Department of Physics
IIT Kanpur, Semester II, 2016-17

PSO201A: Quantum Physics

Homework # 8

Problem 8.1: Gaussian Wave-packet

The eigen-solutions to the free particle hamiltonian are the plane waves. One of the superpositions of the plane waves is a Gaussian wave-packet. The position-basis wave-function for a Gaussian wave-packet at $t = 0$ is given as

$$\psi(x,0) = e^{ip_0x/\hbar}\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{1/2}\exp\left[-\frac{x^2}{4\sigma^2}\right]$$

(a) What is the mean velocity with which this wave-packet is moving at $t = 0$?

(b) Find out the probability density $P(x,t)$ at time t.

(c) What is the mean velocity with which this wave-packet is moving at time t?

(d) The width of a wave-packet is defined as the position uncertainty Δx associated with the wave-function. The width of the wave-packet at $t = 0$ is $\Delta x(0) = \sigma$. Find the width $\Delta x(t)$ of the wave-packet at time t.

(e) why does the width of the wave-packet increase with time while its speed remain the same?

Problem 8.2: The finite square-well potential

In the class, we worked out the bound states for the even solutions of the finite square-well potential:

$$\begin{aligned} V(x) &= -V_0 \qquad \text{for} \quad -a < x < a \\ &= 0 \qquad \text{for} \quad |x| > a \end{aligned}$$

This problem is regarding the odd solutions:

(a) Work out the transcendental equation for the allowed energies of the odd bound states.

(b) Solve the transcendental equation graphically to calculate the bound state energies in the limit $V_0 \to \infty$.

(c) Will there always be a bound state even if V_0 is made very small.

Problem 8.3: The "transparent" finite square-well potential

Consider again the potential of Problem 8.2. Suppose a particle of mass m and energy E approaches this potential form left. Calculate the energy value(s) for which the potential becomes transparent for the particle (Ramsauer-Townsend effect).

Problem 8.4: Finite-Infinite square-well potential

Suppose a particle of mass m is in the potential:

$$\begin{aligned} V(x) &= \infty \qquad \text{for} \quad x < 0 \\ &= -\frac{32\hbar^2}{ma^2} \qquad \text{for} \quad 0 \le x \le a \\ &= 0 \qquad \text{for} \quad |x| > a \end{aligned}$$

Find out all the possible bound states.

Problem 8.5: The double delta-function potential

Consider a particle of mass m in a double Dirac delta-function potential.

$$V(x) = -\alpha[\delta(x+a) + \delta(x-a)],$$

where α and a are positive constants.

Find out the bound states that the particle can have.