End-Semester Examination

April 30th, 2017 Time: 9:00 am -12:00 pm Maximum Marks: 100

(Answer all 8 questions. Calculators are not allowed.)

NAME .... **ROLL NO.** ....

CLASS .... **SECTION** ....

SUBJECT ....

DATE ....

Marks Obtained	
Q-01	
Q-02	
Q-03	
Q-04	
Q-05	
Q-06	
Q-07	
Q-08	
Q-09	
Q-10	
Total	

I PLEDGE MY HONOUR AS A GENTLEMAN / LADY THAT DURING THE EXAMINATION I HAVE NEITHER GIVEN ASSISTANCE NOR RECEIVED ASSISTANCE

....

Signature

Problem 1:

(a) Consider a particle in the n^{th} stationary state of a potential at time $t = t_1$. Find the probability that the particle is found in the $(n+1)^{\text{th}}$ stationary state at time $t = t_2$. **(2 marks)**

(b) The expectation value of an operator $\hat{\mathrm{A}}$ is defined as $\langle \hat{\mathrm{A}} \rangle \equiv \langle \psi | \hat{\mathrm{A}} | \psi \rangle$. Express $\langle \psi | \hat{\mathrm{A}} | \psi \rangle$ in the position basis. **(2 marks)**

(c) If two operators $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ have complete set of common eigenfunctions, Evaluate the commutator $[\hat{\mathrm{A}}, \hat{\mathrm{B}}]$. **(3 marks)**

(d) Find out whether the expectation value of an anti-hermitian operator $\hat{\mathrm{Q}}$ ($\hat{\mathrm{Q}}^\dagger = -\hat{\mathrm{Q}}$) is real or imaginary. **(3 marks)**

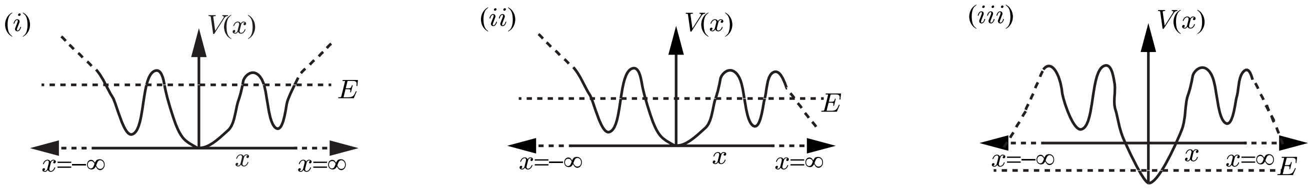
Problem 2:

(a) If $\hat{\mathrm{H}}$ is a Hermitian matrix, then show that $e^{i\hat{\mathrm{H}}}$ is a unitary matrix. **(3 marks)**

(b) If $\psi(x) = e^{ip_0 x/\hbar}$ is the position-basis wave function of a quantum system with p_0 being a constant, find the momentum-basis wave function of the quantum system. **(3 marks)**

(c) For the harmonic potential $V(x) = \frac{1}{2}m\omega^2 x^2$, the ground-state wave function is given by: $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$ with energy $E_0 = \frac{1}{2}\hbar\omega$. Calculate the first-order correction to the ground-state energy for the potential $V(x) = \frac{1}{2}m\omega^2 x^2 - cx$, where $c \ll \sqrt{m\omega^2}$. **(3 marks)**

(d) For each of the three potentials $V(x)$ shown below, state whether a particle of energy E will be in a bound state or in a scattering state. **(3 marks)**



Problem 3:

Consider a free particle of mass m in the 3-dimensional cartesian space.

(a) Solve the 3-dimensional Schrödinger equation and derive the most general solution for the wave function. **(8 marks)**

(b) From the general solution, find out the solution representing a wave moving along a direction with all positive directions cosines. **(4 marks)**

Problem 4:

Consider a particle of mass m in an infinite well potential of width a. The intial wave function $\psi(x, 0)$ at time $t = 0$ of the particle is given by:

$$\begin{aligned} \psi(x,0) &= Ax & \text{for } a \leq x \leq \frac{a}{2} \\ &= A(a-x) & \text{for } \frac{a}{2} \leq x \leq a. \end{aligned}$$

(a) Find the normalization constant A of the wave function $\psi(x, 0)$. **(3 marks)**

(b) Find the wave function $\psi(x, t)$ at a later time t. **(10 marks)**

(c) What is the probability that the particle is found to have Energy E_1, where E_n is the energy corresponding to the n^{th} stationary state of the potential. **(3 marks)**

Problem 5:

The Hamiltonian $\hat{H}$ for a three-level system is given by $\hat{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}$, where a, b, and c are real and $a - c \neq \pm b$.

(a) Find out the stationary-state eigenvectors and the corresponding stationary-state energies. **(8 marks)**

(b) If the state of the system at time $t = 0$ is given by $|\psi(0)\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, find out the state $|\psi(t)\rangle$ at time t. **(7 marks)**

Problem 6:

Consider a particle of mass m with the following wave function

$$\psi(x,t) = A\sin(2\pi x/a)e^{-iEt/\hbar} \quad \text{for } 0 \leq x \leq a$$
$$= 0 \quad \text{otherwise,}$$

where a and A are constants. $\left[\text{Useful Formula: } \int_0^a x^2 \sin\left(\frac{2\pi x}{a}\right) dx = \frac{a^3}{6} - \frac{a^3}{16\pi^2}\right]$.

(a) Assuming the potential $V(x) = 0$ for $0 \leq x \leq a$, express the particle energy E in terms of a and m. **(3 marks)**

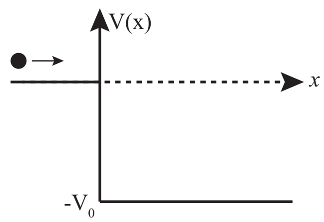
(b) What is the value of the normalization constant A. **(2 marks)**

(c) Calculate the uncertainties ΔX and ΔP. **(8 marks)**

(d) Find out the factor by which the uncertainty product is greater than the minimum uncertainty product. **(2 marks)**

Problem 7:

A free particle of mass m and kinetic energy $E = V_0/3$ approaches a potential from left as shown in the figure. Calculate the probability that the particle gets reflected back. **(8 marks)**



Problem 8:

Consider a particle of mass m in a Dirac-delta potential $V(x)$ given by $V(x) = -\alpha\delta(x)$, where α is a positive real number. If the energy of the particle $E < 0$:

(a) Find the stationary state wave function $\psi(x)$ of the particle. **(8 marks)**

(b) Calculate the probability that the particle is found between $x = -a$ and $x = a$. **(4 marks)**