Entangled Photons

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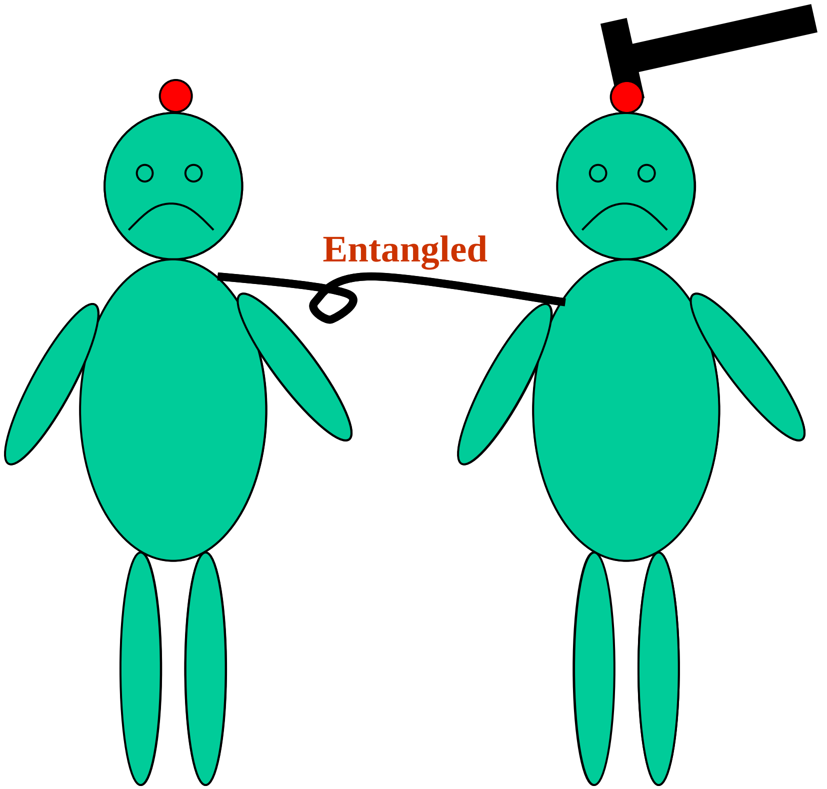
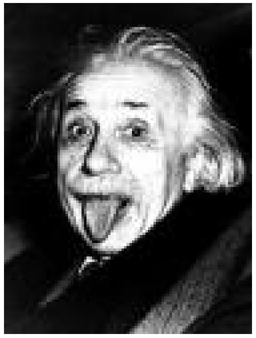
December 13th, 2014

Quantum Entanglement





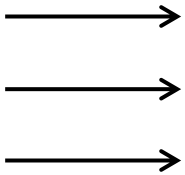
Einstein objected to this kind of phenomenon

EPR Paradox [A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935)]

One photon system:

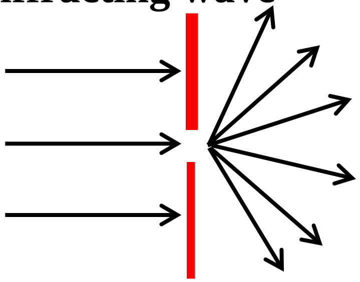
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Plane-wave



Momentum is the Physical Reality

Diffracting wave

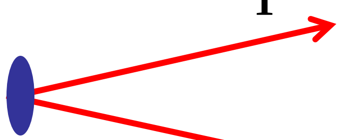


Position is the Physical Reality

"When the operators corresponding to two physical quantities do not commute the two quantifies cannot have simultaneous reality." -- EPR rephrasing the uncertainty relation.

Two-photon system (Entangled):

1



2

$$\Delta x^{(1)}_{\text{cond}} \Delta p^{(1)}_{\text{cond}} < \frac{\hbar}{2}$$

Non-local correlation ???

EPR's Questions: **(1) Is Quantum mechanics incomplete??**

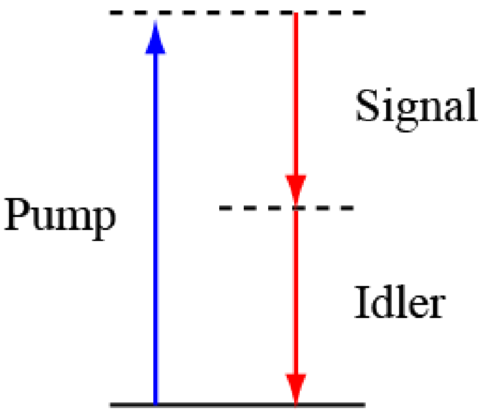
(2) Does it require additional "hidden variables" to explain the measurement results.

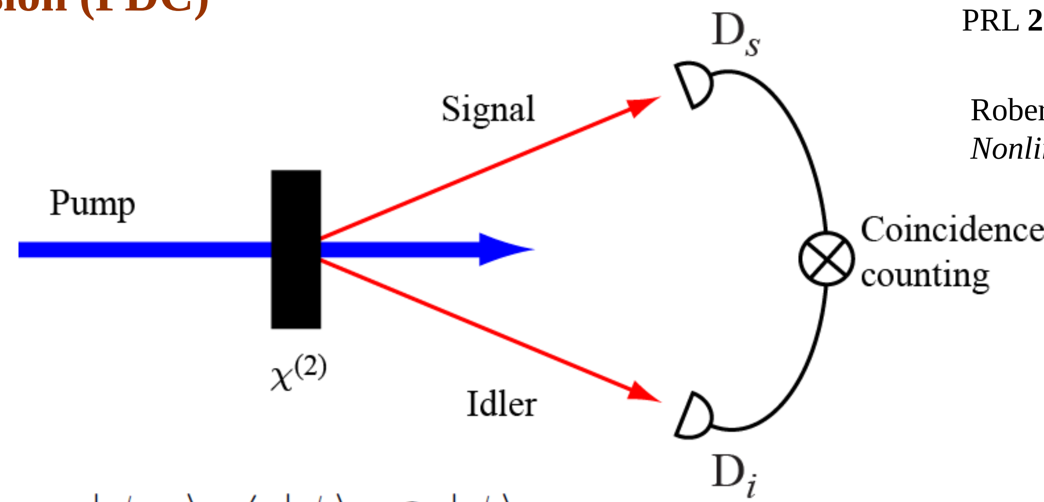
Sources of Entangled Photons

Parametric down-conversion (PDC)

Burnham and Weinberg, PRL **25**, 85 (1970)

Robert W. Boyd, *Nonlinear Optics*, 2nd ed.





two-photon field $|\psi_{\rm tp}\rangle \neq |\psi\rangle_s \otimes |\psi\rangle_i$

$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$ **Conservation of momentum**

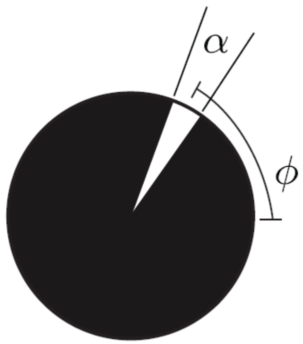
$\omega_p = \omega_s + \omega_i$ **Conservation of Energy**

$l_p = l_s + l_i$ **Conservation of Orbital Angular Momentum**

Other method: Four-wave Mixing

Orbital Angular momentum of a photon

Angular position



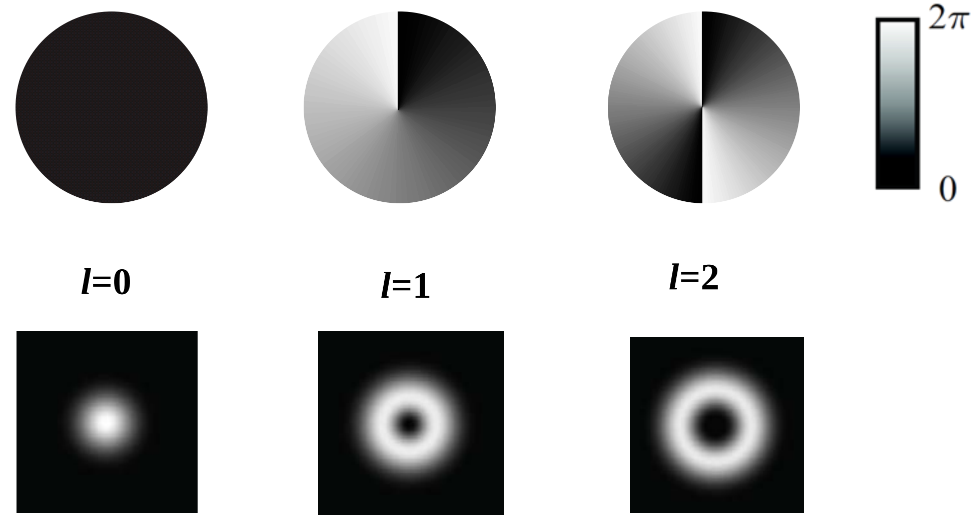
$$A_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi(\phi) \exp(-il\phi)$$

$$\Psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} A_l \exp(il\phi)$$

Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

Laguerre-Gauss basis LG_p^l

$$\mathbf{A} = \hat{x} u(\rho, z) e^{-ikz} e^{il\phi}$$



$$\frac{J_z}{W} = \frac{\iint \rho d\rho d\phi (\boldsymbol{\rho} \times \langle \mathbf{E} \times \mathbf{B} \rangle)_z}{c \iint \rho d\rho d\phi \langle \mathbf{E} \times \mathbf{B} \rangle_z} = \frac{\hbar l}{\hbar \omega}$$

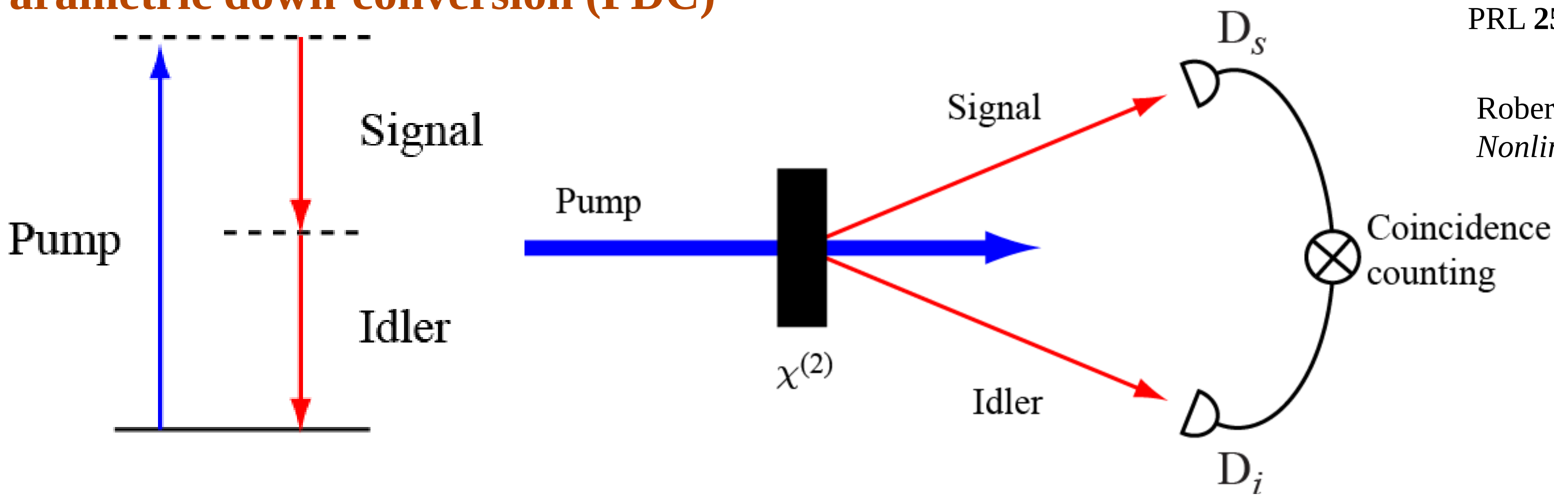
Allen et al., PRA **45**, 8185 (1992)

Types of Entanglement

Parametric down-conversion (PDC)

Burnham and Weinberg, PRL **25**, 85 (1970)

Robert W. Boyd, *Nonlinear Optics*, 2nd ed.



Entanglement in position and momentum $\Delta x^{(1)}_{\text{cond}} \Delta p^{(1)}_{\text{cond}} < \frac{\hbar}{2}$

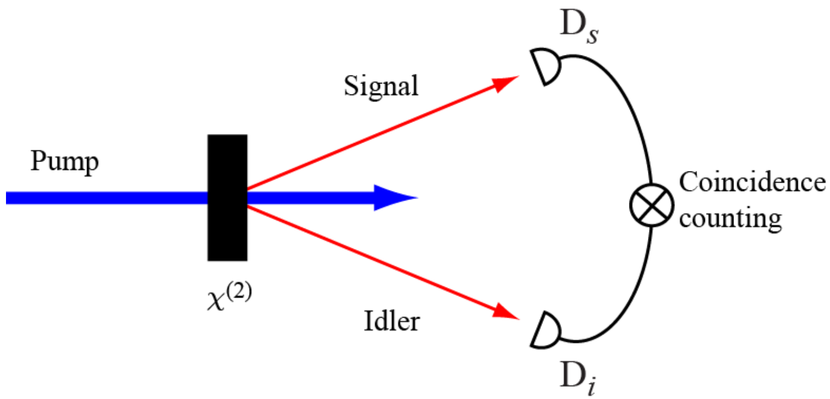
Entanglement in time and energy $\Delta t^{(1)}_{\text{cond}} \Delta E^{(1)}_{\text{cond}} < \frac{\hbar}{2}$

Entanglement in angular position and orbital angular momentum $\Delta \phi^{(1)}_{\text{cond}} \Delta L^{(1)}_{\text{cond}} < \frac{\hbar}{2}$

} **Continuous-variable entanglement**

Entanglement in Polarization } **Two-dimensional entanglement**

What is Polarization Entanglement?

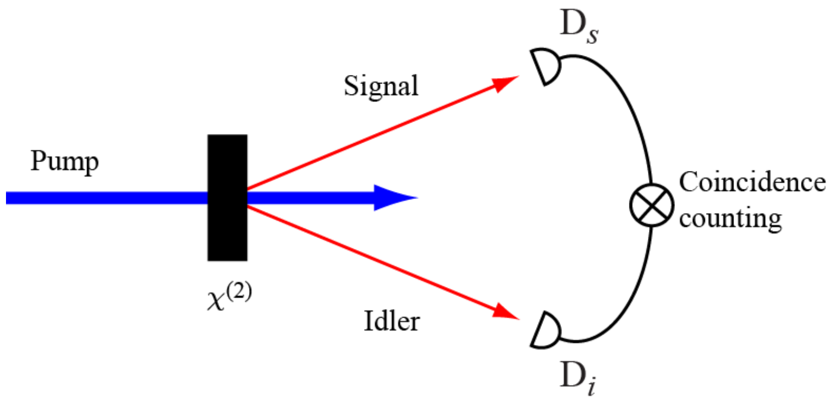


(1) If signal photon has horizontal (vertical) polarization, idler photon is guaranteed to have horizontal (vertical) polarization

--- Is this entanglement ?? NO

--- Two independent classical sources can also produce such correlations

What is Polarization Entanglement?



(1) If signal photon has horizontal (vertical) polarization, idler photon is guaranteed to have horizontal (vertical) polarization

--- Is this entanglement ?? NO

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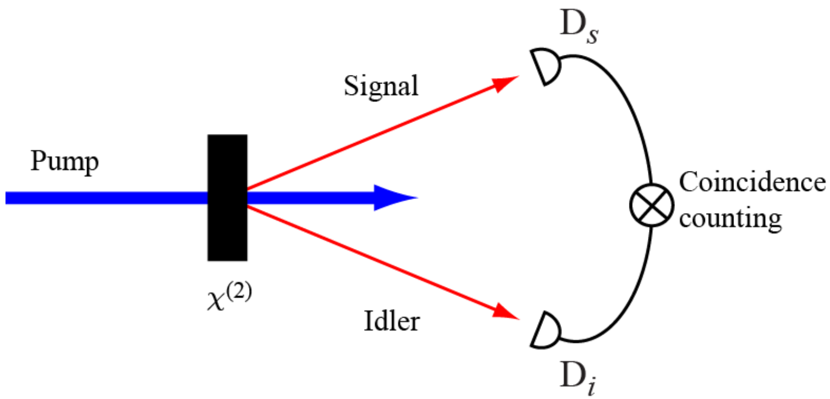
(2) If signal photon has 45^0 (-45^0) polarization, idler photon is guaranteed to have 45^0 (-45^0) polarization

--- Is this entanglement ?? NO

--- Two independent classical sources can also produce such correlations

What is Polarization Entanglement?



(1) If signal photon has horizontal (vertical) polarization, idler photon is guaranteed to have horizontal (vertical) polarization

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(2) If signal photon has 45^0 (-45^0) polarization, idler photon is guaranteed to have 45^0 (-45^0) polarization

--- Is this entanglement ?? NO

--- Two independent classical sources can also produce such correlations

If correlations (1) and (2) exist simultaneously, then that is entanglement

Quantum Entanglement and hidden variables

- **1950s: hidden variable quantum mechanics by David Bohm**

  D. Bohm, Phys. Rev. 85, 166 (1952);
  D. Bohm, Phys. Rev. 85, 180 (1952).

- **1964: Bell's Inequality--- A proposed test for quantum entanglement**

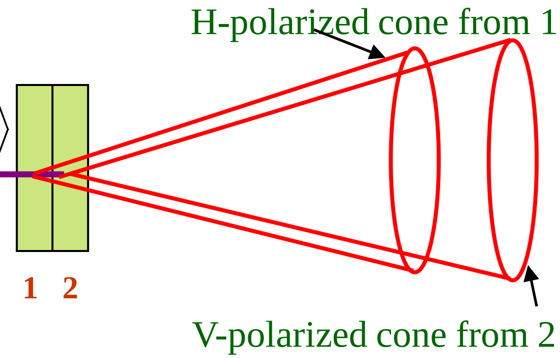
  J. S. Bell, Physics 1, 195 (1964).

- **1980s -90s --- Experimental violations of Bell's inequality**

  Aspect et al., Phys. Rev. Lett. 47, 460 (1981).
  Brendel et al., Phys. Rev. Lett. **66**, 1142 (1991)
  Kwiat et al., Phys. Rev. A **47**, R2472 (1993)
  Strekalov et al., Phys. Rev. A **54**, R1 (1996)
  Barreiro et al., Phys. Rev. Lett. **95**, 260501 (2005)

Bell’s Inequality for Polarization-Entangled Photons

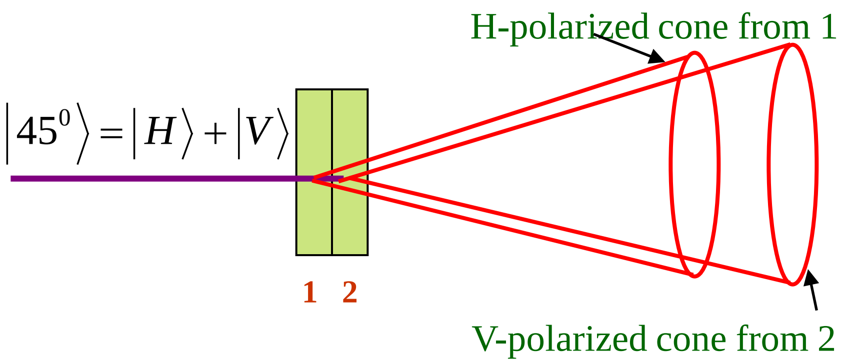
$$\left|45^0\right\rangle = \left|H\right\rangle + \left|V\right\rangle$$



$$\left|\psi\right\rangle = \left|H_s\right\rangle\left|H_i\right\rangle + \left|V_s\right\rangle\left|V_i\right\rangle$$

$$\left|\psi\right\rangle = \left|45\right\rangle_s\left|45\right\rangle_i + \left|-45\right\rangle_s\left|-45\right\rangle_i$$

Bell's Inequality for Polarization-Entangled Photons



$$|\psi\rangle = |H_s\rangle|H_i\rangle + |V_s\rangle|V_i\rangle$$

$$|\psi\rangle = |45\rangle_s|45\rangle_i + |-45\rangle_s|-45\rangle_i$$

Phys. Rev. Lett. 47, 460 (1981).
Phys. Rev. Lett. **66**, 1142 (1991)
Phys. Rev. A **47**, R2472 (1993)
Phys. Rev. A **54**, R1 (1996)
Phys. Rev. Lett. **95**, 260501 (2005)

Bell Parameter: $S = E(a,b) - E(a,b') + (a',b) + E(a',b')$

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta_\perp)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta_\perp)}$$

$$\alpha = -45^0;\ \alpha' = 0^0;\ \alpha_\perp = 45^0;\ \alpha'_\perp = 90^0$$

$$\beta = -22.5^0;\ \beta' = 22.5^0;\ \beta_\perp = 67.5^0;\ \beta'_\perp = 112.5^0$$

$\lvert S \rvert \le 2$	For hidden variable theories
$\lvert S \rvert \le 2\sqrt{2}$	For quantum correlations

Types of Entanglement

Parametric down-conversion (PDC)

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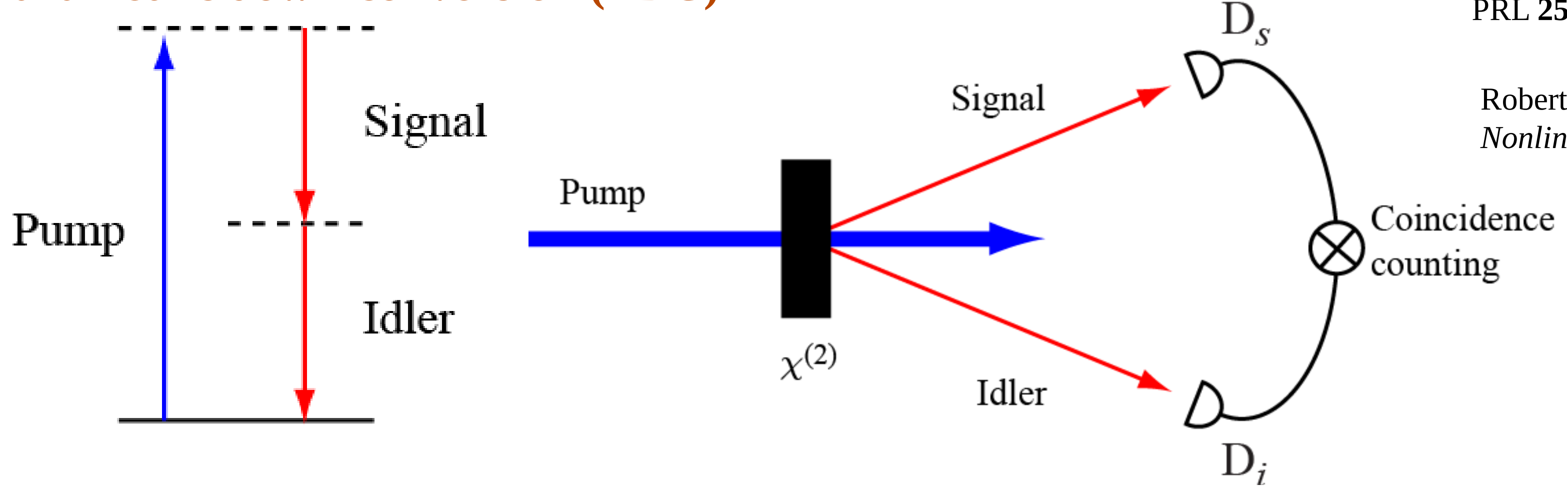


Entanglement in position and momentum $\Delta x^{(1)}_{\mathrm{cond}} \Delta p^{(1)}_{\mathrm{cond}} < \frac{\hbar}{2}$

Entanglement in time and energy $\Delta t^{(1)}_{\mathrm{cond}} \Delta E^{(1)}_{\mathrm{cond}} < \frac{\hbar}{2}$

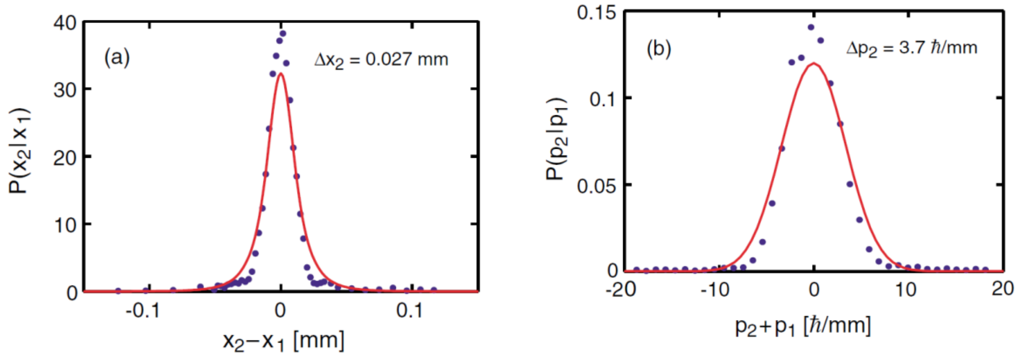
Entanglement in angular position and orbital angular momentum $\Delta \phi^{(1)}_{\mathrm{cond}} \Delta L^{(1)}_{\mathrm{cond}} < \frac{\hbar}{2}$

} **Continuous-variable entanglement**

Entanglement in Polarization } **Two-dimensional entanglement**

Verifying continuous variable entanglement

Position-momentum Entanglement [Phys. Rev. Lett. 92, 210403 (2004)]

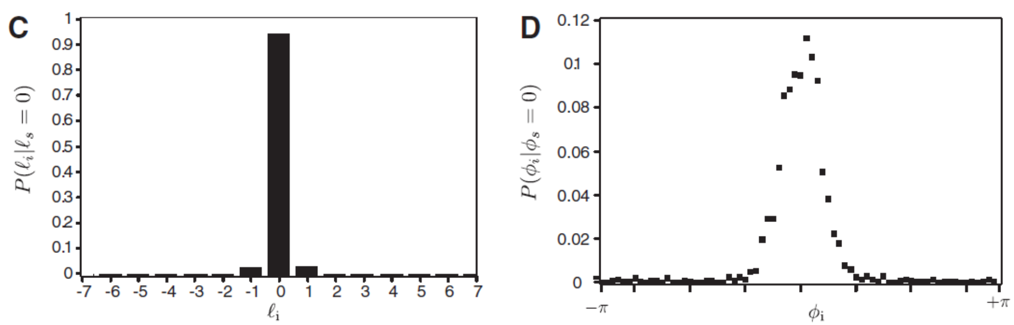


$$\Delta x^{(1)}_{\text{cond}} \Delta p^{(1)}_{\text{cond}} < 0.06\hbar$$

Time-energy Entanglement

Phys. Rev. A 73, 031801(R), 2006
Nature Physics 9, 19 (2013)

Angular-position Orbital-angular-momentum Entanglement [Science 329, 662 (2010).]

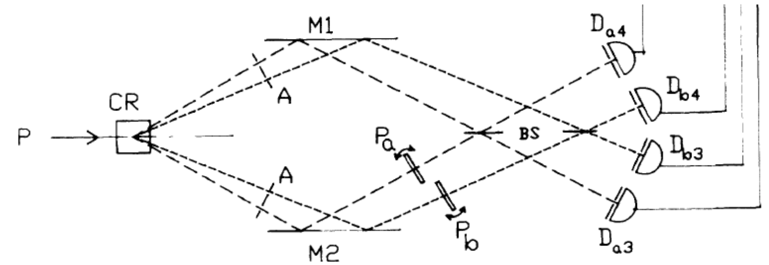


$$\Delta \phi^{(1)}_{\text{cond}} \Delta L^{(1)}_{\text{cond}} < 0.15\hbar$$

Bell inequality violation in 2D state space of continuous variables

Position-momentum Entanglement [Phys. Rev. Lett. 64, 2495 (1990)]

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|p_1\rangle_s|p_2\rangle_i + |p_2\rangle_s|p_1\rangle_i]$$

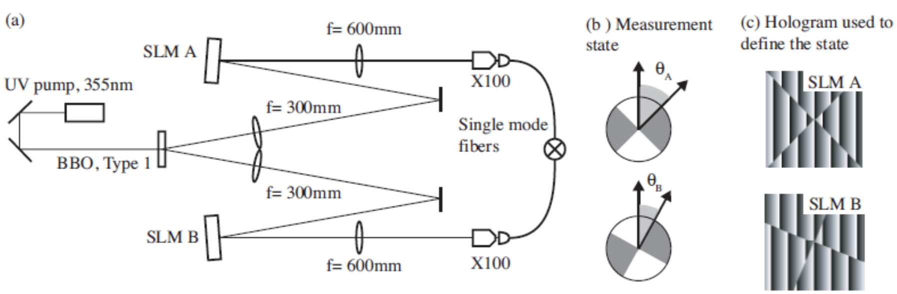


Time-energy Entanglement [Phys. Rev. Lett. 103, 253601 (2009)]

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\omega_1\rangle_s|\omega_2\rangle_i + |\omega_2\rangle_s|\omega_1\rangle_i]$$

Angular-position Orbital-angular-momentum Entanglement [Optics Express 17, 8287 (2009)]

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|l_1\rangle_s|l_2\rangle_i + |l_2\rangle_s|l_1\rangle_i]$$

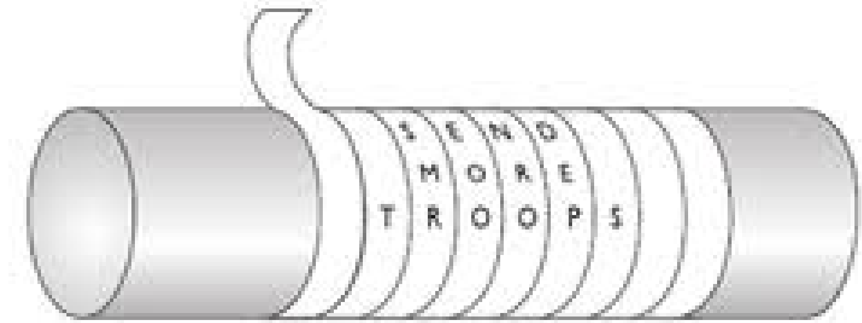


Quantum Cryptography (Quantum Key Distribution)

Older Method

(scylate)

A



B

Modern Method

Message:	**OPTICS**		Encrypted message:	**OQTJDS**
Encrypt with Key:	**010110**	→	Decrypt with Key:	**010110**
Encrypted message:	**OQTJDS**		Decrypted Message:	**OPTICS**

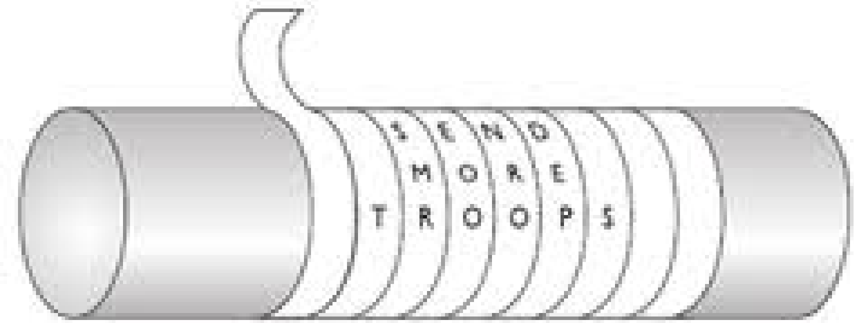
Image source: Wikepedia and google images

Quantum Cryptography (Quantum Key Distribution)

Older Method

(scylate)

A



B

Modern Method

Message:	**OPTICS**		Encrypted message:	**OQTJDS**
Encrypt with Key:	**010110**	→	Decrypt with Key:	**010110**
Encrypted message:	**OQTJDS**		Decrypted Message:	**OPTICS**

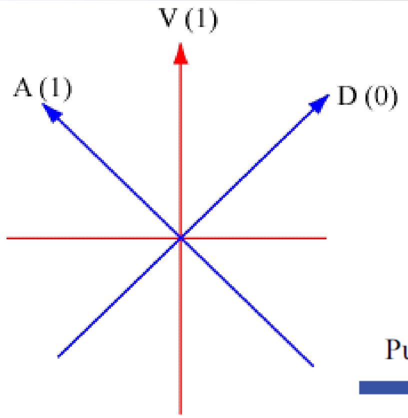
Main issue: Security

Future?

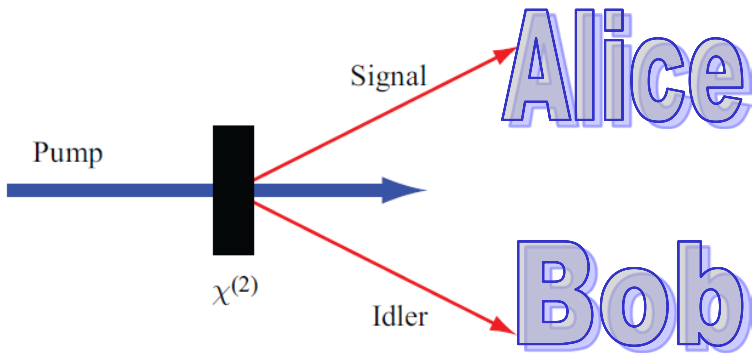
Quantum Key Distribution

Image source: Wikepedia and google images

Ekert91 Protocol: [Phys. Rev. Lett. **67**, 661 (1991)]



- Alice sends a bit to Bob by measuring her bit; whatever bit she measures becomes the incoming bit for Bob.



$$|\psi_{ab}\rangle = |H\rangle_a|H\rangle_b + |V\rangle_a|V\rangle_b$$

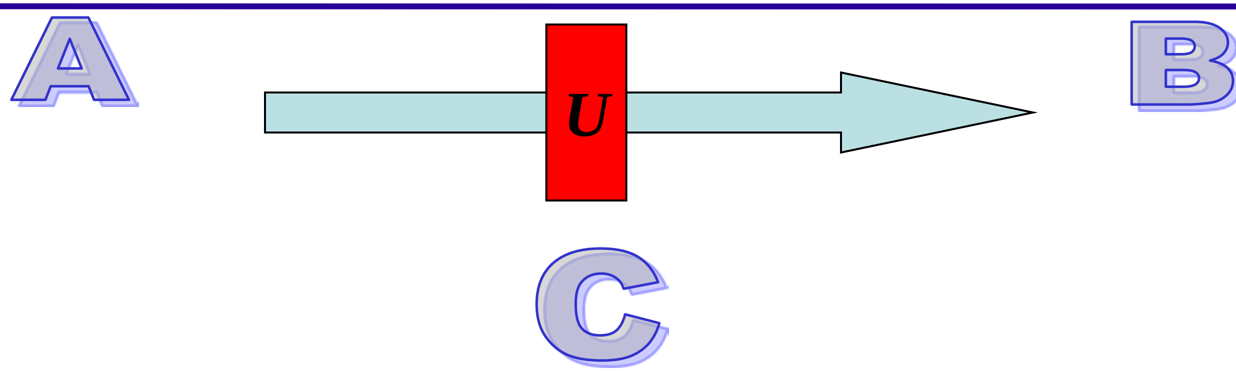
$$|\psi_{ab}\rangle = |D\rangle_a|D\rangle_b + |A\rangle_a|A\rangle_b$$

Alice's Bases	DA	HV	DA	HV	HV	HV	DA	DA	HV	HV	DA
Alice's random bits	1	0	0	1	0	1	1	0	0	1	0
Sifted bits		0			0		1			1	

Bob's Bases	HV	HV	HV	DA	HV	DA	DA	HV	DA	HV	HV
Bob's random bits	0	0	1	1	0	0	1	0	1	1	1
Sifted bits		0			0		1			1	

perfectly secure because of the laws of quantum mechanics

Quantum Superposition: Application (Quantum Cryptography)



What are those laws ?

1. **Measurement in an incompatible basis changes the quantum state**
2. **No Cloning Theorem:**

$$\hat{U}|S\rangle|H\rangle \rightarrow |0\rangle|HH\rangle$$

$$\hat{U}|S\rangle|V\rangle \rightarrow |0\rangle|VV\rangle$$

$$\hat{U}|S\rangle(|H\rangle + |V\rangle) \rightarrow |0\rangle(|HH\rangle + |VV\rangle)$$

$$\neq |0\rangle|(H+V)(H+V)\rangle$$

- C cannot clone an arbitrary quantum state sent out by A

Quantum Computation / Entanglement Quantification

Quantum Computation:

Shor's Factoring Algorithm [Proc. 35th Ann. Symp. Found. Comp. Sci. (IEEE Comp. Soc. Press, California, 1994) p. 124]

Grover's Search Algorithm Phys. Rev. Lett. **79**, 325 (1997)

The basic building block for quantum computation:

two-qubit state, or more generally N-qudit state

Polarization Two-qubit state: $|\psi\rangle = |H_s\rangle|H_i\rangle + |V_s\rangle|V_i\rangle$

OAM Two-qubit state: $$|\psi\rangle = \frac{1}{\sqrt{2}}[|l_1\rangle_s|l_2\rangle_i + |l_2\rangle_s|l_1\rangle_i]$$

Entanglement Quantification

Most general Two-qubit state:

$$\rho_{\text{qubit}} = \begin{pmatrix} \rho 11 & \rho 12 & \rho 13 & \rho 14 \\ \rho 21 & \rho 22 & \rho 23 & \rho 24 \\ \rho 31 & \rho 32 & \rho 33 & \rho 34 \\ \rho 41 & \rho 42 & \rho 43 & \rho 44 \end{pmatrix}$$

What is the entanglement of such a two-qubit state:

The most widely accepted quantifier is Wootter's Concurrence, which ranges from 0 to 1.

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho_{\text{qubit}}(\sigma_y \otimes \sigma_y)\rho^*_{\text{qubit}}(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\text{qubit}}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

Entanglement quantifier for a general N-qudit state is yet to be found

Quantum Entanglement (Current Status of the Field)

Questions related to Foundations

- Non-locality and physical reality
- Physical origin of correlations between entangled particles
- Decay of correlation between entangled photons
- Quantification of entanglement in a quantum states

Applications

- Quantum Information, Quantum Cryptography, Quantum Teleportation
- Preparation of entangled states: Two-Qubit state, N-Qudit state
- Improved ways of making entangled quantum states
- Quantum Metrology, Quantum remote sensing

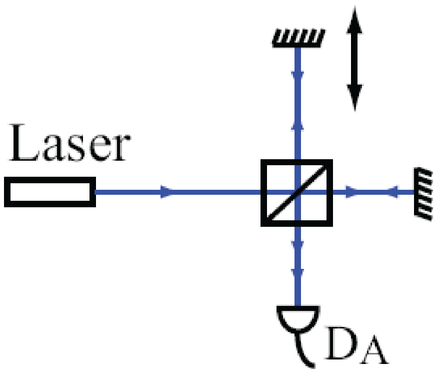
Two-photon Coherence: an alternative approach to Studying Entanglement

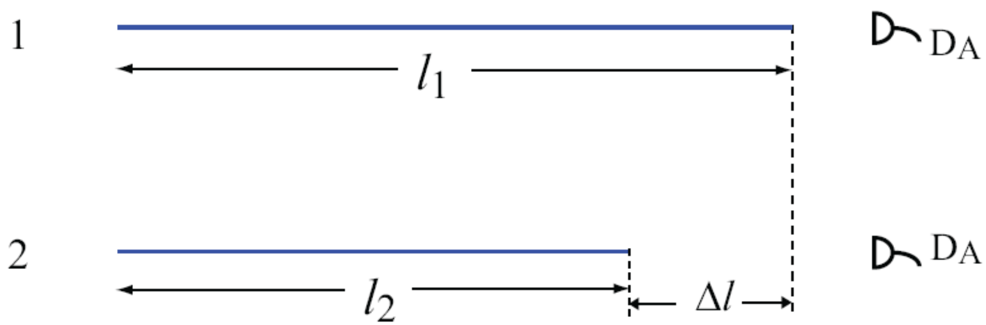
What is one-photon coherence?

What is two-photon coherence?

How is two-photon coherence connected to two-photon entanglement?

One-Photon Interference: “A photon interferes with itself ” - Dirac





$$\Delta l = l_1 - l_2$$

$$I_A \propto \langle V_A^*(t) V_A(t) \rangle_t$$

$$I_{\mathrm{A}} \propto 1 + \gamma(\Delta l) \cos(k_0 \Delta l)$$

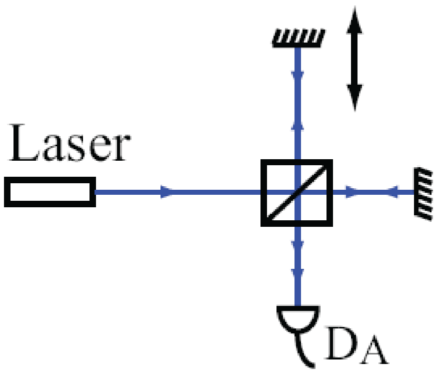
$$\gamma(\Delta l) = \frac{\langle V_1^*(t) V_2(t - \Delta l / c) \rangle_t}{\sqrt{|V_1(t)|^2 |V_2(t)|^2}}$$

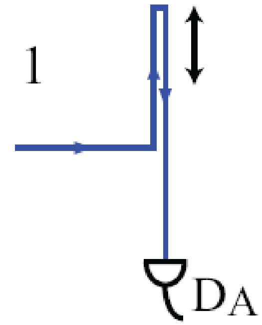
Necessary condition for interference:

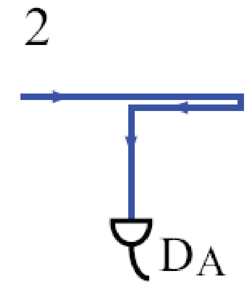
$$\Delta l < l_{\mathrm{coh}}$$

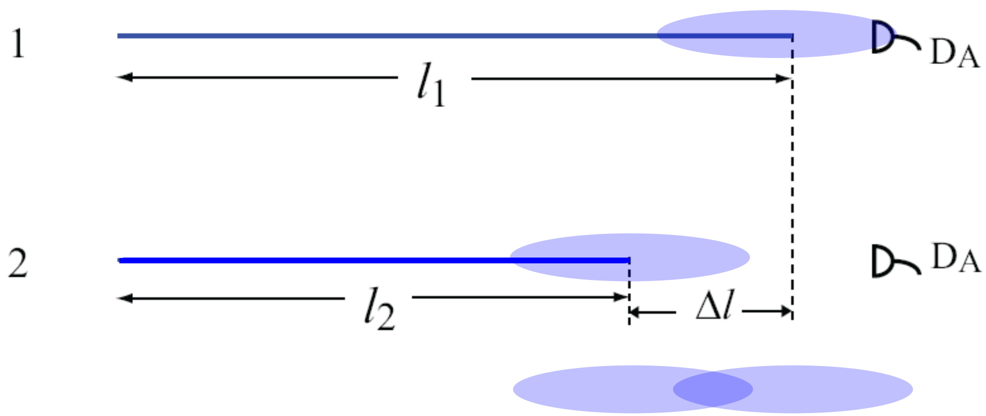
Mandel and Wolf,
Optical Coherence and Quantum Optics

One-Photon Interference: “A photon interferes with itself ” - Dirac



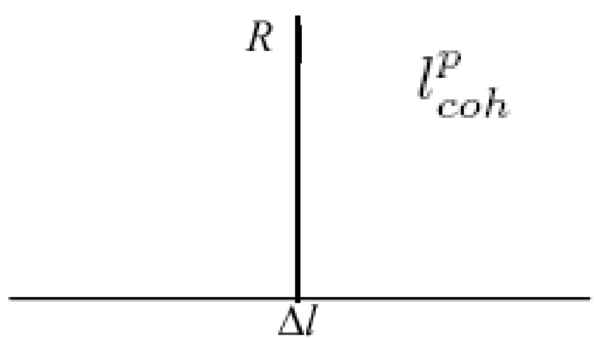






$$\Delta l = l_1 - l_2$$

$$I_A \;\propto\; 1 + \gamma(\Delta l)\cos(k_0 \Delta l)$$



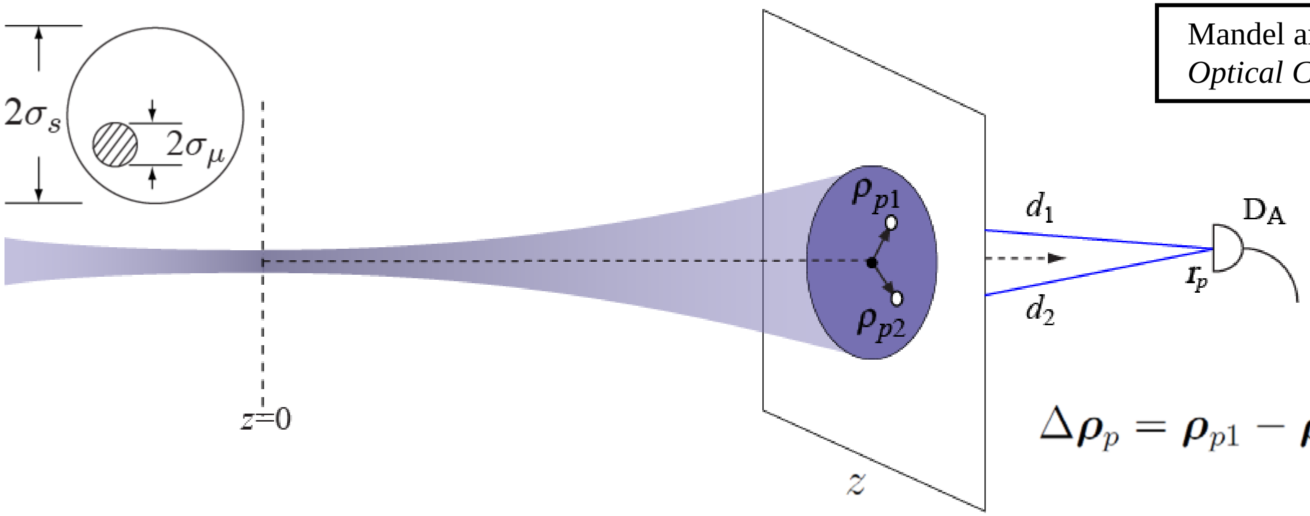
Necessary condition for interference:

$$\Delta l < l_{\mathrm{coh}}$$

Mandel and Wolf,
Optical Coherence and Quantum Optics

A photon interferes with itself: Spatial

Mandel and Wolf,
Optical Coherence and Quantum Optics



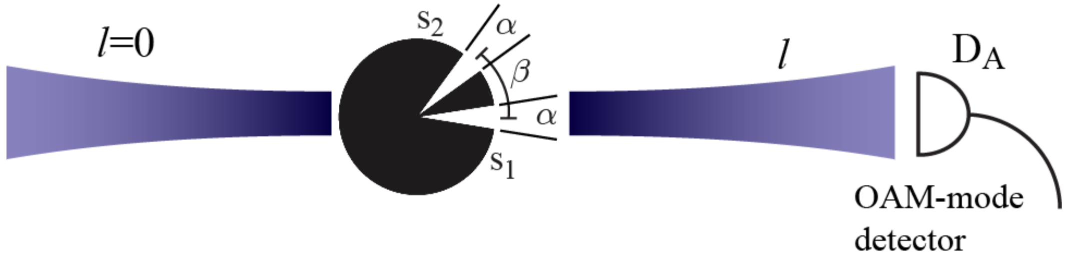
$$\Delta\boldsymbol{\rho}_p = \boldsymbol{\rho}_{p1} - \boldsymbol{\rho}_{p2}$$

$$I_A(x) = k_1^2 S(x_1, z) + k_2^2 S(x_2, z) + 2k_1 k2 \sqrt{S(x_1, z) S(x_2, z)}\mu(\Delta x, z)\cos(k_0 \Delta l)$$

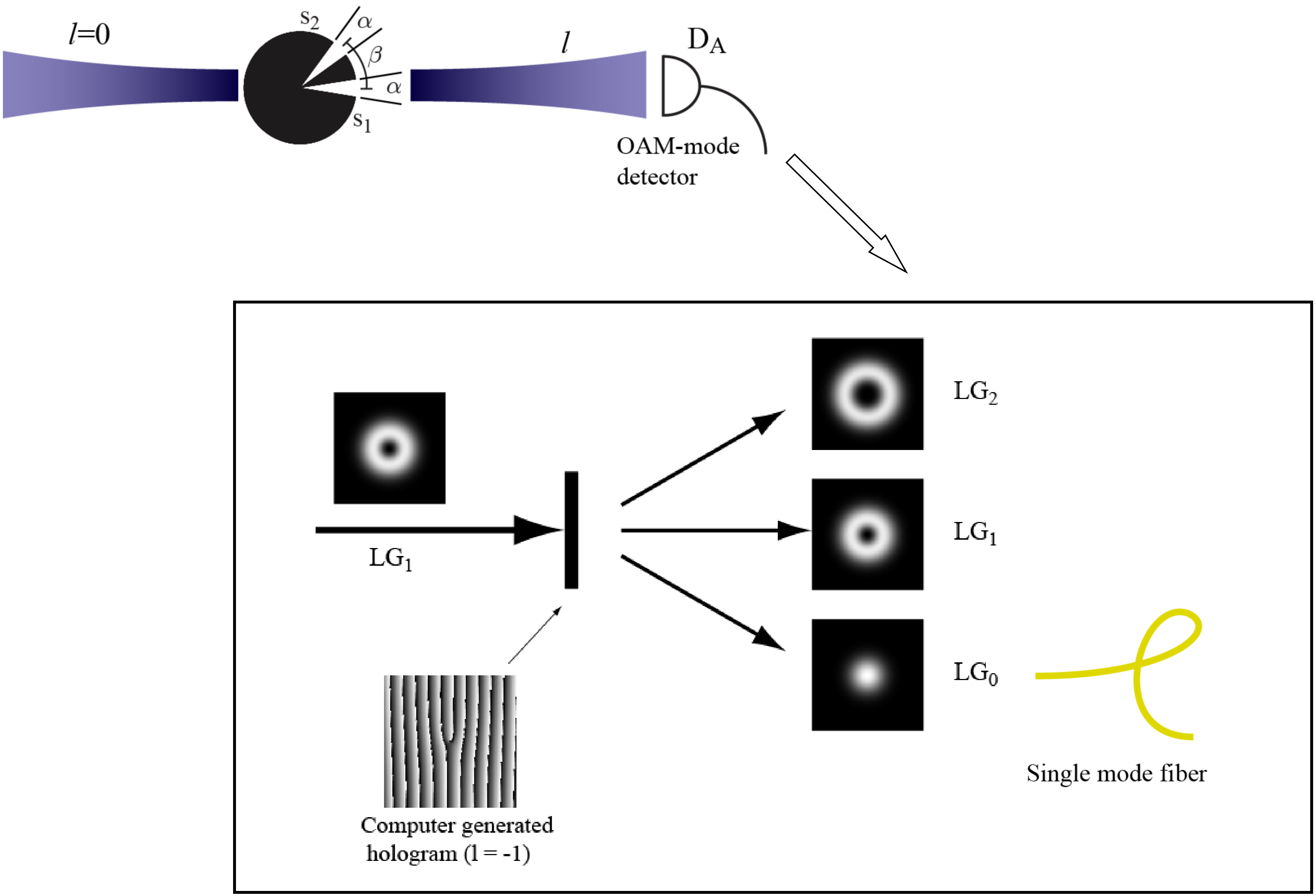
Necessary condition for interference:

$$|\Delta\boldsymbol{\rho}_p| < \sigma_\mu(z)$$

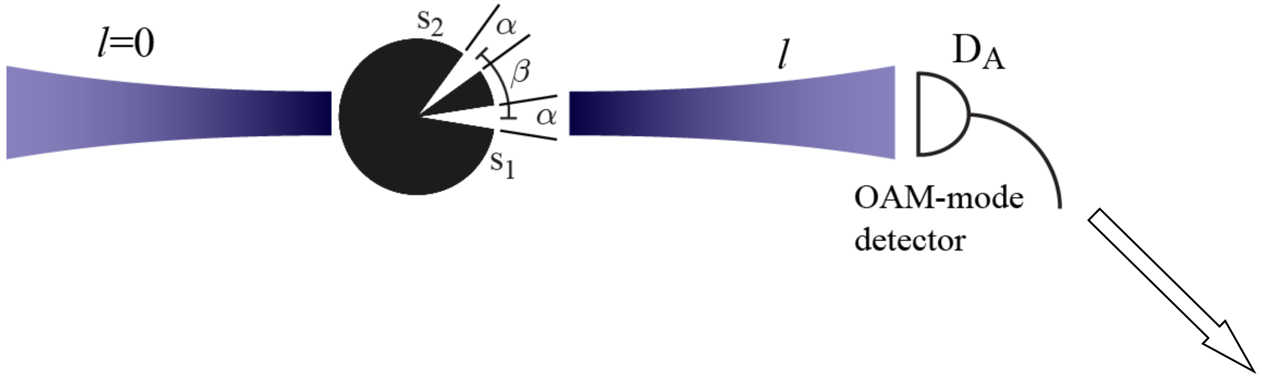
A photon interferes with itself: Angular

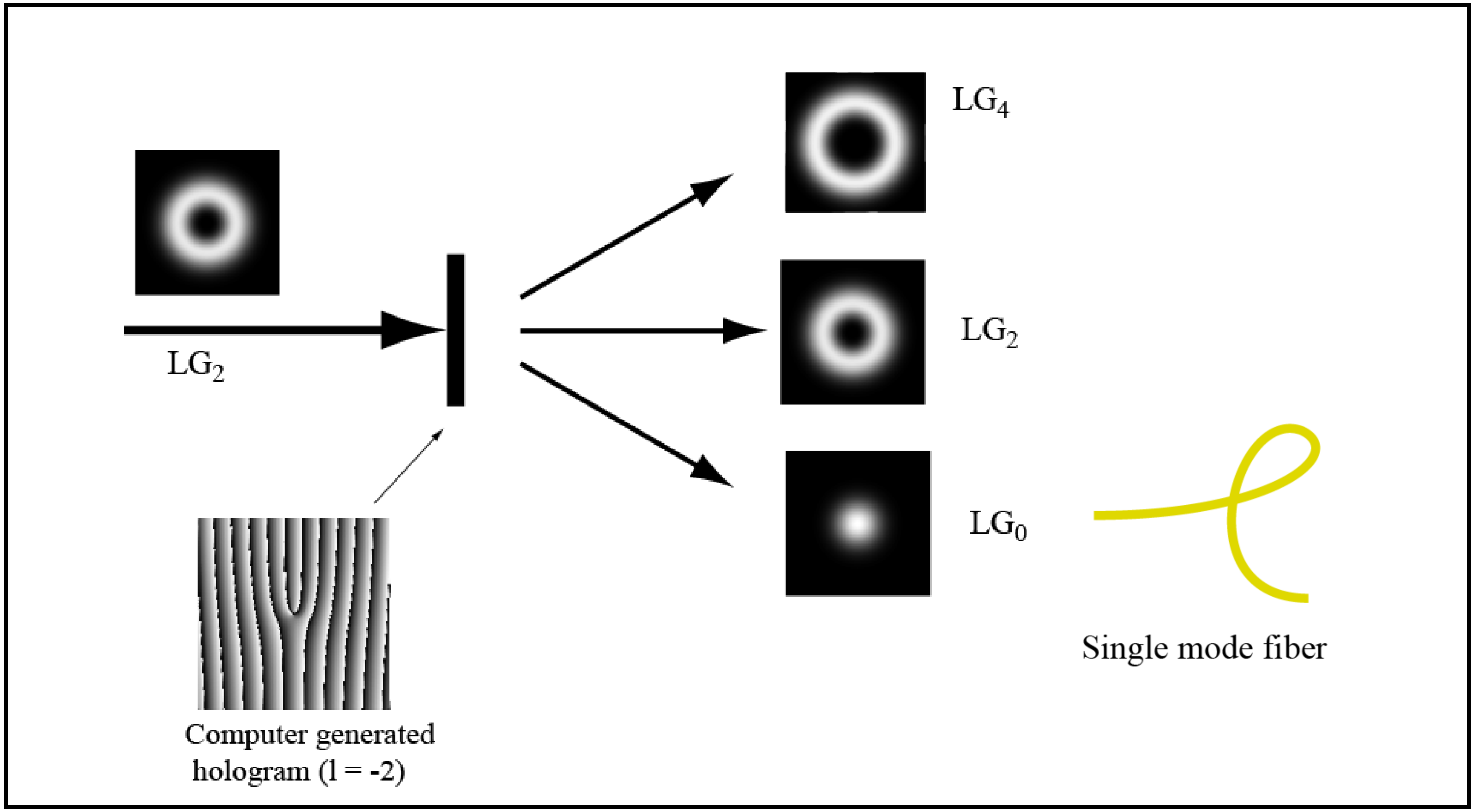


A photon interferes with itself: Angular



A photon interferes with itself: Angular





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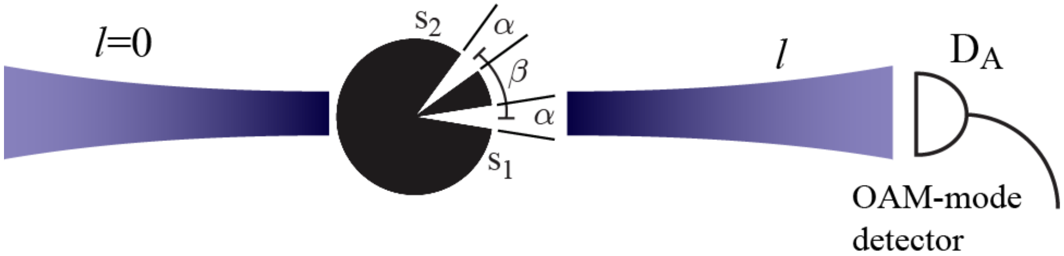
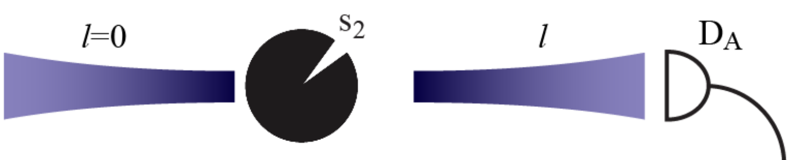


1

$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$

$$= \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right)$$

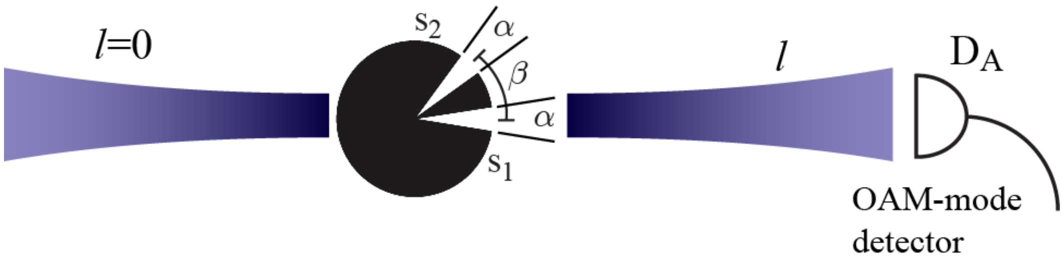
2



$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

E. Yao et al., Opt. Express **14**, 13089 (2006)
A. K. Jha, et al., PRA **78**, 043810 (2008)

A photon interferes with itself: Angular



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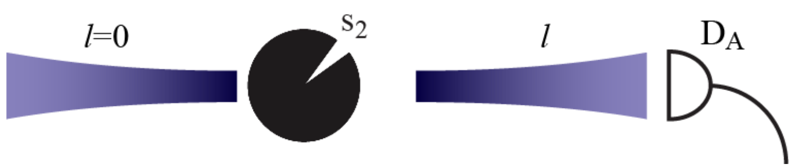
2



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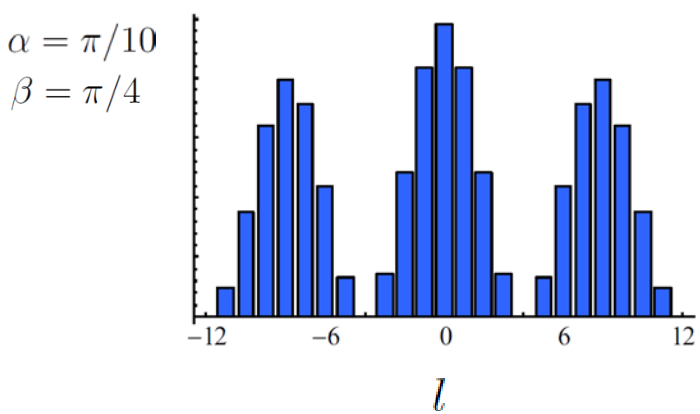


OAM-mode distribution:

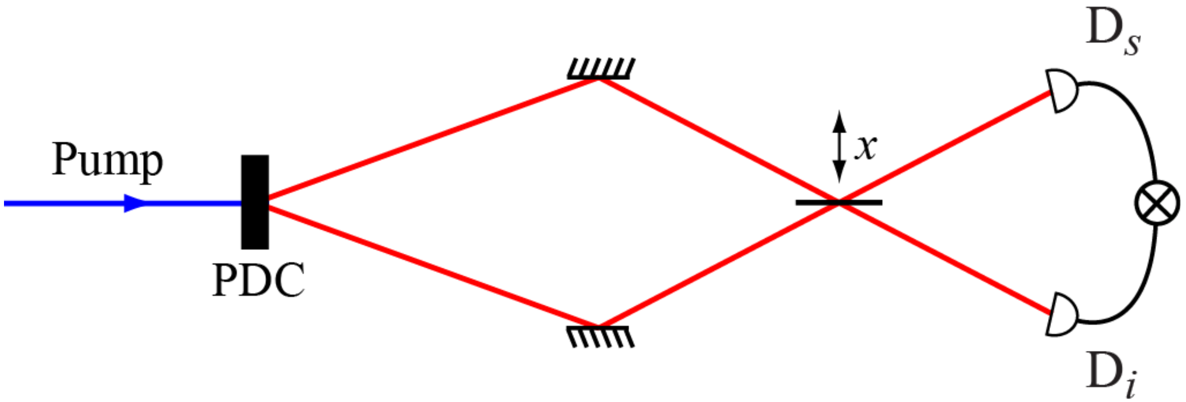
$$I_A = C\frac{\alpha^2}{\pi} \mathrm{sinc}^2\left(\frac{l\alpha}{2}\right)[1 + \cos(l\beta)]$$

E. Yao et al., Opt. Express **14**, 13089 (2006)
A. K. Jha, et al., PRA **78**, 043810 (2008)

Two-photon interference (an example)

Hong-Ou-Mandel Effect

C. K. Hong et al., PRL 59, 2044 (1987)



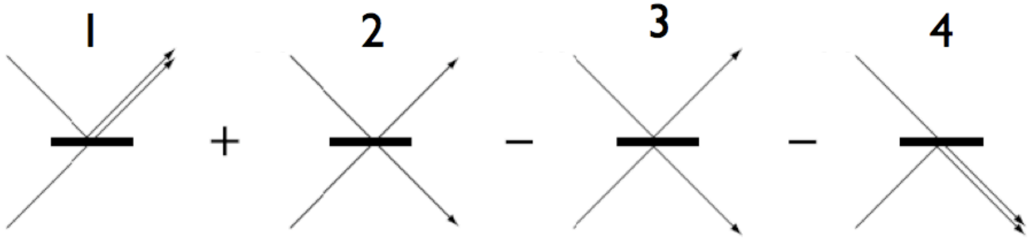


Image source: Wikepedia and google images

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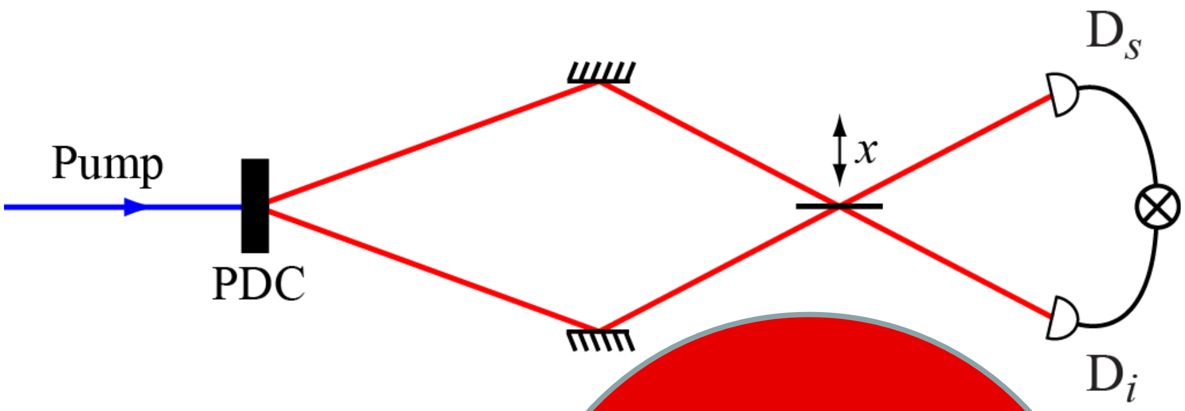
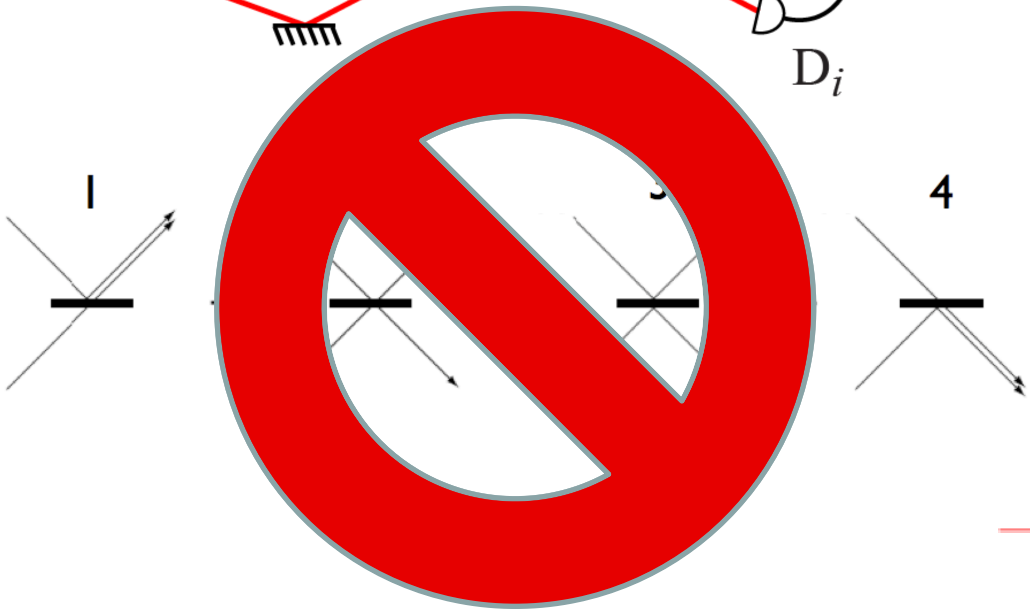
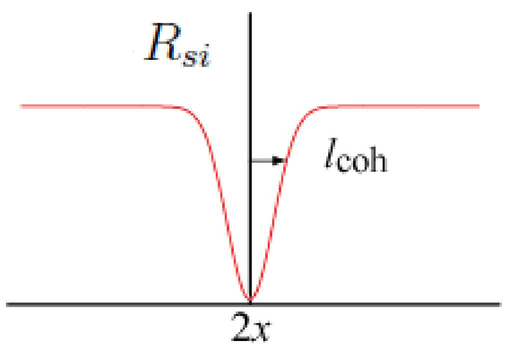
Hong-Ou-Mandel Effect

C. K. Hong et al., PRL 59, 2044 (1987)







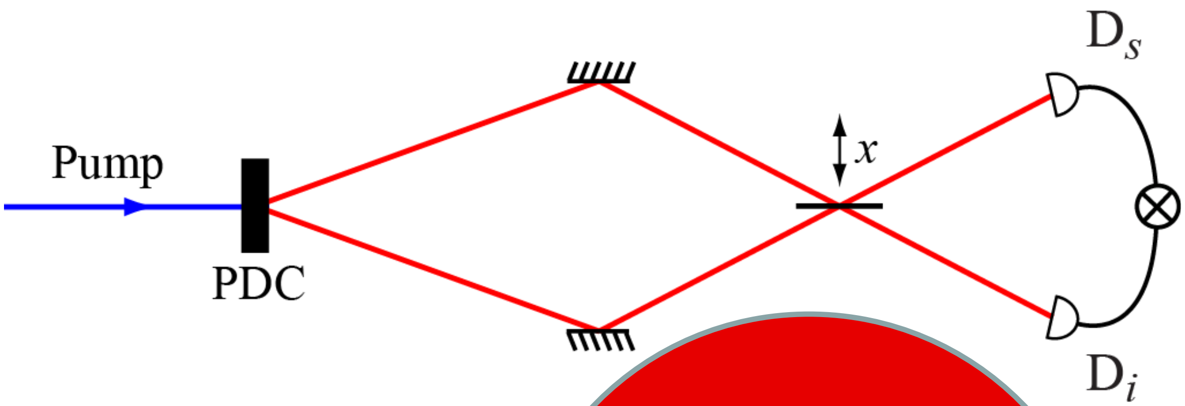
Bunching of photons at a beam splitter

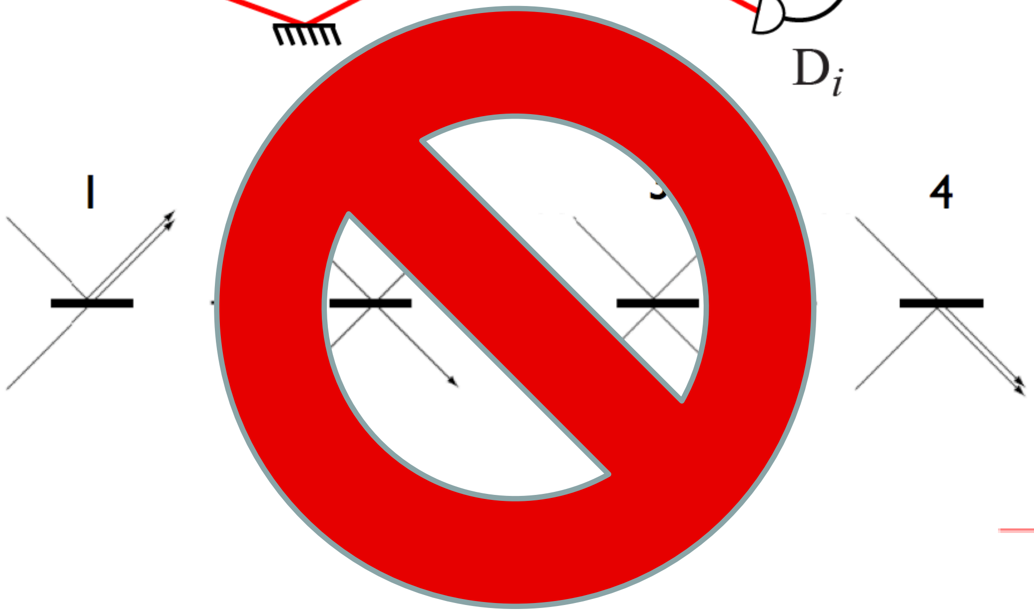
Image source: Wikepedia and google images

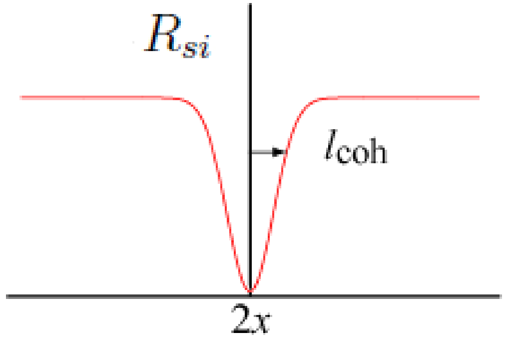
Two-photon interference (an example)

Hong-Ou-Mandel Effect

C. K. Hong et al., PRL 59, 2044 (1987)







Is this interference between two different phtons?

Is Dirac's statement incorrect? - NO

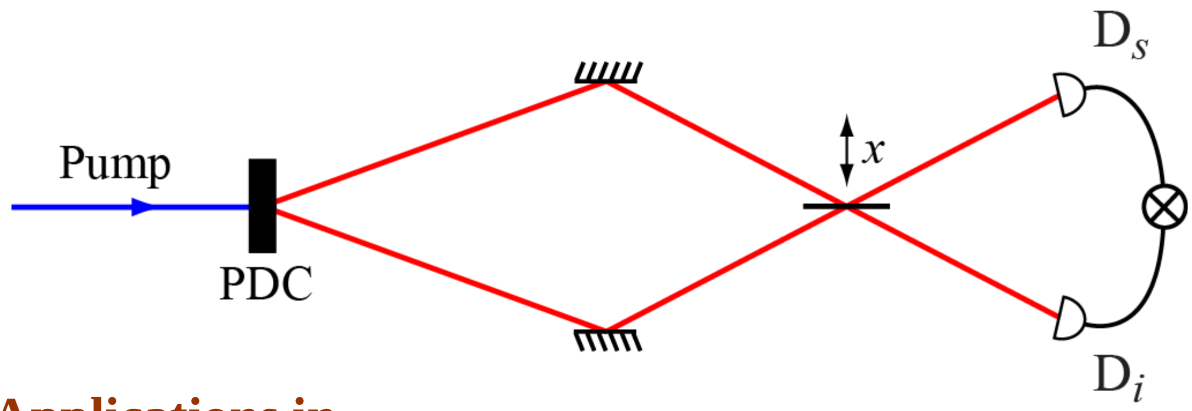
Here, a two-photon is interfering with itself

Image source: Wikepedia and google images

Two-photon interference (an example)

Hong-Ou-Mandel Effect

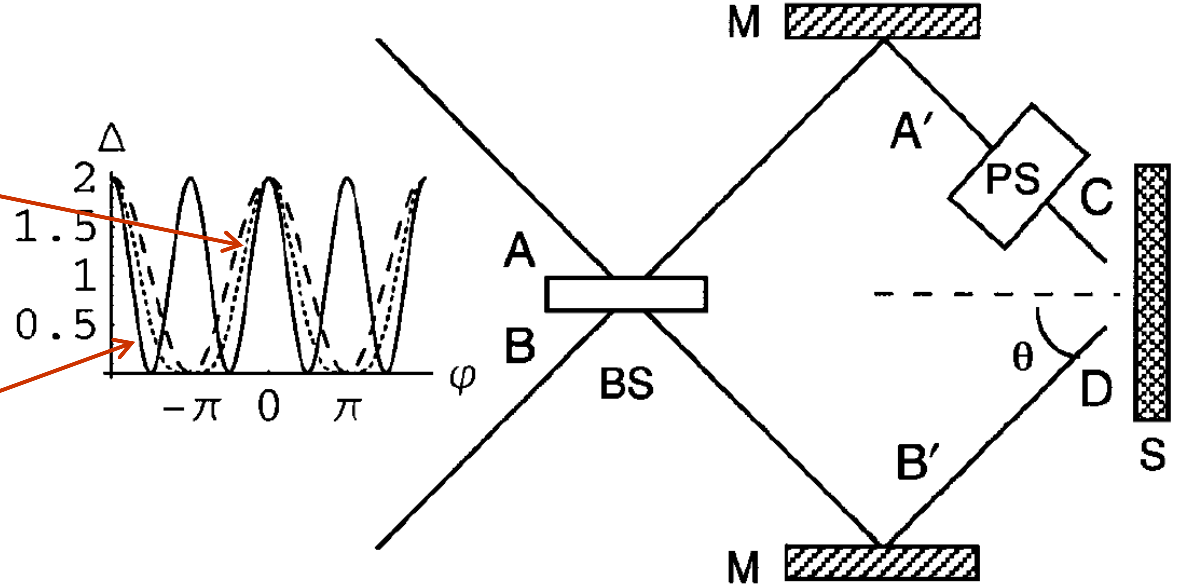
C. K. Hong et al., PRL 59, 2044 (1987)



Applications in quantum metrology

$1 + \cos 2\varphi$

$1 + \cos 4\varphi$

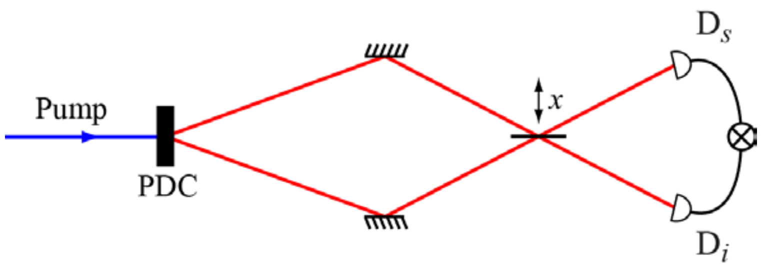


Phys. Rev. Lett. 85, 2733 (2000).

Two-Photon Interference (Other examples)

- **Hong-Ou-Mandel effect**

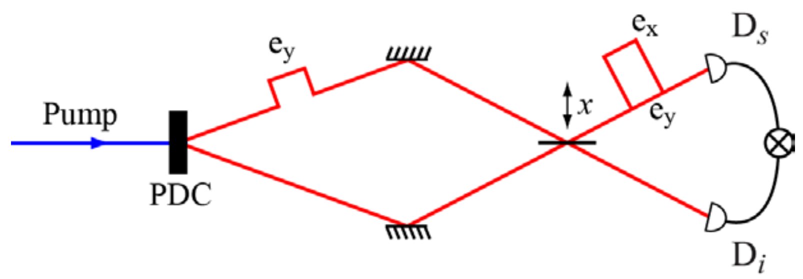
C. K. Hong et al., PRL 59, 2044 (1987)



Quantum Optical Lithography
Phys. Rev. Lett. 85, 2733 (2000).

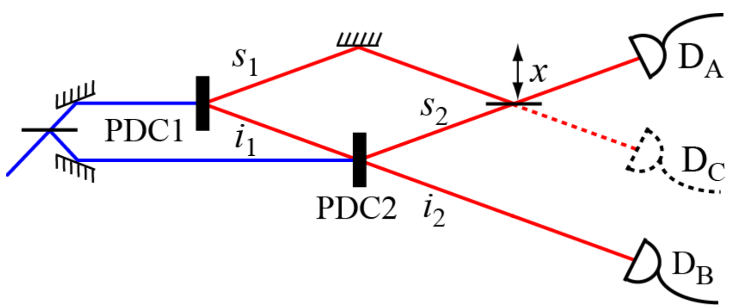
- **Postponed Compensation Experiment**

T. B. Pittman, PRL 77, 1917 (1996)



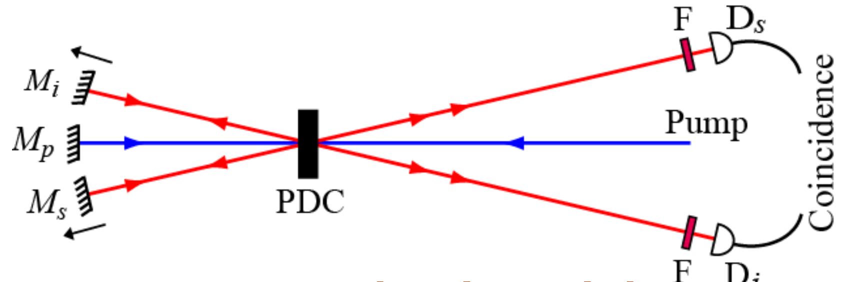
- **Induced Coherence**

X. Y. Zou et al., PRL 67, 318 (1991)



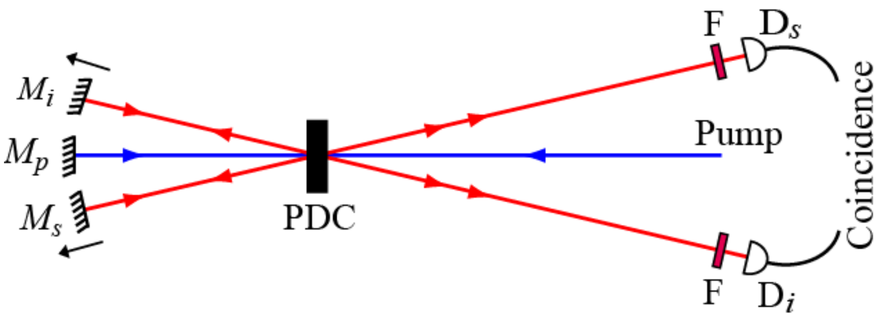
- **Frustrated two-photon Creation**

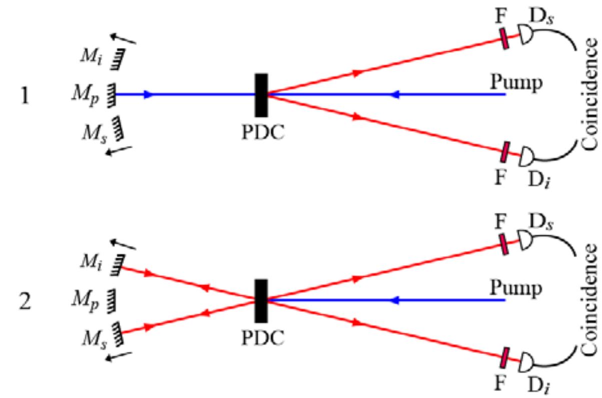
T. J. Herzog et al., PRL 72, 629 (1994)

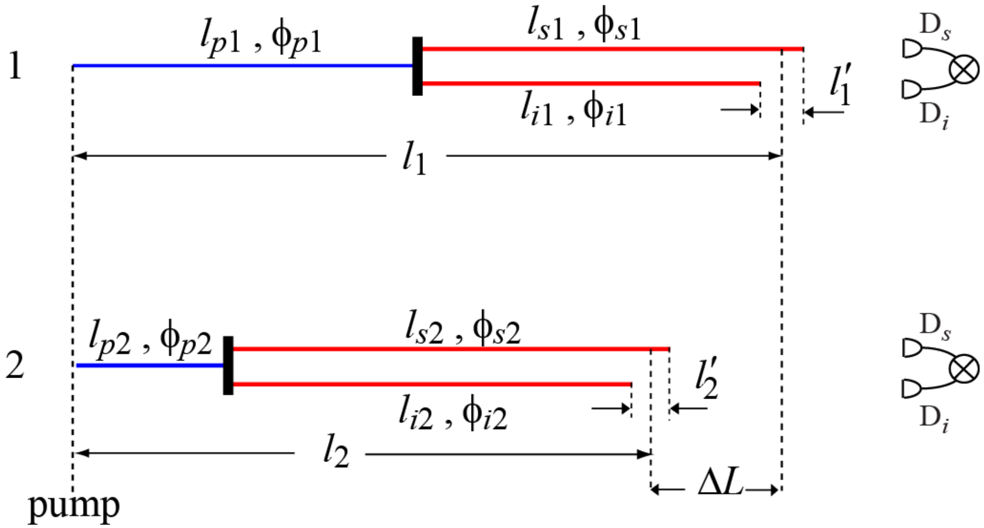


Quantum Imaging with undetected photons
,Nature 512, 409 (2014)

Two-Photon Interference: A two-photon interferes with itself







$$\Delta L \equiv l_1 - l_2$$

two-photon path-length difference

$$\Delta L' \equiv l_1' - l_2'$$

two-photon path-asymmetry length difference

$$\Delta\phi \equiv (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L + \Delta\phi)]$$

$$\gamma(\Delta L) = \frac{\langle v_1(t) v_2^*(t + \Delta L/c)\rangle_t}{\sqrt{|v_1|^2 |v_2|^2}} \qquad \gamma'(\Delta L') = \frac{\langle g_1^*(\tau) g_2(\tau - \Delta L'/c)\rangle_\tau}{\sqrt{|g_1|^2 |g_2|^2}}$$

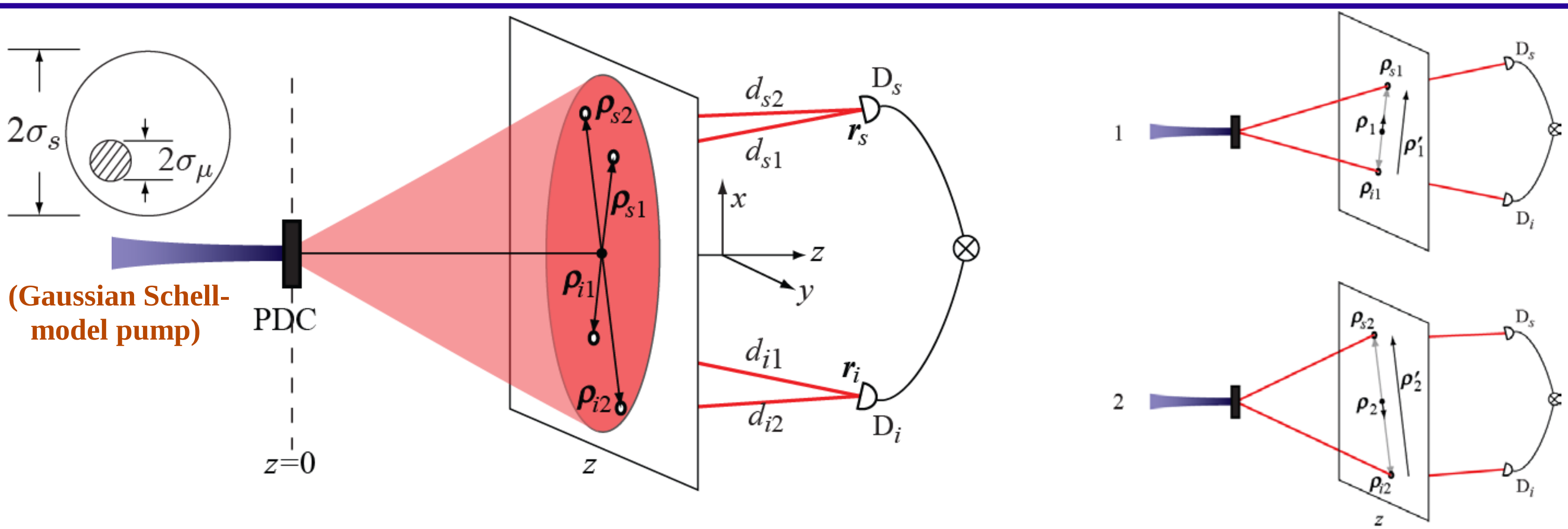
Necessary conditions for two-photon interference:

$$\Delta L < l_{coh}^p$$

$$\Delta L' < l_{coh}$$

Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

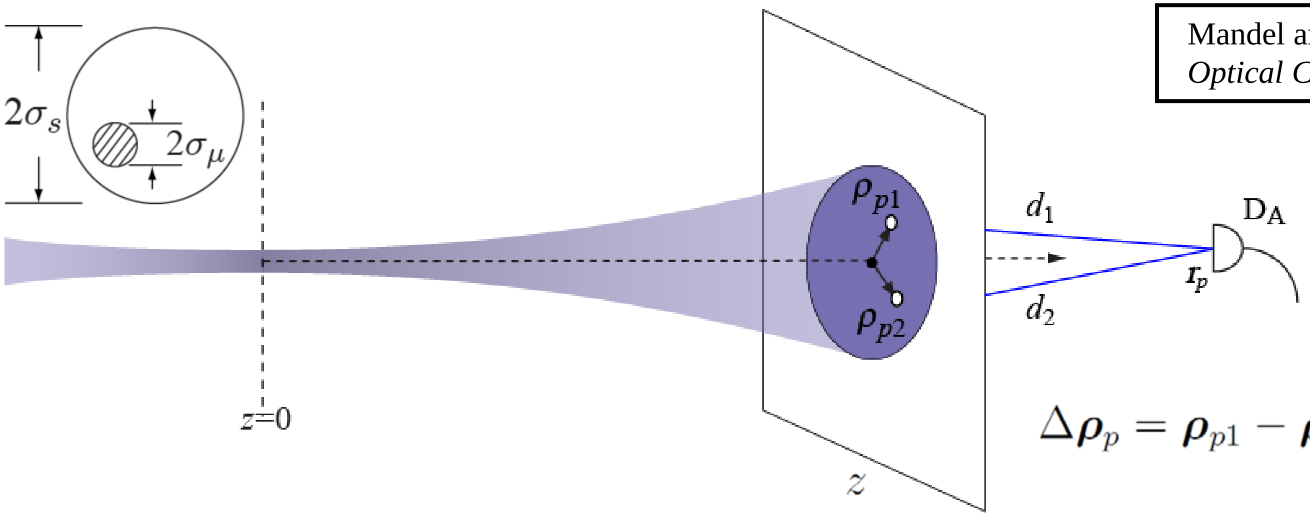
Two-Photon Coherence and Entanglement



Coincidence Rate $R_{si}(\boldsymbol{r}_s, \boldsymbol{r}_i) = k_1^2 S^{(2)}(\boldsymbol{\rho}_{s1}, \boldsymbol{\rho}_{i1}, z) + k_2^2 S^{(2)}(\boldsymbol{\rho}_{s2}, \boldsymbol{\rho}_{i2}, z) + k_1 k_2 W^{(2)}(\boldsymbol{\rho}_{s1}, \boldsymbol{\rho}_{i1}, \boldsymbol{\rho}_{s2}, \boldsymbol{\rho}_{i2}, z) e^{i[\omega_s(t_{s1}-t_{s2})+\omega_i(t_{i1}-t_{i2})]} + \text{c.c.}$

A photon interferes with itself: Spatial

Mandel and Wolf,
Optical Coherence and Quantum Optics



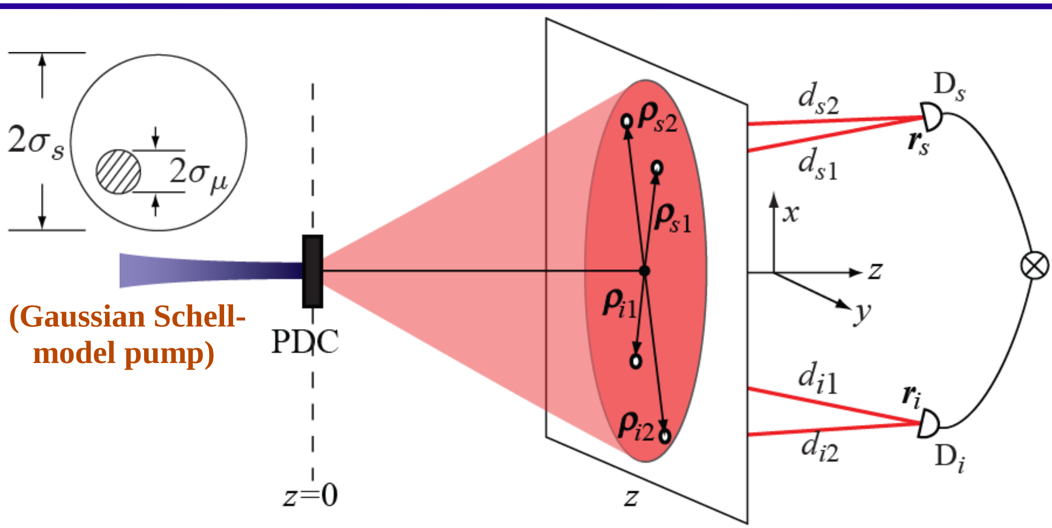
$$\Delta\boldsymbol{\rho}_p = \boldsymbol{\rho}_{p1} - \boldsymbol{\rho}_{p2}$$

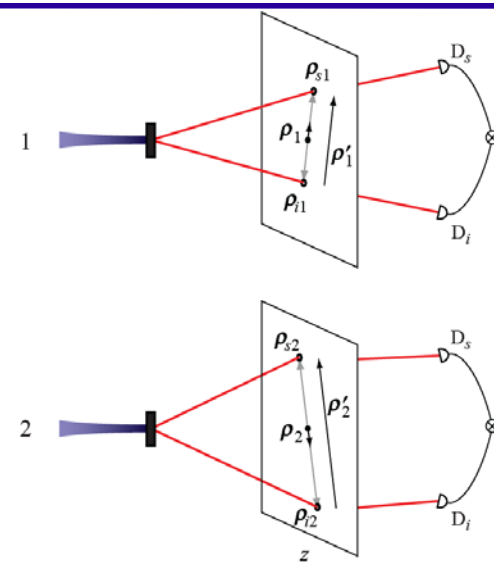
$$I_A(x) = k_1^2 S(x_1, z) + k_2^2 S(x_2, z) + 2k_1 k2 \sqrt{S(x_1, z) S(x_2, z)}\mu(\Delta x, z)\cos(k_0 \Delta l)$$

Necessary condition for interference:

$$|\Delta\boldsymbol{\rho}_p| < \sigma_\mu(z)$$

Two-Photon Coherence and Entanglement





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Entangled two-qubit state

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}$$

$$a = \eta S^{(2)}(\boldsymbol{\rho}_1, z)$$

$$b = \eta S^{(2)}(\boldsymbol{\rho}_2, z)$$

$$c = d^* = \eta W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$$

$$\eta = 1/[S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z)]$$

O'Sullivan et al., PRL **94**, 220501 (2005)
Neves et al., PRA **76**, 032314 (2007)
Walborn et al., PRA **76**, 062305 (2007)
Taguchi et al., PRA **78**, 012307 (2008)

Entanglement of the state (Concurrence) :

$$C(\rho_{\text{qubit}}) = 2|c| = 2\eta |W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)|$$

$$C(\rho_{\text{qubit}}) = \mu^{(2)}(\Delta\boldsymbol{\rho}, z)$$ (**with** $a = b$)

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

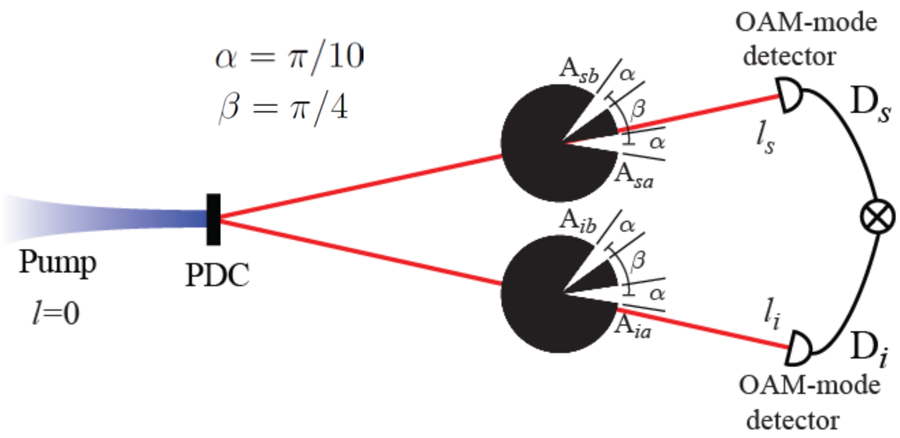
$$\zeta = \rho_{\text{qubit}}(\sigma_y \otimes \sigma_y)\rho^*_{\text{qubit}}(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\text{qubit}}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

A. K. Jha and R.W. Boyd, PRA **81,** 013828 (2010)

A. K. Jha, G.A. Tyler and R.W. Boyd, **PRA 81**, 053832 (2010)

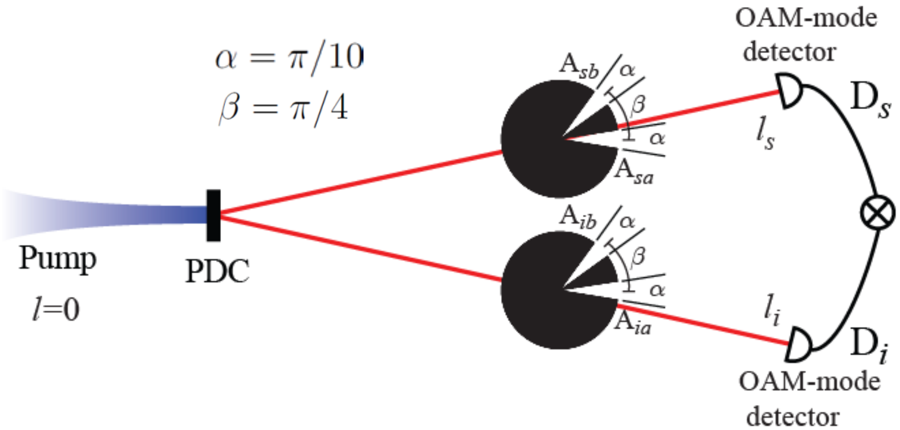
Angular Two-Photon Interference

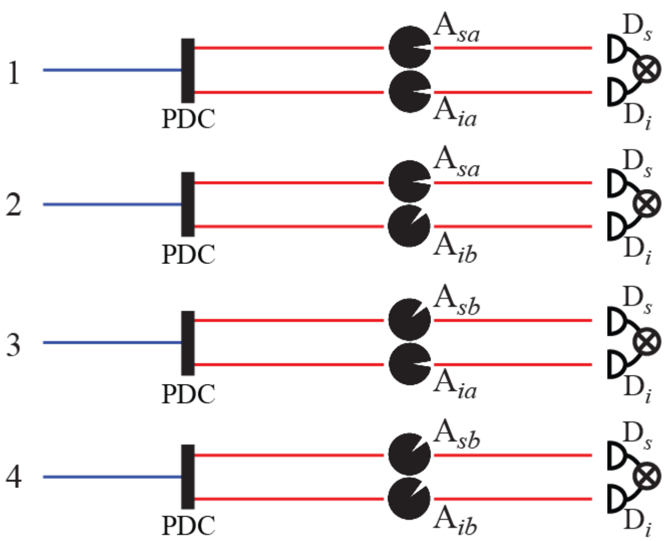


State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

Angular Two-Photon Interference





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$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

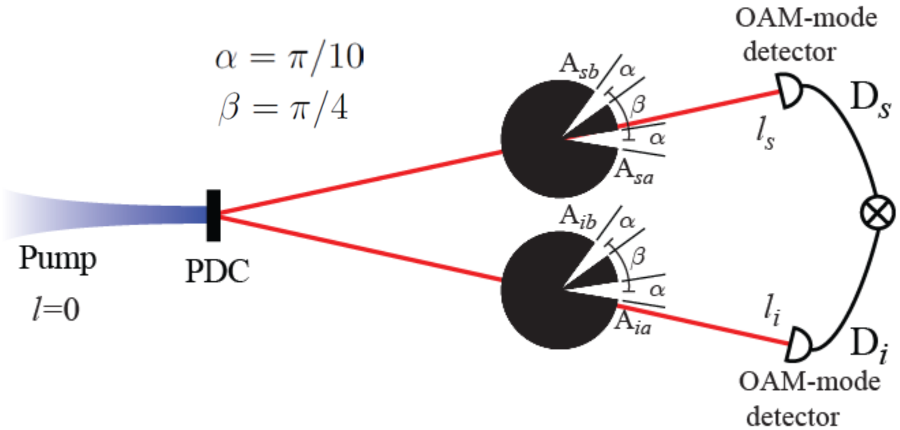
State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^{*} = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Angular Two-Photon Interference



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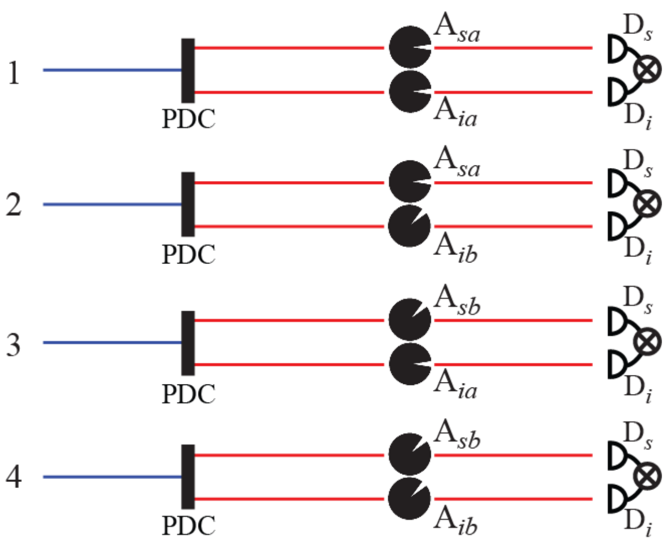


Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2$$

$$\times \left\{\rho_{11} + \rho_{44} + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos\left[(l_s + l_i)\beta + \theta\right]\right\}$$

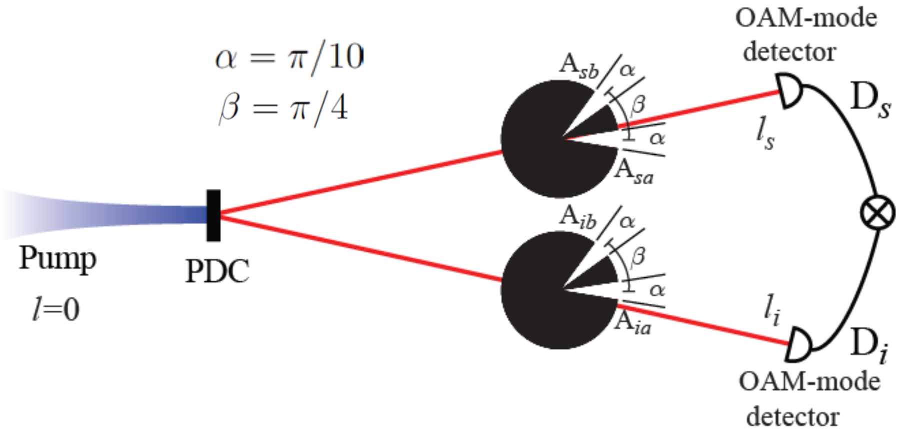
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

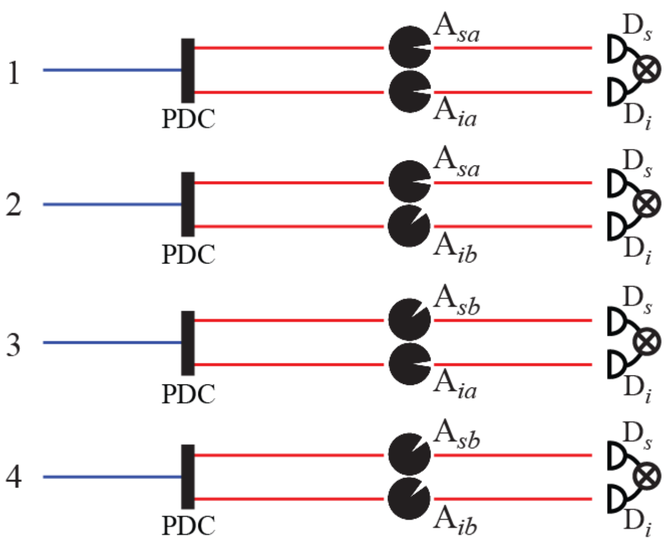
Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho_{\rm qubit}(\sigma_y \otimes \sigma_y)\rho_{\rm qubit}^*(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\rm qubit}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s | -l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^* = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$
$$\times \{\rho_{11} + \rho_{44} + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho_{\rm qubit}(\sigma_y \otimes \sigma_y)\rho_{\rm qubit}^*(\sigma_y \otimes \sigma_y)$$

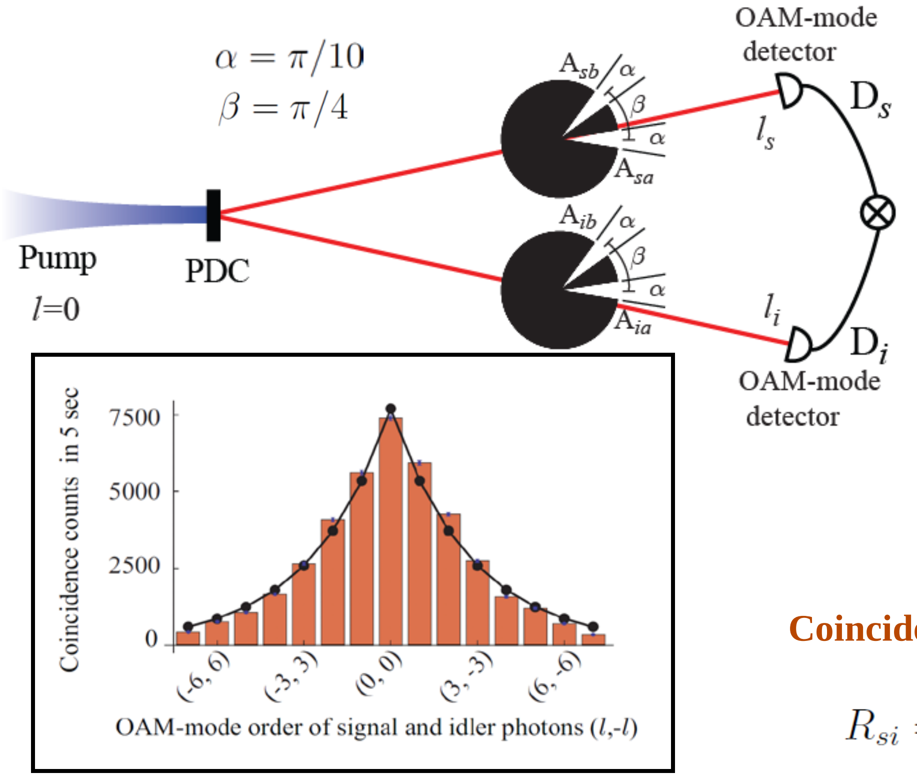
$$C(\rho_{\rm qubit}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

Concurrence of the two-qubit state:

$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu = V$$

A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference



State of the two photons produced by PDC:

$$|\psi_{\text{tp}}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\text{qubit}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \quad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\, \mu e^{i\theta} \end{aligned}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,\text{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \text{sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2$$
$$\times \{\rho_{11} + \rho_{44} + 2\sqrt{\rho_{11}\rho_{44}}\,\mu \cos[(l_s + l_i)\beta + \theta]\}$$

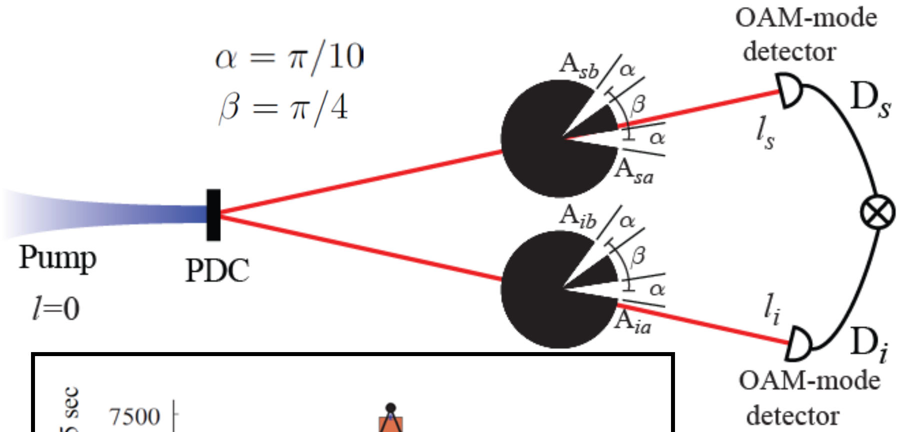
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

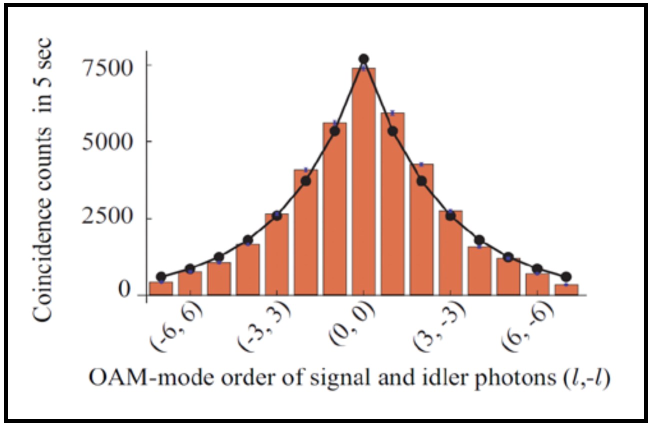
Concurrence of the two-qubit state:

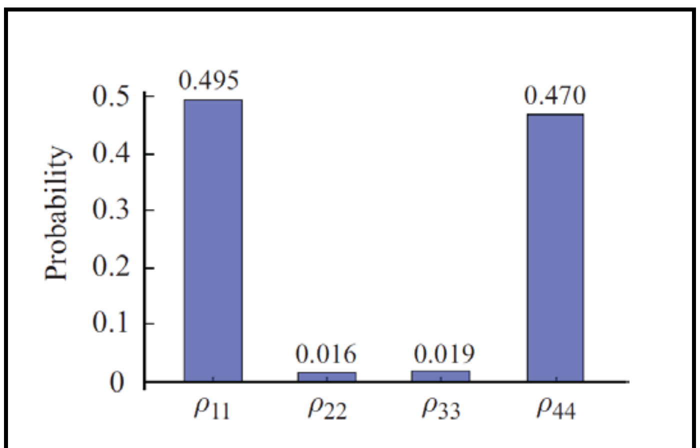
$$C(\rho_{\text{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu \;= V$$

A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference







State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta} \end{aligned}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,{\rm sinc}\left[(l_s - l)\frac{\alpha}{2}\right] {\rm sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2$$

$$\times \{\rho_{11} + \rho_{44} + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

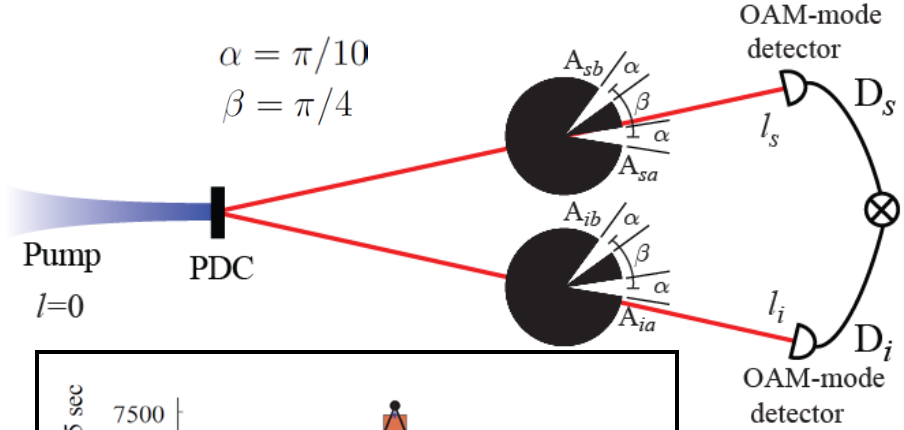
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

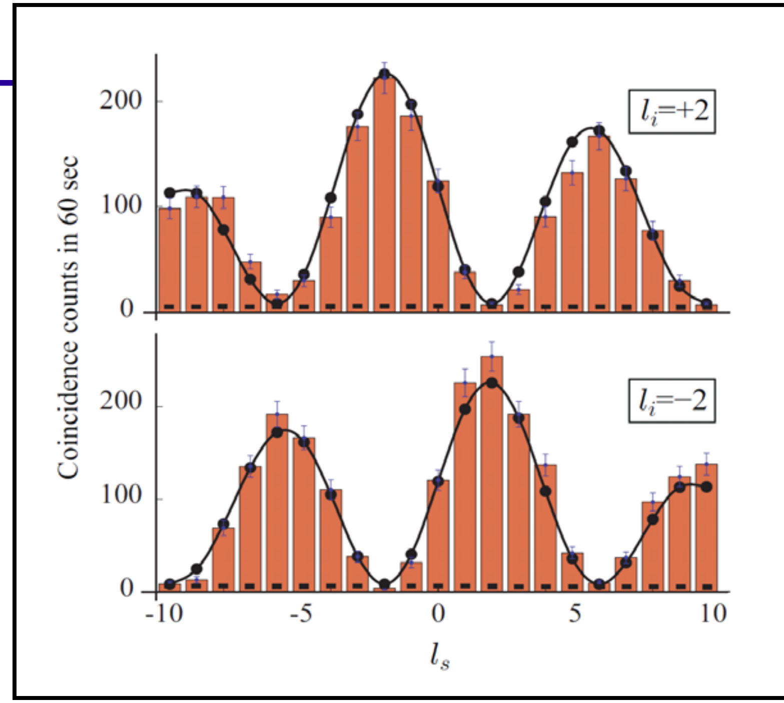
Concurrence of the two-qubit state:

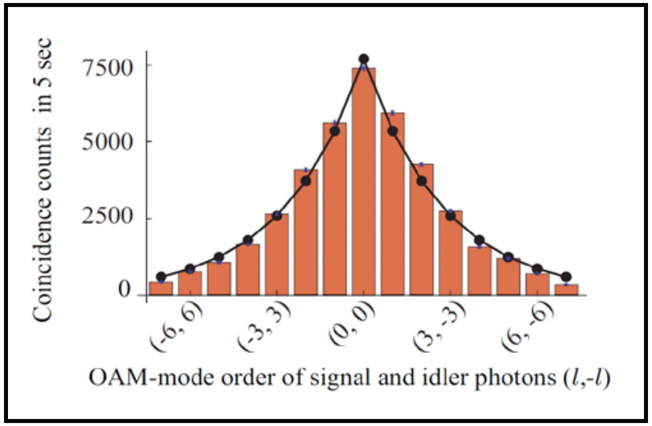
$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu \ = V$$

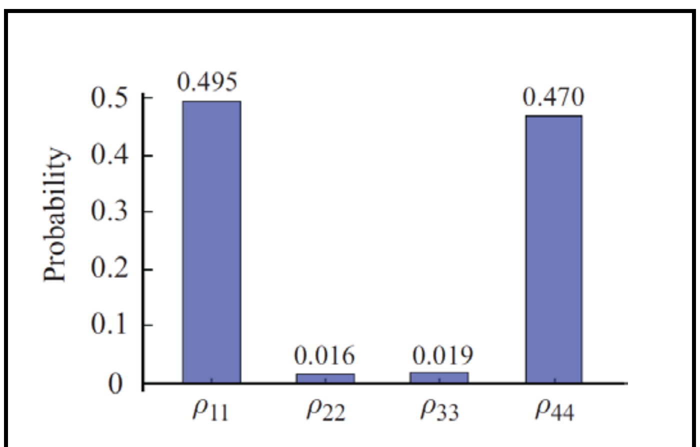
A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference









Coincidence count rate:

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Concurrence of the two-qubit state:

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A. K. Jha et al., PRL **104,** 010501 (2010)

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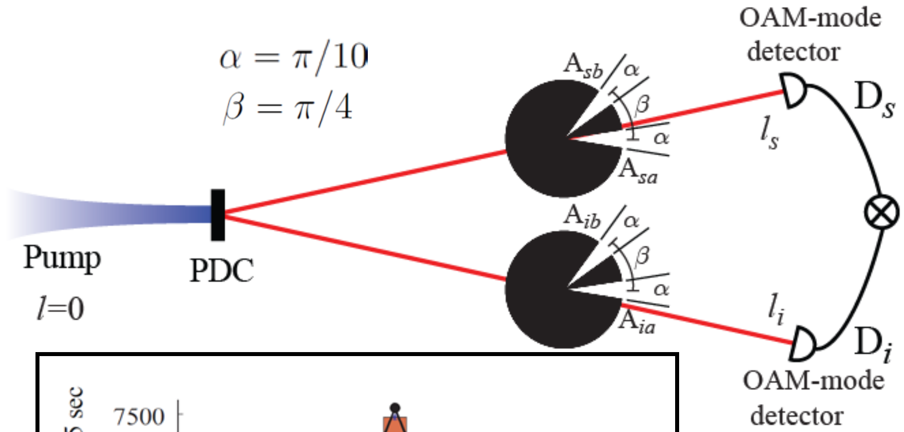
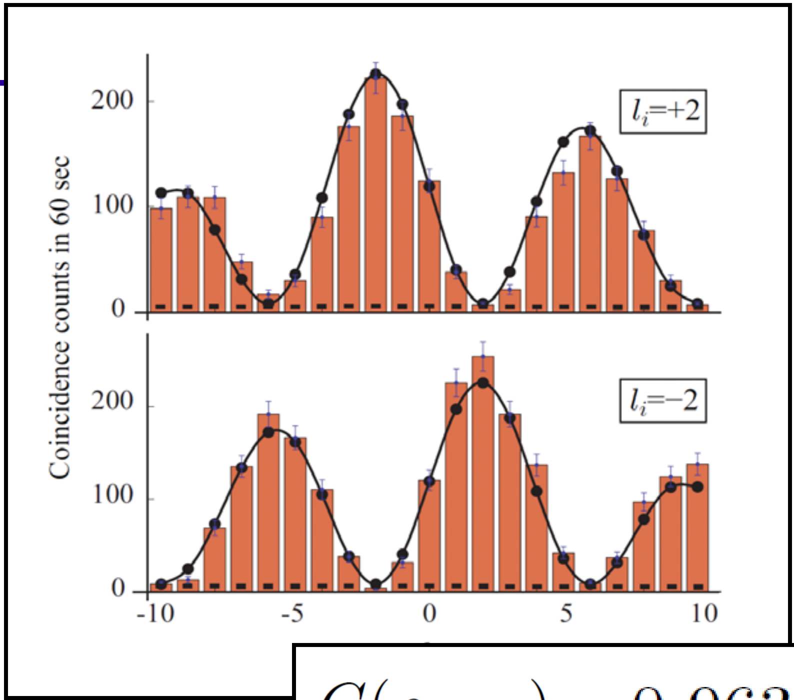
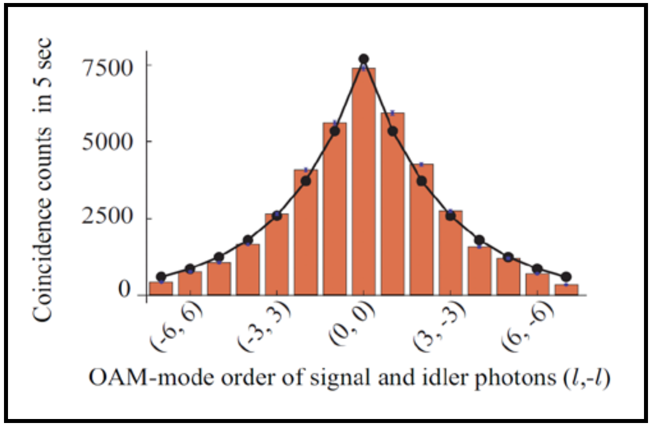
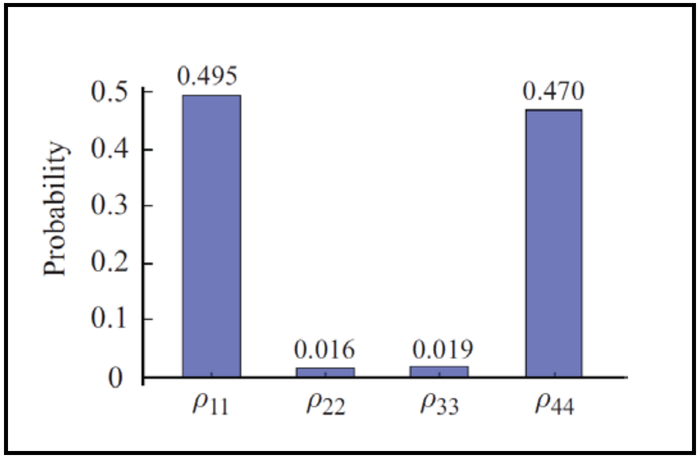




$$C(\rho_{\text{qubit}}) = 0.963$$





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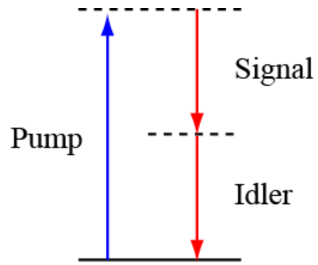
A. K. Jha et al., PRL **104,** 010501 (2010)
Jha, Agarwal, and Boyd, PRA**84**, 063847 (2011)

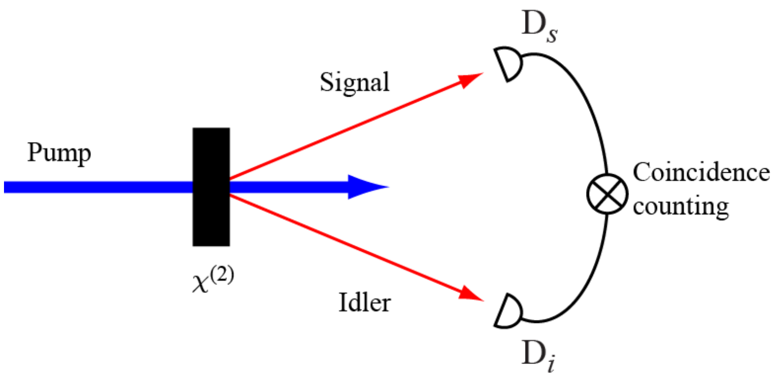
Summary

Parametric down-conversion (PDC)

Bunrham and Weinberg,
Phys. Rev. Lett. **25,** 85 (1970)

Robert W. Boyd,
Nonlinear Optics, 2nd ed.





variable	Conservation law	Entanglement	EPR Paradox	Two-photon coherence
Energy	$\omega_p = \omega_s + \omega_i$	Time and energy	$\Delta t^{(1)}_{\text{cond}} \Delta E^{(1)}_{\text{cond}} < \frac{\hbar}{2}$	Temporal
Transverse Momentum	$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$	Position and momentum	$\Delta x^{(1)}_{\text{cond}} \Delta p^{(1)}_{\text{cond}} < \frac{\hbar}{2}$	Spatial
Orbital angular momentum	$l_p = l_s + l_i$	Angular position and orbital angular momentum	$\Delta \phi^{(1)}_{\text{cond}} \Delta L^{(1)}_{\text{cond}} < \frac{\hbar}{2}$	Angular

Entangled Photons: Future directions

1. **Foundations of Quantum Mechanics. (Theory + Experiment)**

- Questions related to non-locality and physical reality.
- Complete description of two-photon entanglement in terms of coherence measures
- Extension of coherence-based measure for quantifying high-dimensional entanglement.
- Photon-statistics of entangled photons.
- Correlated-noise measurements of entangled photons

2. **Applications of Quantum Entanglement. (Theory + Experiment)**

- Developing sources of entangled photon based on parametric down-conversion
- Use of OAM-entangled photons for high-dimensional Quantum information processing.
- Use of entangled photons for high-resolution imaging, remote sensing and communication through turbulent atmosphere.

Entangled Photons: Open Problems!

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PhD and Post-Doc positions available within the group

Thank you for your attention