Department of Physics, IIT Kanpur, Semester II, 2016-17

PSO201A: Quantum Physics **Quiz # 2** **Time: 35 Minutes** **Max Marks: 100**

Name: .... **Roll No.:** .... **Section**..........

Problem 1: The position-space wave function for a particle in the lowest-energy stationary state of a harmonic potential is given by $\psi_0(x) = \left(\frac{1}{2\pi\sigma_x^2}\right)^{1/4} \exp\left(-\frac{x^2}{4\sigma_x^2}\right)$, where $\sigma_x = \sqrt{\hbar/(2m\omega)}$.

(a) Find the momentum-space wave function for the particle. (Hint: you could make use of the standard integral: $\int_{-\infty}^{\infty} e^{-\alpha x^2+\beta x} dx = \exp\left(\frac{\beta^2}{4\alpha}\right)\left(\frac{\pi}{\alpha}\right)^{1/2}$.) **(15 marks)**

(b) Taking the standard deviation of a probability-density curve to be the uncertainty, calculate the position and momentum uncertainly product for the above wave-function. **(15 marks)**

Problem 2: Let $\hat{H} = \begin{pmatrix} \hbar\omega & 0 \\ 0 & 2\hbar\omega \end{pmatrix}$ be the Hamiltonian and $\hat{A} = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix}$ be an observable of a two-level system. Let $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be the state of this system at $t = 0$.

(a) Calculate the normalized eigenvectors and the eigenvalues of the Hamiltonian. **(10 marks)**

(b) Calculate the expectation value of energy and of the observable $\hat{A}$ at $t = 0$. **(10 marks)**

(c) Find the 2×2 matrix representation for the propagator $\hat{U}(t, 0)$. **(10 marks)**

(d) Calculate the expectation value of energy and of the observable $\hat{A}$ at time t. **(10 marks)**

Problem 3: For the potential :

$$\begin{aligned} V(x) &= -V_0 \qquad \text{for} \quad -a < x < a \\ &= 0 \qquad\quad \text{for} \quad |x| > a \end{aligned}$$

(a) Work out the transcendental equation for the allowed energies of the odd bound states. **(20 marks)**

(b) What is the minimum number of bound states that this potential can allow? **(10 marks)**