectron is independent of the intensity of light. From the definition, we know that intensity is energy falling on a unit area per unit time. So, if the intensity is increased, the energy falling per unit area increases. But why this increase in energy per unit area is not sufficient to increase the kinetic energy of photoelectrons?

(b) Explain Bohr's complementarity principle using Young's double-slit setup?

(c) What is the main difference between the wave-function $\psi(x,t)$ representing a quantum particle and the function $E(x,t)$ representing the wave-amplitude of a classical wave?

(d) In Young's double slit experiment with single photons, if each photon passes through one of the two slits without getting divided between the slits then what interferes with what to give the interference pattern observed experimentally?