PSO201A: Quantum Physics **Semester II, 2016-17; IIT Kanpur**

Mid-Semester Examination

March 3rd, 2017 Time: 1:00-3:00 pm Maximum Marks: 100

(Answer all 6 questions. Calculators are not allowed. Some important constants are provided below.)

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1. Stefan's-Boltzmann's constant: $\sigma = 5.67 \times 10^{-8}$ W/m^2–K^4
2. Wein's constant = 2.898×10^{-3} m-K
3. Planck's constant: $h = 6.626 \times 10^{-34}$ joule-sec

NAME .... ROLL NO. ....

CLASS .... SECTION ....

SUBJECT ....

DATE ....

I PLEDGE MY HONOUR AS A GENTLEMAN / LADY THAT DURING THE EXAMINATION I HAVE NEITHER GIVEN ASSISTANCE NOR RECEIVED ASSISTANCE

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Signature

Marks Obtained

Q-01	
Q-02	
Q-03	
Q-04	
Q-05	
Q-06	
Q-07	
Q-08	
Q-09	
Q-10	
Total	

Problem 1:

(a) In a nuclear reaction the temperature of the rising fireball reaches 10^7 K at some instant. What is the wavelength at which the emitted radiation is a maximum at that instant. **(3 marks)**

(b) If the surface temperature of the Sun is 5000 K, calculate the approximate energy lost to radiation per unit time per unit area of the Sun's surface. **(3 marks)**

(c) Write the following in the position basis: $\langle\psi_1|\hat{\Omega}|\psi_2\rangle$. **(3 marks)**

(d) Find the adjoint of the equation: $|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle\langle\psi_3|\psi_4\rangle + \sum_i |\psi_5\rangle\langle\psi_6|\hat{\Omega}|v_i\rangle\langle w_i|\psi_7\rangle$. **(3 marks)**

(e) Show that the uncertainty in the energy ΔE of a particle in a stationary state is zero. **(3 marks)**

Problem 2:

(a) Explicitly work out the commutator: $[\hat{X}, \hat{P}]$, where $\hat{X}$ is the position operator and $\hat{P}$ is the momentum operator. **(10 marks)**

(b) Assuming no degeneracy and using Dirac notations, prove that the eigenvectors of a Hermitian operator are mutually orthogonal. **(10 marks)**

Problem 3: If $|x\rangle$ and $|p\rangle$ are the position and momentum eigetkets with eigenvalues x and p, respectively.

(a) Work out the inner product $\langle x|p\rangle$. **(8 marks)**

(b) For a quantum state $|\psi\rangle$, express the momentum-basis wave function $\psi(p)$ in terms of the position-basis wave function $\psi(x)$. **(3 marks)**

(c) write the following completeness relationship in the position basis: $\int_{\infty}^{\infty} |p\rangle\langle p|dp = \mathrm{I}$. **(4 marks)**

Problem 4: Suppose the position-basis wave function $\Psi(x,t)$ of a particle is given by $\Psi(x,t) = \frac{1}{\sqrt{2}}[\Psi_1(x,t)+\Psi_2(x,t)]$, where $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are the two normalized stationary-state solutions to the Schrödinger equation with energies E_1 and E_2, respectively, with $\Psi_1(x,0) = \psi_1(x)$ and $\Psi_2(x,0) = \psi_2(x)$.

(a) Find out the position probability density of the particle at time t in terms of ψ_1, ψ_2, E_1, and E_2. **(7 marks)**

(b) What is the expectation value $\langle E \rangle$ for energy? **(8 marks)**

Problem 5: The position-basis wave function $\psi(x)$ of a particle is given by

$$\begin{aligned}\psi(x) &= A; \quad \text{if} \quad -\left(\frac{R+d}{2}\right) < x < -\left(\frac{R-d}{2}\right) \quad \text{and} \quad \left(\frac{R-d}{2}\right) < x < \left(\frac{R+d}{2}\right) \\ &= 0; \quad \text{otherwise.}\end{aligned}$$

(Take $R \gg 2d$)

(a) Find the normalization constant A. **(2 marks)**

(b) Calculate and plot the position probability density that a particle is found at position x. **(2 marks)**

(c) Calculate and plot the momentum probability density that the particle is found with momentum p. **(6 marks)**

Problem 6: Consider an infinite potential well of width a centered at $x = 0$. The position-basis wave function of the n^{th} stationary state $\psi_n(x)$ is given by

$$\begin{aligned}
&\text{For} \quad n = 1, 3, 5, ... \qquad && \psi_n(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right); \qquad && \text{if} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \\
& && = 0 && \text{else.} \\
&\text{For} \quad n = 2, 4, 6, ... \qquad && \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right); \qquad && \text{if} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \\
& && = 0 && \text{else.}
\end{aligned}$$

Suppose that the particle is in the ground state ($n = 1$) of the potential,

(a) Find the probability that the particle is found between $x = -a/4$ and $x = a/4$. **(5 marks)**

(b) What is the expectation value $\langle x \rangle$ for position? **(3 marks)**

(c) What is the expectation value $\langle p \rangle$ for momentum? **(3 marks)**

(d) Now, suppose that the potential well suddenly expands symmetrically to twice its size. Calculate the probability of finding the particle in the ground state of the new potential well. **(14 marks)**