Coherence Properties of the Entangled Two-Photon Field Produced by Parametric Down-Conversion

Anand Kumar Jha

Robert W. Boyd Research Group
The Institute of Optics, University of Rochester, Rochester, NY

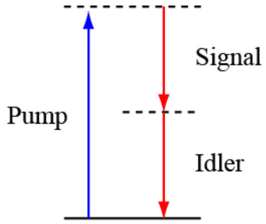
Institute for Quantum Computing
University of Waterloo, ON, January 7, 2010

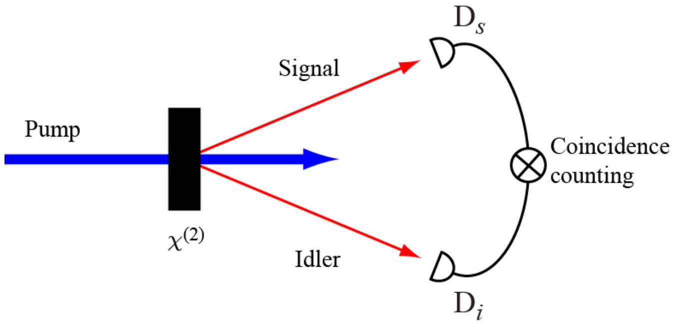
Introduction and Outline

Quantum Entanglement

- EPR paradox and non-locality, Hidden variable theories, Bell inequalities …
- Quantum cryptography, Quantum dense coding, Quantum lithography…

Parametric down-conversion provides a source of entangled photons



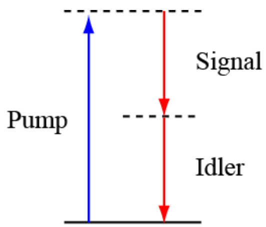


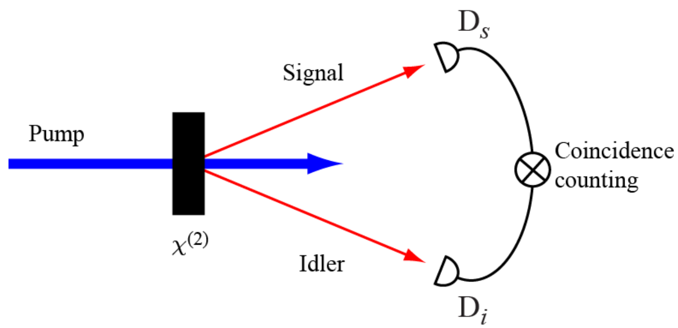
Introduction and Outline

Quantum Entanglement

- EPR paradox and non-locality, Hidden variable theories, Bell inequalities …
- Quantum cryptography, Quantum dense coding, Quantum lithography…

Parametric down-conversion provides a source of entangled photons





$$\omega_p = \omega_s + \omega_i$$

Entanglement in time and energy

"Temporal" two-photon coherence

$$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$$

Entanglement in position and momentum

"Spatial" two-photon coherence

$$l_p = l_s + l_i$$

Entanglement in angular position and orbital angular momentum

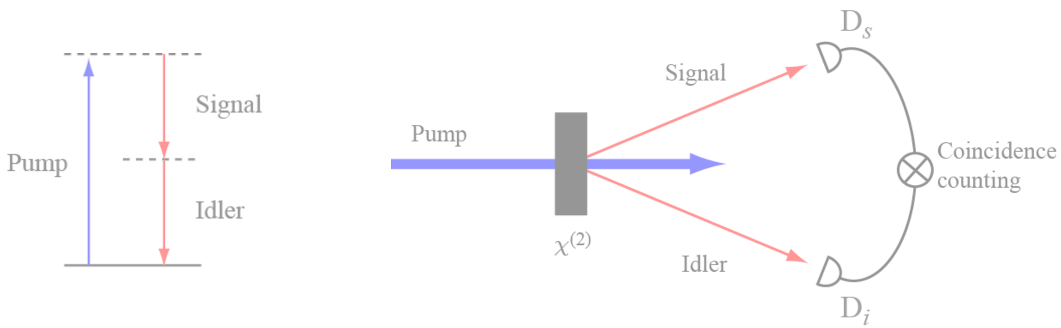
"Angular" two-photon coherence

Introduction and Outline

Quantum Entanglement

- EPR paradox and non-locality, Hidden variable theories, Bell inequalities …
- Quantum cryptography, Quantum dense coding, Quantum lithography…

Parametric down-conversion provides a source of entangled photons



$\omega_p = \omega_s + \omega_i$ **Entanglement in time and energy**

“Temporal” two-photon coherence

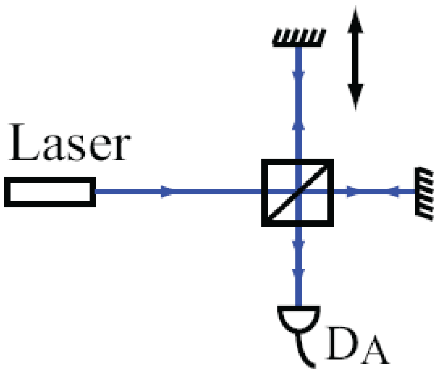
$\mathbf{q}_p = \mathbf{q}_s + \mathbf{q}_i$ **Entanglement in position and momentum**

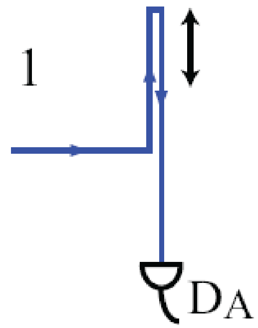
“Spatial” two-photon coherence

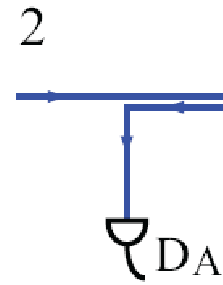
$l_p = l_s + l_i$ **Entanglement in angular position and orbital angular momentum**

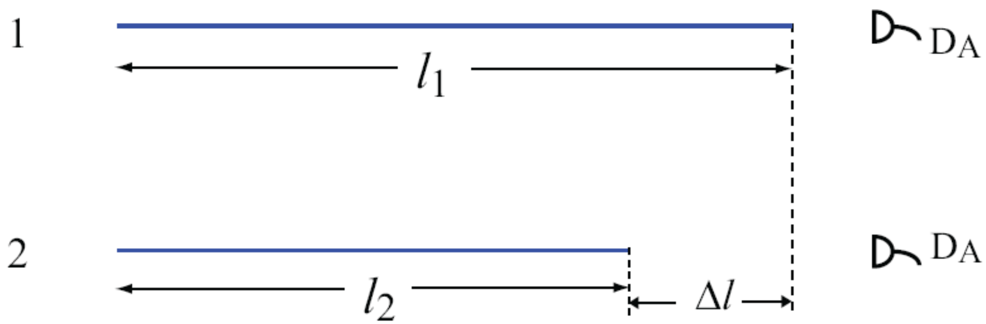
“Angular” two-photon coherence

One-Photon Interference: “A photon interferes with itself ” - Dirac









$$\Delta l = l_1 - l_2$$

$$I_A \propto 1 + \gamma(\Delta l)\cos(k_0 \Delta l)$$

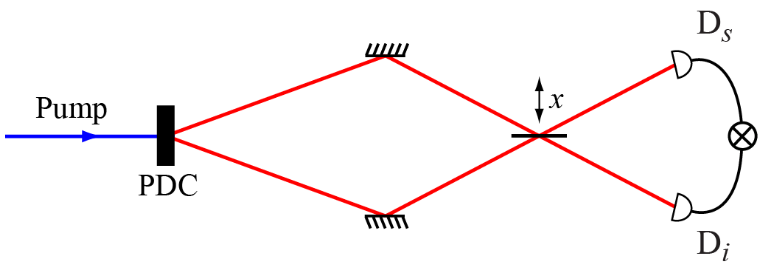
Necessary condition for interference:

$$\boxed{\Delta l < l_{coh}}$$

Two-Photon Interference

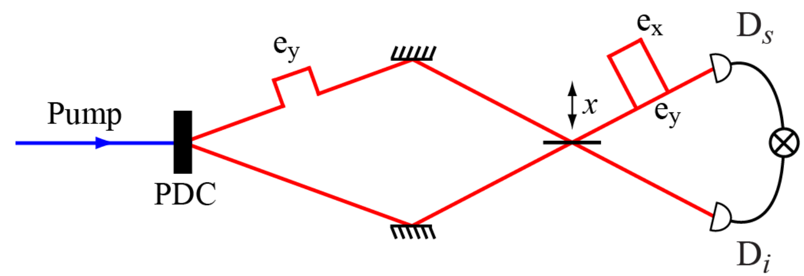
- **Hong-Ou-Mandel effect**

C. K. Hong et al., PRL 59, 2044 (1987)



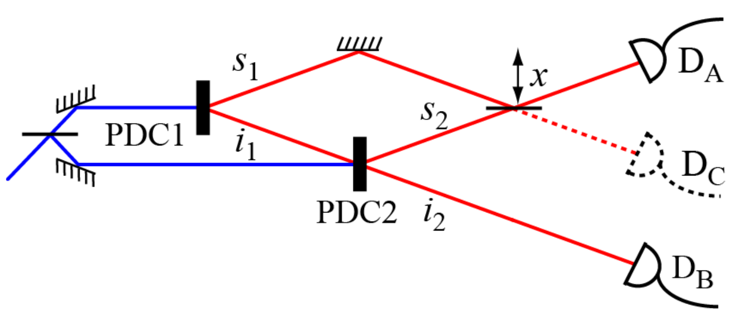
- **Postponed Compensation Experiment**

T. B. Pittman, PRL 77, 1917 (1996)



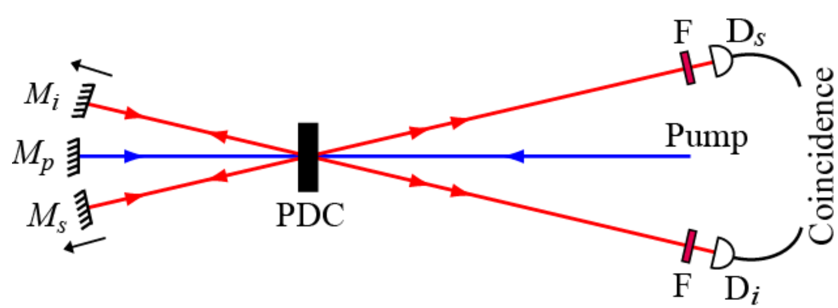
- **Induced Coherence**

X. Y. Zou et al., PRL 67, 318 (1991)

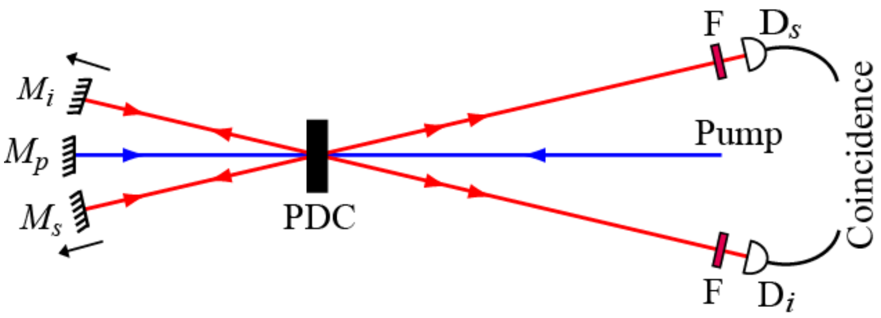


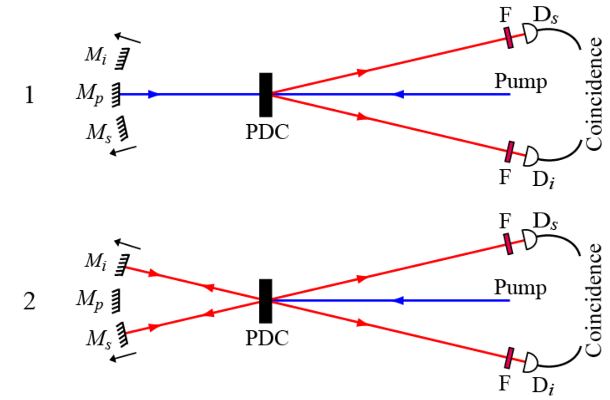
- **Frustrated two-photon Creation**

T. J. Herzog et al., PRL 72, 629 (1994)



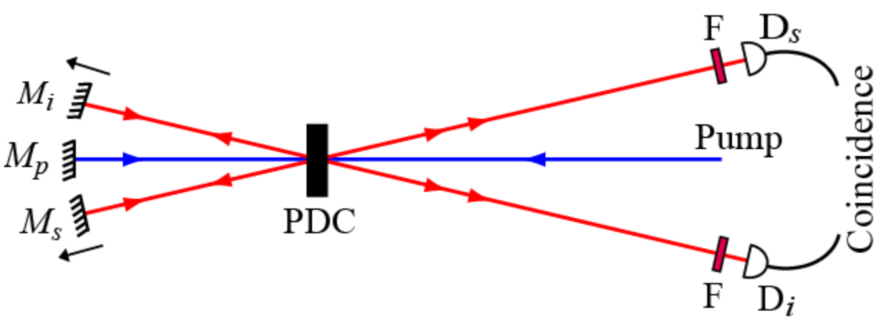
Two-Photon Interference

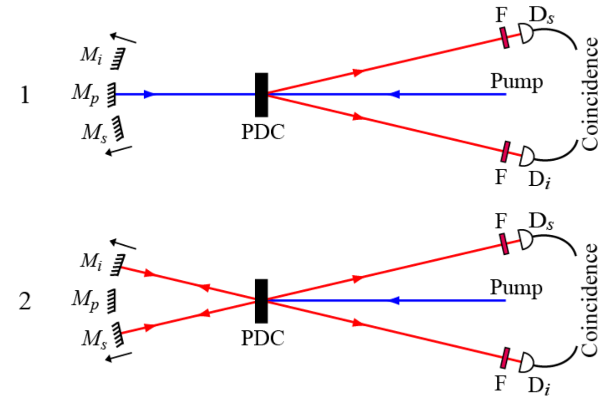


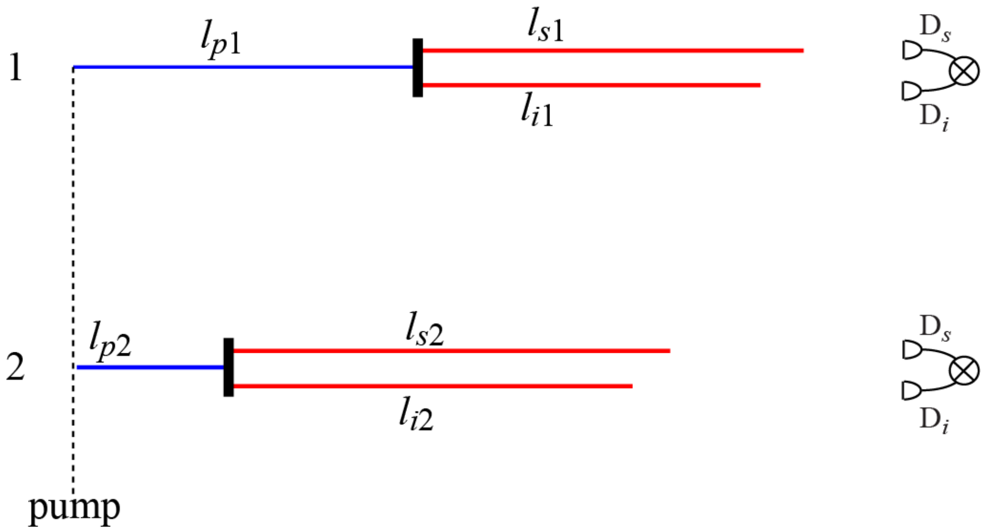


Herzog et al., PRL **72**, 629 (1994)

Two-Photon Interference

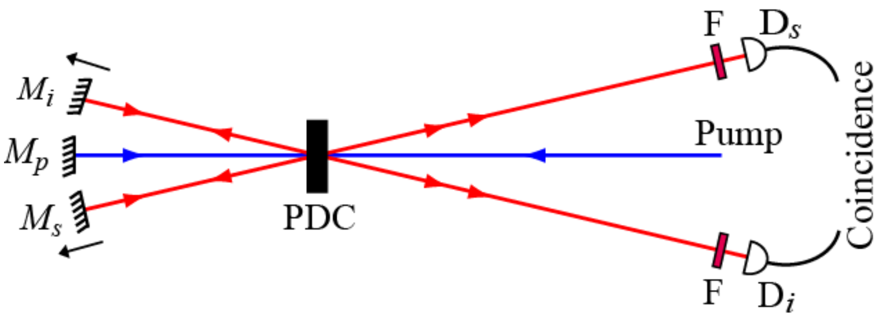


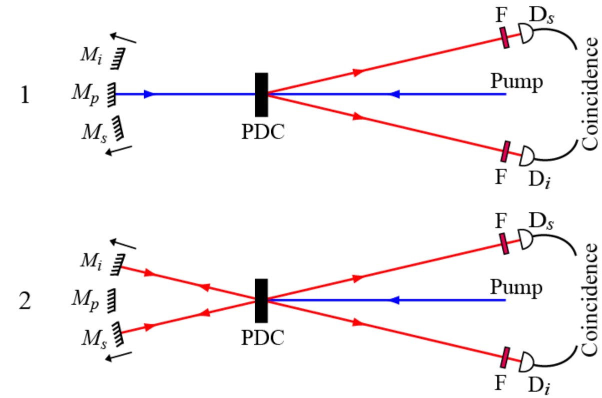


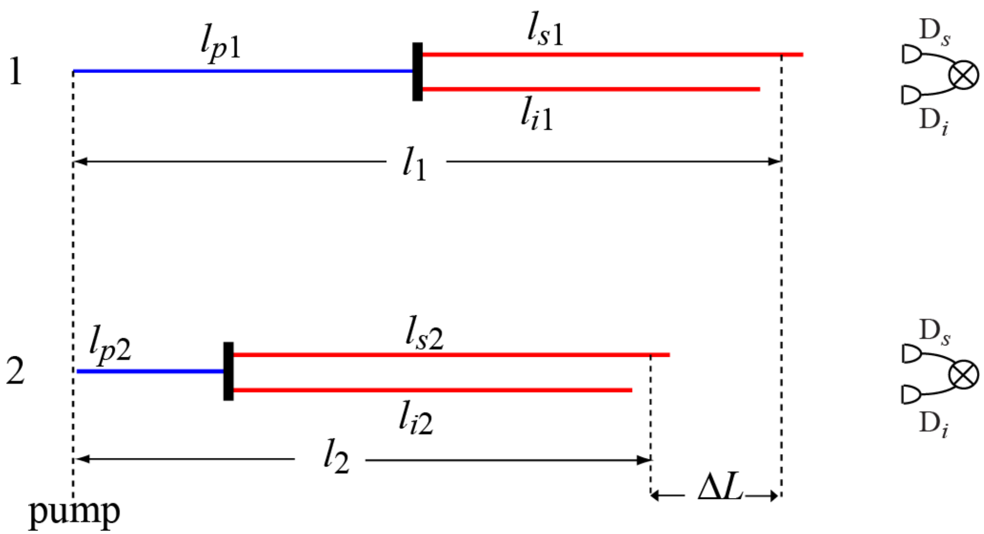


Herzog et al., PRL **72**, 629 (1994)

Two-Photon Interference





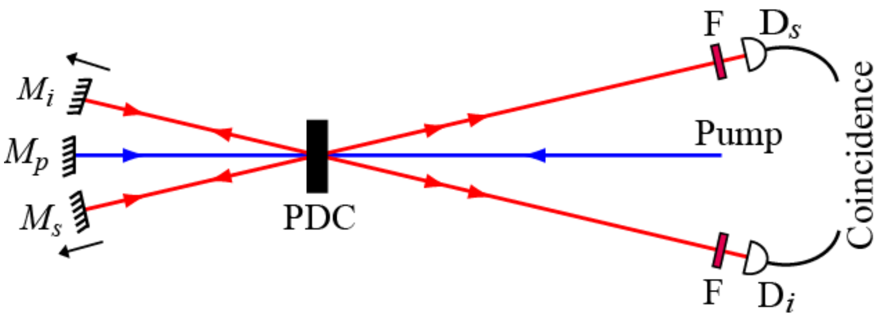


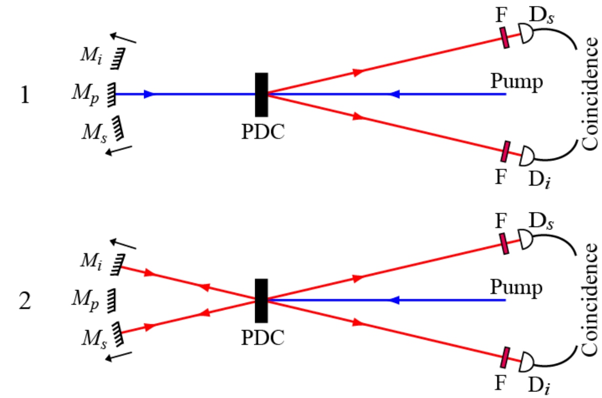
$$\Delta L \equiv l_1 - l_2$$

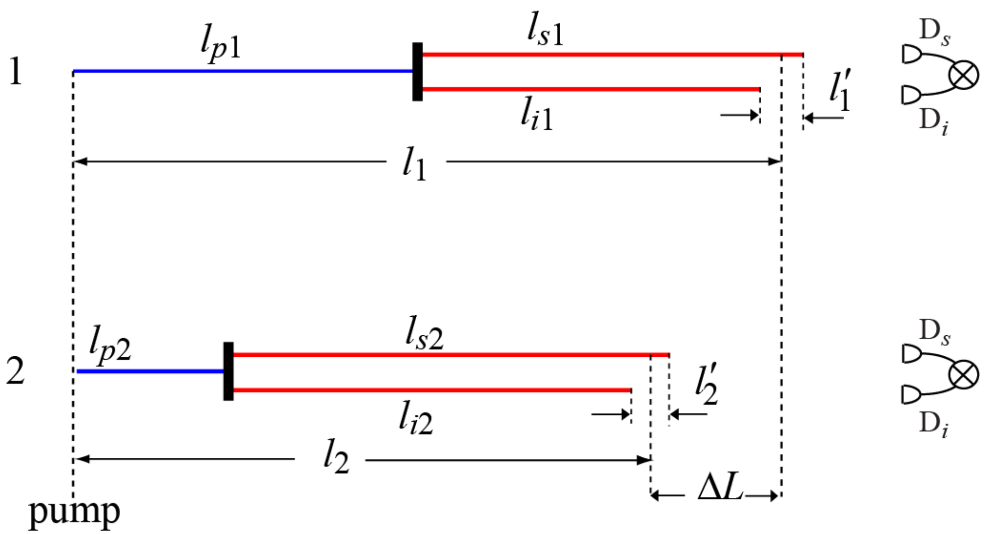
two-photon path-length

Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Two-Photon Interference







$$\Delta L \equiv l_1 - l_2$$

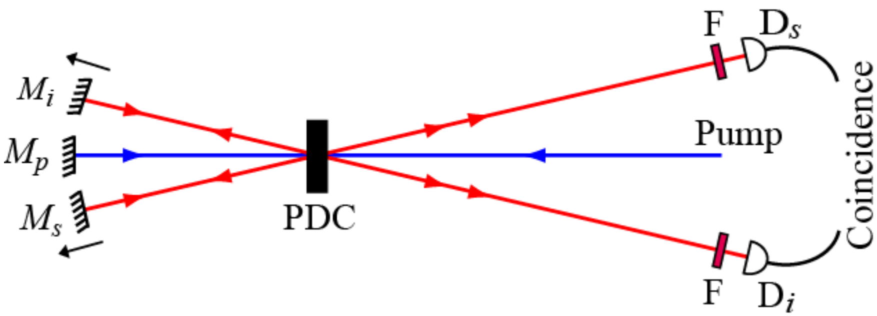
two-photon path-length

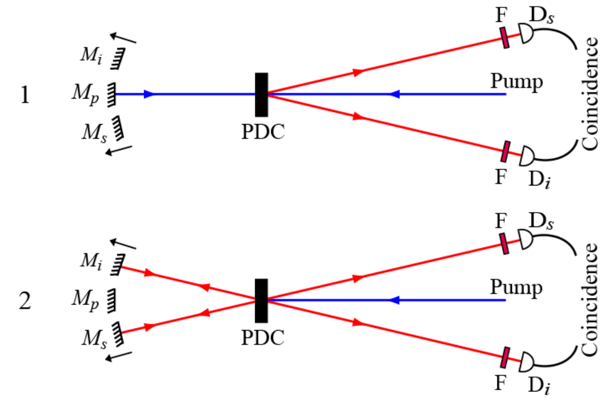
$$\Delta L' \equiv l'_1 - l'_2$$

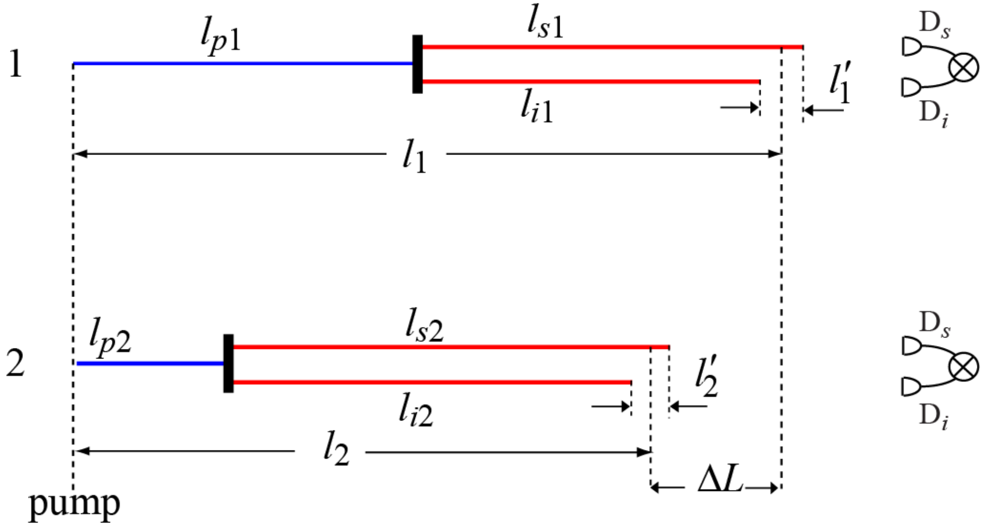
two-photon path-asymmetry length

Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Two-Photon Interference







$$\Delta L \equiv l_1 - l_2$$

two-photon path-length

$$\Delta L' \equiv l'_1 - l'_2$$

two-photon path-asymmetry length

Necessary conditions for two-photon interference:

$$\Delta L < l^p_{\text{coh}} \sim 10 \text{ cm}$$

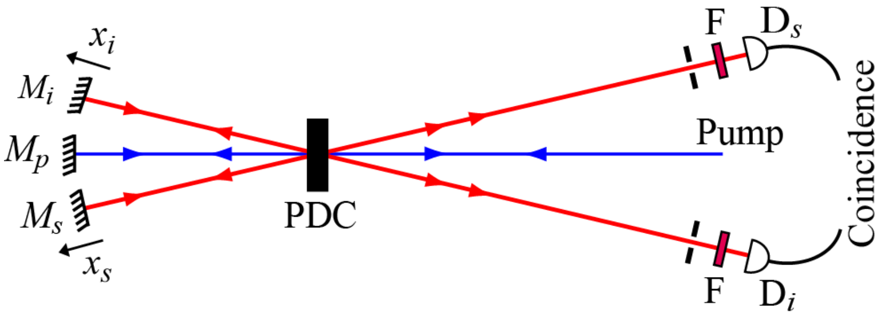
$$\Delta L' < l_{\text{coh}} = \frac{c}{\Delta\omega} \sim 100 \ \mu\text{m}$$

$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L)]$$

R. J. Glauber, Phys. Rev. **130**, 2529 (1963)

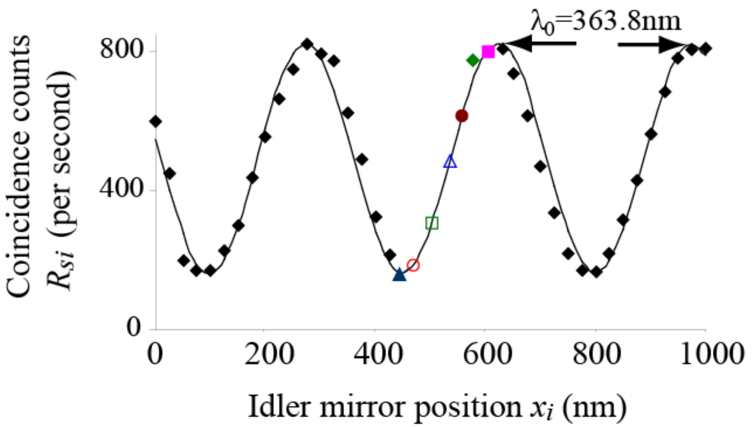
Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

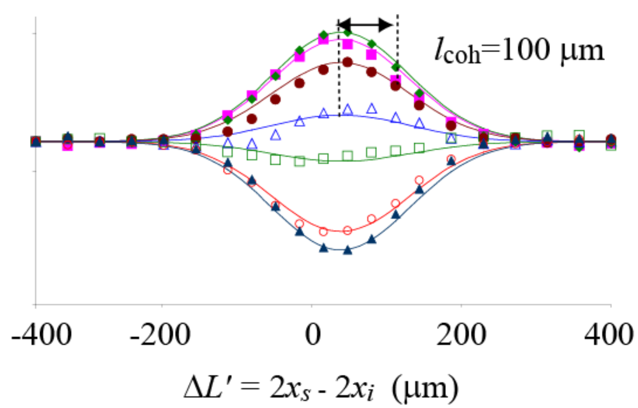
Experimental Verification



$$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$$

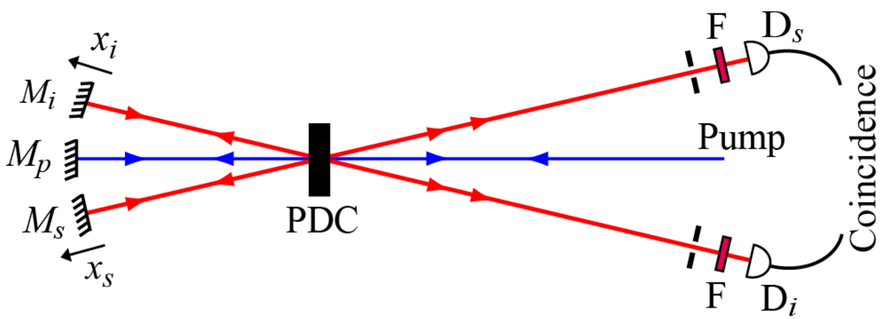
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos\left[k_0(x_s + x_i)\right]\}$$





Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Experimental Verification

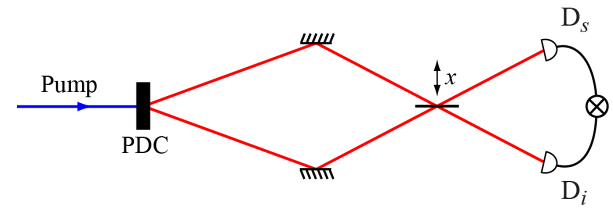


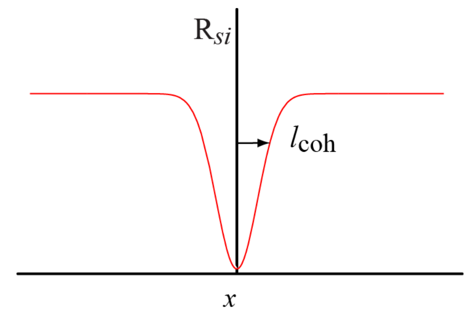
$$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$$

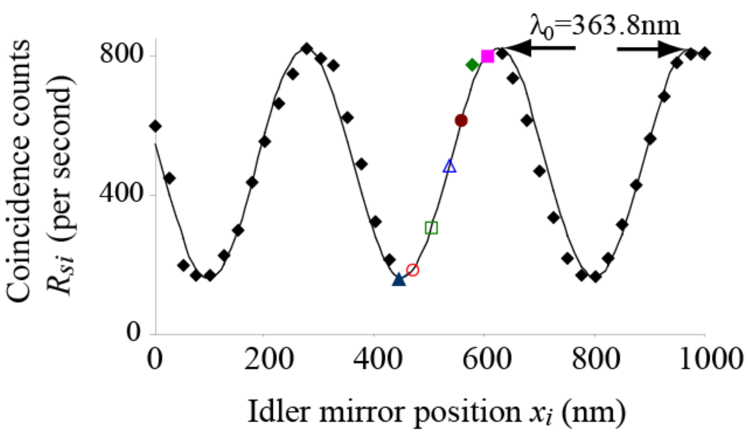
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos[k_0(x_s + x_i)]\}$$

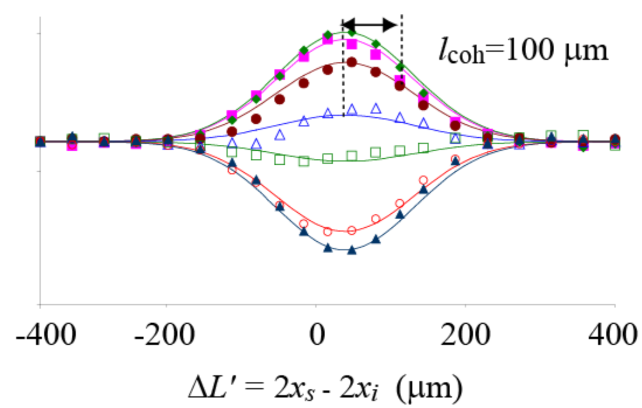
- **Hong-Ou-Mandel effect**

C. K. Hong et al., PRL 59, 2044 (1987)



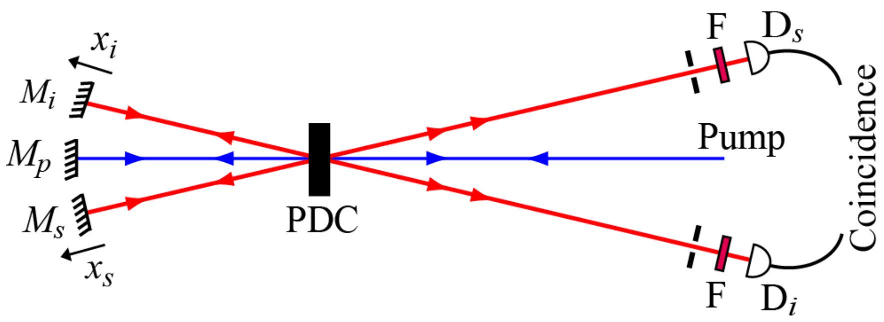






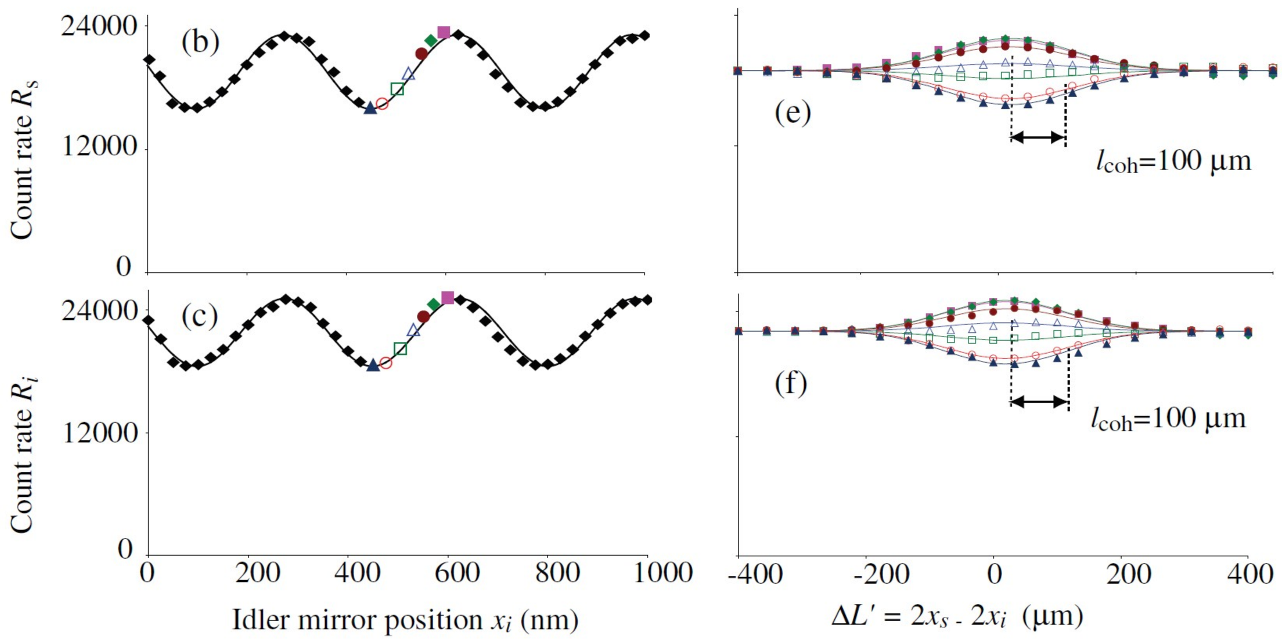
Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Some One-Photon Interference Effects



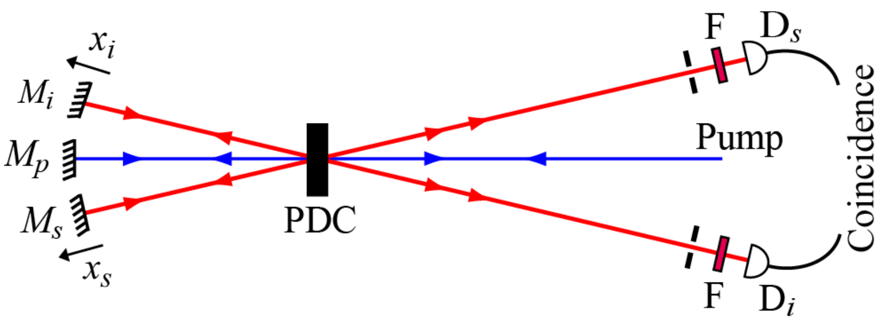
$$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$$

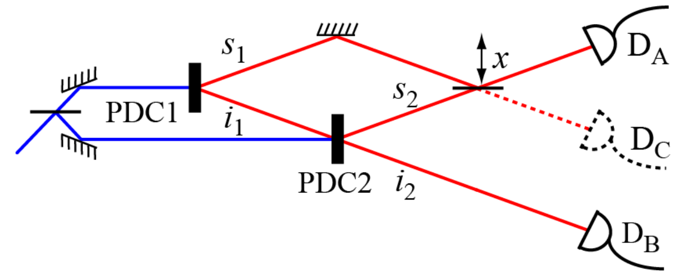
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos\left[k_0(x_s + x_i)\right]\}$$



Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Some One-Photon Interference Effects



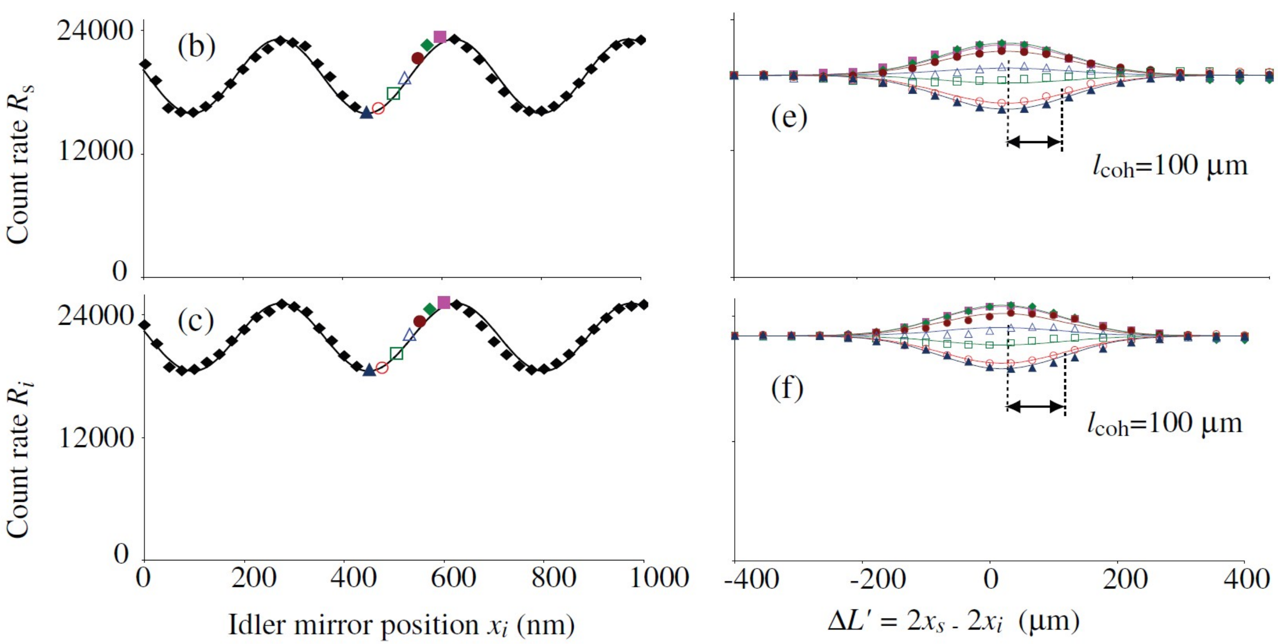


- **Induced Coherence**

X. Y. Zou et al., PRL 67, 318 (1991)

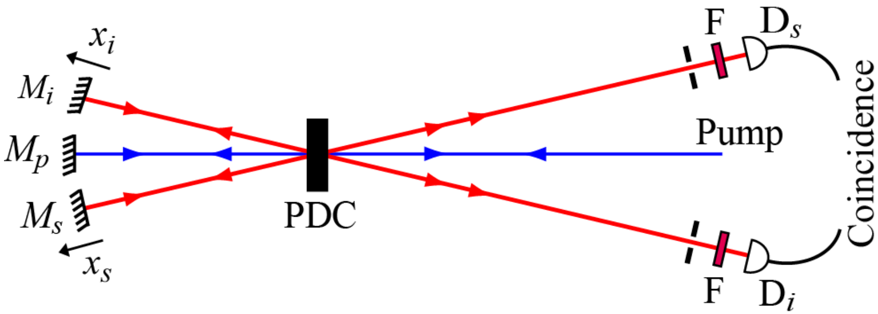
$$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$$

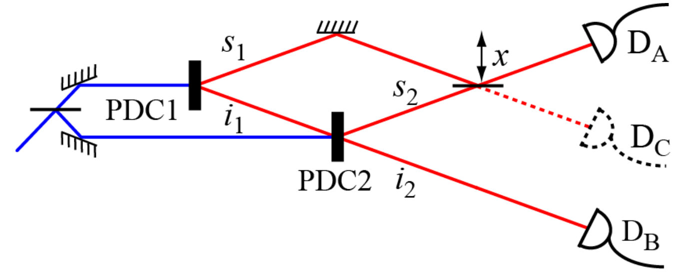
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos\left[k_0(x_s + x_i)\right]\}$$



Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Some One-Photon Interference Effects



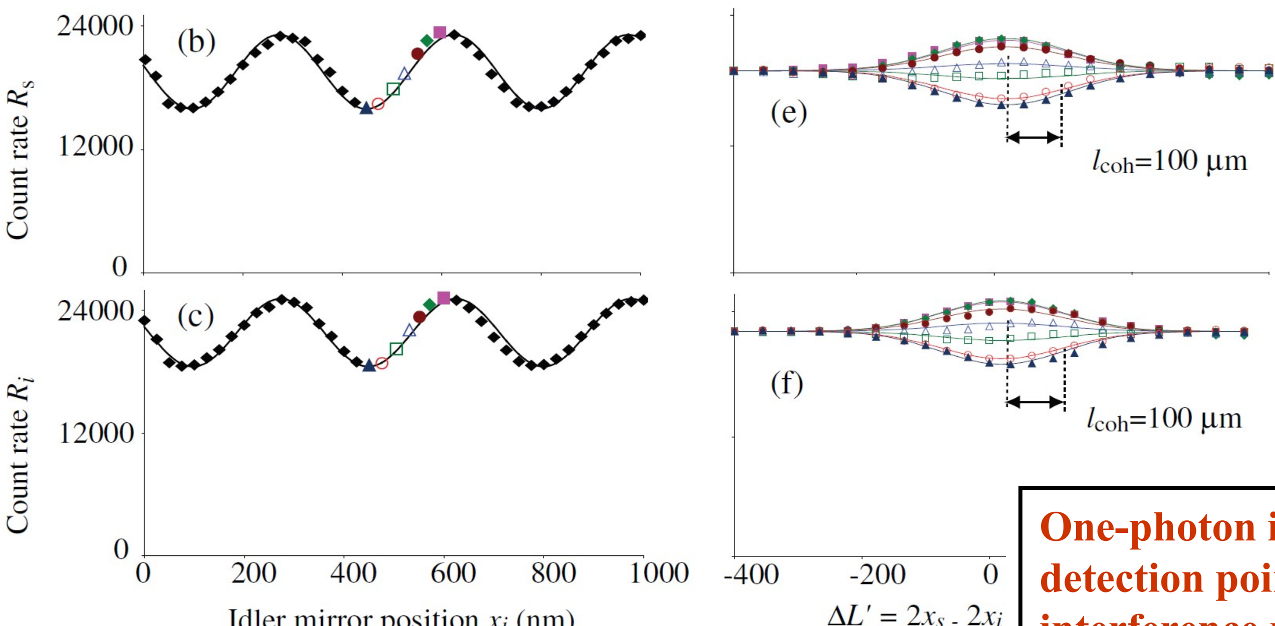


- **Induced Coherence**

X. Y. Zou et al., PRL 67, 318 (1991)

$\Delta L = x_s + x_i$ $\quad \Delta L' = 2x_s - 2x_i$ $\quad \gamma(\Delta L) \sim 1$

$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos[k_0(x_s + x_i)]\}$$



$$R_X = \sum_i R_{XY_i}$$

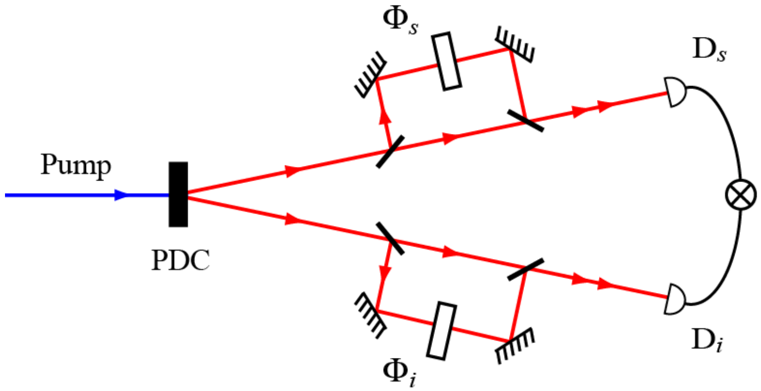
$$R_s = R_i = R_{si}$$

One-photon interference profile at a given detection point is the sum of the two-photon interference profiles

Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Exploring time-energy entanglement using geometric phase

Franson Interferometer J. D. Franson, PRL **62**, 2205 (1989)



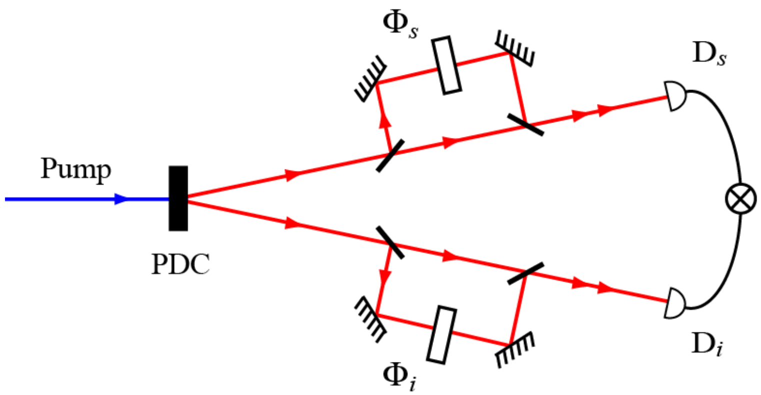
$$R_{si} = C[1 + \cos(\Phi_s + \Phi_i)]$$

Violation of CHSH Bell Inequality using dynamic phase

Brendel et al., PRL **66**, 1142 (1991)
Kwiat et al., PRA **47**, R2472 (1993)
Strekalov et al., PRA **54**, R1 (1996)
Barreiro et al., PRL **95**, 260501 (2005)

Exploring time-energy entanglement using geometric phase

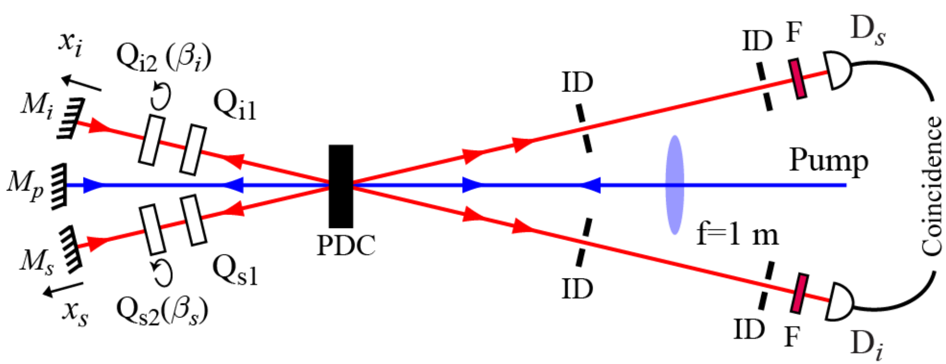
Franson Interferometer J. D. Franson, PRL **62**, 2205 (1989)

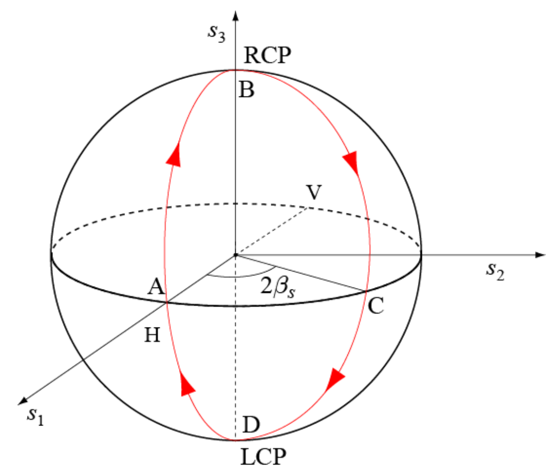


$$R_{si} = C[1 + \cos(\Phi_s + \Phi_i)]$$

Violation of CHSH Bell Inequality using dynamic phase

Brendel et al., PRL **66**, 1142 (1991)
Kwiat et al., PRA **47**, R2472 (1993)
Strekalov et al., PRA **54**, R1 (1996)
Barreiro et al., PRL **95**, 260501 (2005)





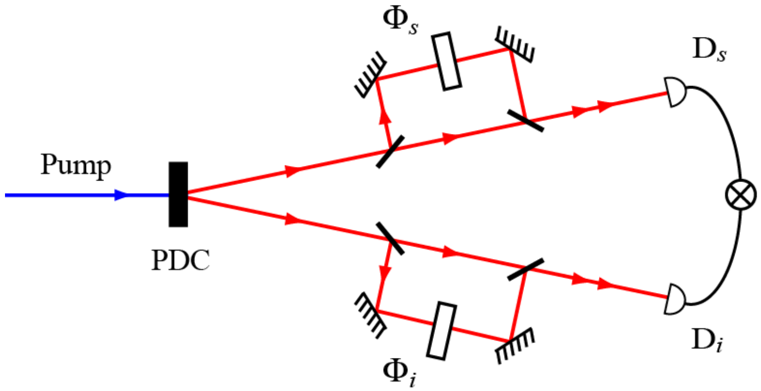
$$R_{si} = C\{1 - \cos[k_0(x_s + x_i) + 2\beta_s + 2\beta_i]\}$$

Violation of CHSH Bell Inequality using geometric (Pancharatnam, Berry) phase

Jha, Malik, and Boyd, PRL **101**, 180405 (2008)

Exploring time-energy entanglement using geometric phase

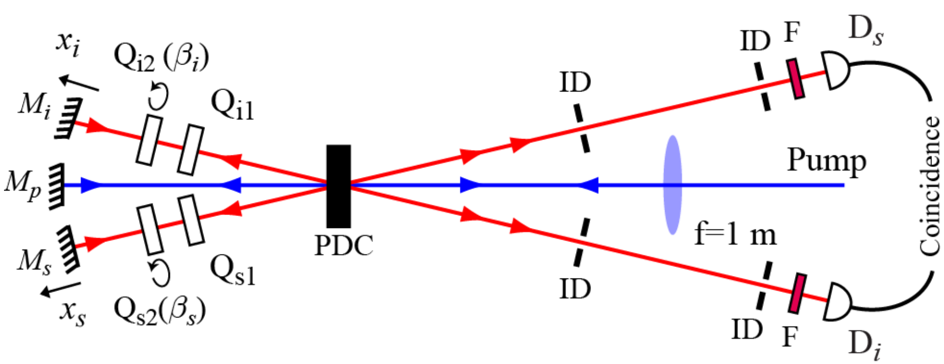
Franson Interferometer J. D. Franson, PRL **62**, 2205 (1989)

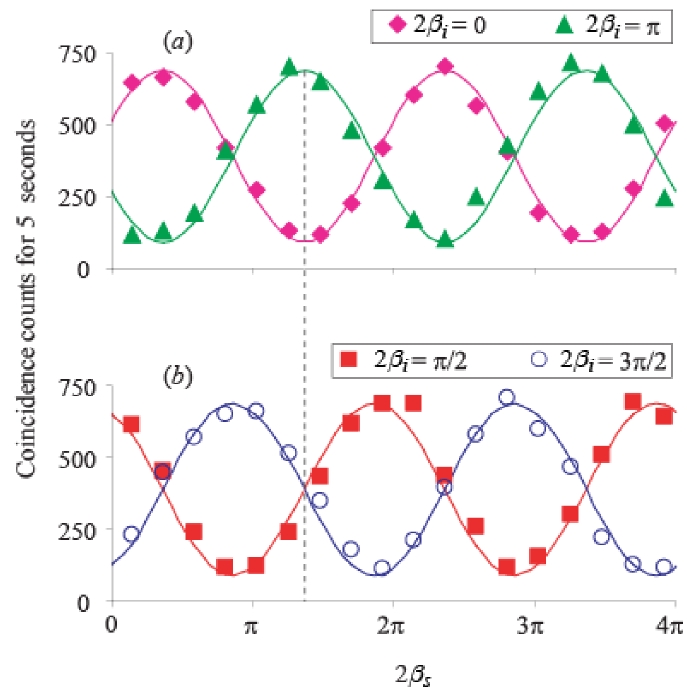


$$R_{si} = C[1 + \cos(\Phi_s + \Phi_i)]$$

Violation of CHSH Bell Inequality using dynamic phase

Brendel et al., PRL **66**, 1142 (1991)
Kwiat et al., PRA **47**, R2472 (1993)
Strekalov et al., PRA **54**, R1 (1996)
Barreiro et al., PRL **95**, 260501 (2005)





$$R_{si} = C\{1 - \cos[k_0(x_s + x_i) + 2\beta_s + 2\beta_i]\}$$

Violation of CHSH Bell Inequality using geometric (Pancharatnam, Berry) phase

Visibility: $V = 77\% \ (> 70.7\%)$

Bell Parameter: $|S| = 2.18 \pm 0.04 \ (> 2.0)$

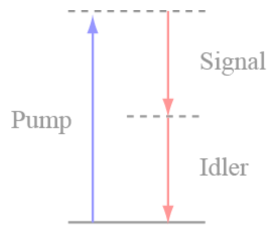
Jha, Malik, and Boyd, PRL **101**, 180405 (2008)

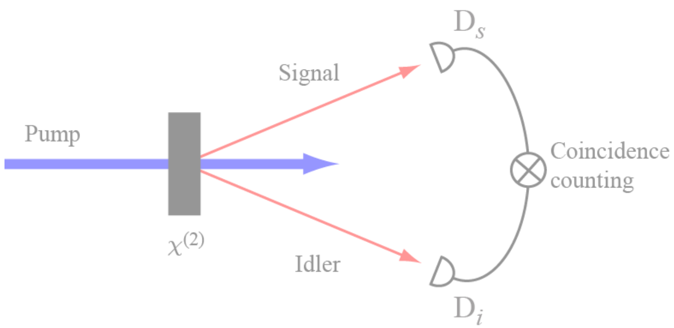
Introduction and Outline

Quantum Entanglement

- EPR paradox and non-locality, Hidden variable theories, Bell inequalities …
- Quantum cryptography, Quantum dense coding, Quantum lithography…

Parametric down-conversion provides a source of entangled photons





$\omega_p = \omega_s + \omega_i$ **Entanglement in time and energy**

"Temporal" two-photon coherence

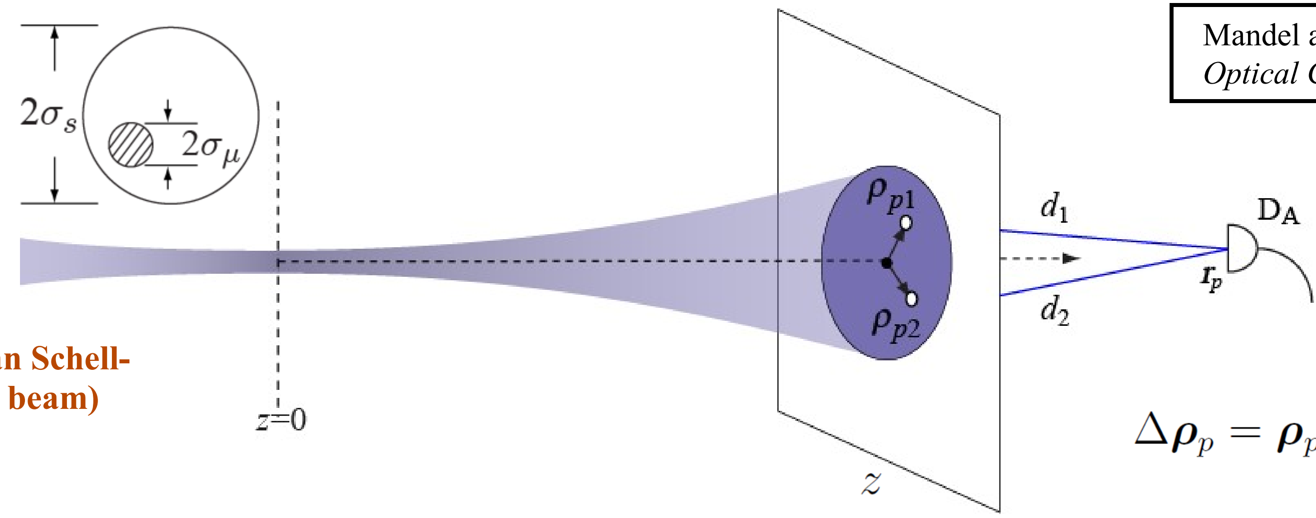
$q_p = q_s + q_i$ **Entanglement in position and momentum**

"Spatial" two-photon coherence

$l_p = l_s + l_i$ **Entanglement in angular position and orbital angular momentum**

"Angular" two-photon coherence

Spatial One-Photon Interference: review



Mandel and Wolf,
Optical Coherence and Quantum Optics

(Gaussian Schell-model beam)

$$\Delta\boldsymbol{\rho}_p = \boldsymbol{\rho}_{p1} - \boldsymbol{\rho}_{p2}$$

Intensity at the detector:

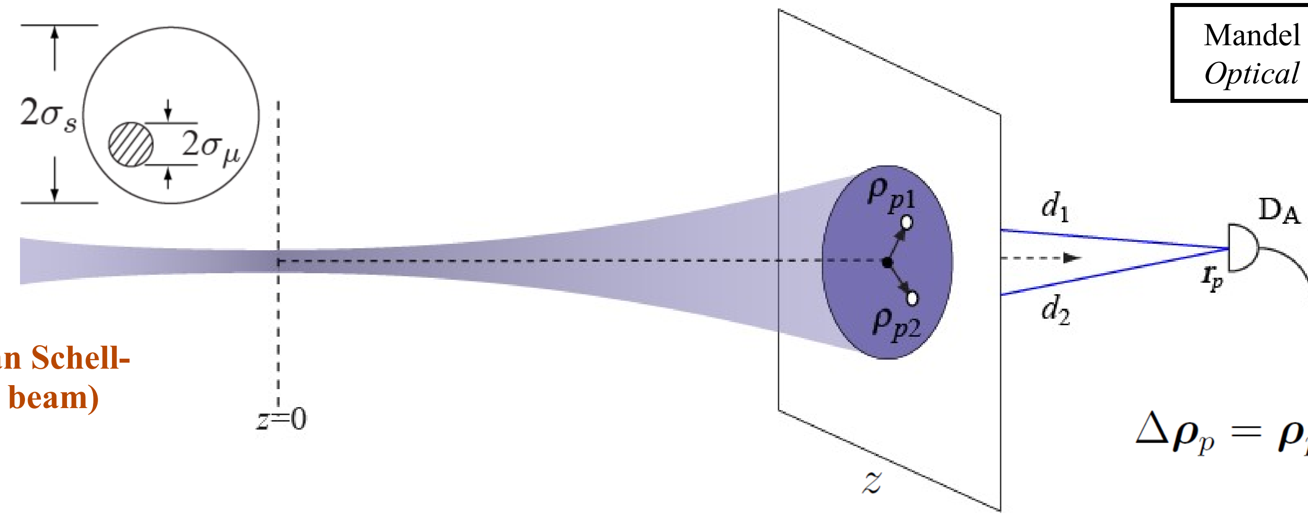
$$I_A(\boldsymbol{r}_p) \propto S(\boldsymbol{\rho}_{p1}, z) + S(\boldsymbol{\rho}_{p2}, z) + W(\boldsymbol{\rho}_{p1}, \boldsymbol{\rho}_{p2}, z)e^{-ik_0(d_1-d_2)} + \text{c.c.}$$

Cross-spectral density:

$$|W(\boldsymbol{\rho}_{p1}, \boldsymbol{\rho}_{p2}, z)| = \sqrt{S(\boldsymbol{\rho}_{p1}, z)S(\boldsymbol{\rho}_{p2}, z)}\mu(\Delta\boldsymbol{\rho}_p, z)$$

Spatial One-Photon Interference: review

Mandel and Wolf,
Optical Coherence and Quantum Optics



(Gaussian Schell-model beam)

$$\Delta\boldsymbol{\rho}_p = \boldsymbol{\rho}_{p1} - \boldsymbol{\rho}_{p2}$$

Intensity at the detector:

$$I_A(\boldsymbol{r}_p) \propto S(\boldsymbol{\rho}_{p1}, z) + S(\boldsymbol{\rho}_{p2}, z) + W(\boldsymbol{\rho}_{p1}, \boldsymbol{\rho}_{p2}, z)e^{-ik_0(d_1-d_2)} + \text{c.c.}$$

Cross-spectral density:

$$|W(\boldsymbol{\rho}_{p1}, \boldsymbol{\rho}_{p2}, z)| = \sqrt{S(\boldsymbol{\rho}_{p1}, z)S(\boldsymbol{\rho}_{p2}, z)}\mu(\Delta\boldsymbol{\rho}_p, z)$$

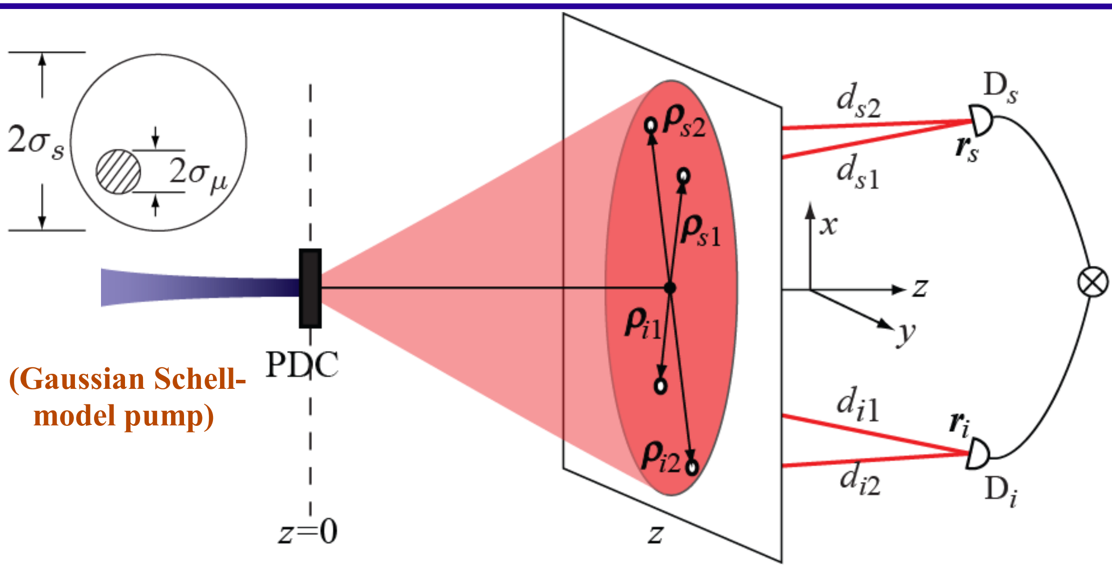
Spectral density:

$$S(\boldsymbol{\rho}_{p1}, z) = C\exp\left\{-(1/2)\left[\boldsymbol{\rho}_{p1}/\sigma_s(z)\right]^2\right\} \qquad \sigma_s(z) = z\sqrt{\sigma_\mu^2 + 4\sigma_s^2}/2k_0\sigma_s\sigma_\mu$$

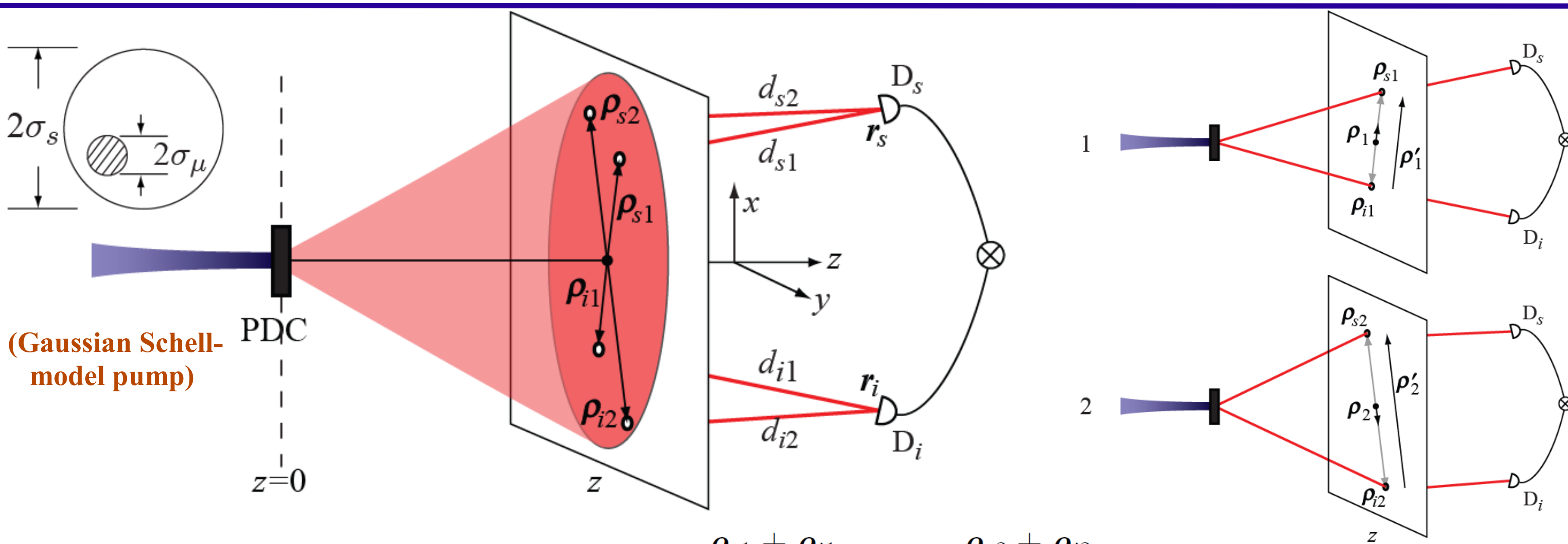
Degree of coherence:

$$\mu(\Delta\boldsymbol{\rho}_p, z) = \exp\left\{-(1/2)\left[\Delta\boldsymbol{\rho}_p/\sigma_\mu(z)\right]^2\right\} \qquad \sigma_\mu(z) = z\sqrt{\sigma_\mu^2 + 4\sigma_s^2}/2k_0\sigma_s^2$$

Spatial Two-photon Interference



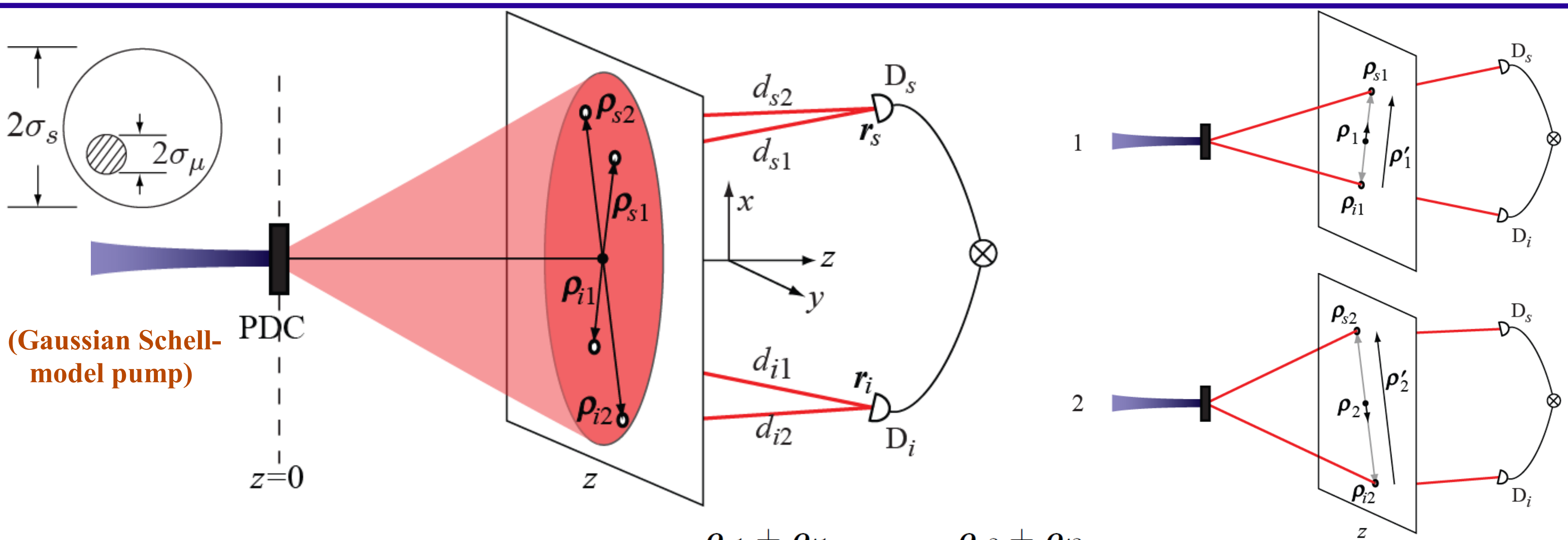
Spatial Two-photon Interference



Two-photon transverse position vector : $\boldsymbol{\rho}_1 \equiv \dfrac{\boldsymbol{\rho}_{s1}+\boldsymbol{\rho}_{i1}}{2}, \quad \boldsymbol{\rho}_2 \equiv \dfrac{\boldsymbol{\rho}_{s2}+\boldsymbol{\rho}_{i2}}{2}; \quad \Delta\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$

Two-photon position-asymmetry vector : $\boldsymbol{\rho}'_1 \equiv \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i1}, \quad \boldsymbol{\rho}'_2 \equiv \boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i2}; \quad \Delta\boldsymbol{\rho}' = \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2$

Spatial Two-photon Interference



Two-photon transverse position vector : $\boldsymbol{\rho}_1 \equiv \dfrac{\boldsymbol{\rho}_{s1} + \boldsymbol{\rho}_{i1}}{2}, \quad \boldsymbol{\rho}_2 \equiv \dfrac{\boldsymbol{\rho}_{s2} + \boldsymbol{\rho}_{i2}}{2}; \quad \Delta\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$

Two-photon position-asymmetry vector : $\boldsymbol{\rho}_1' \equiv \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i1}, \quad \boldsymbol{\rho}_2' \equiv \boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i2}; \quad \Delta\boldsymbol{\rho}' = \boldsymbol{\rho}_1' - \boldsymbol{\rho}_2'$

Coincidence count rate:

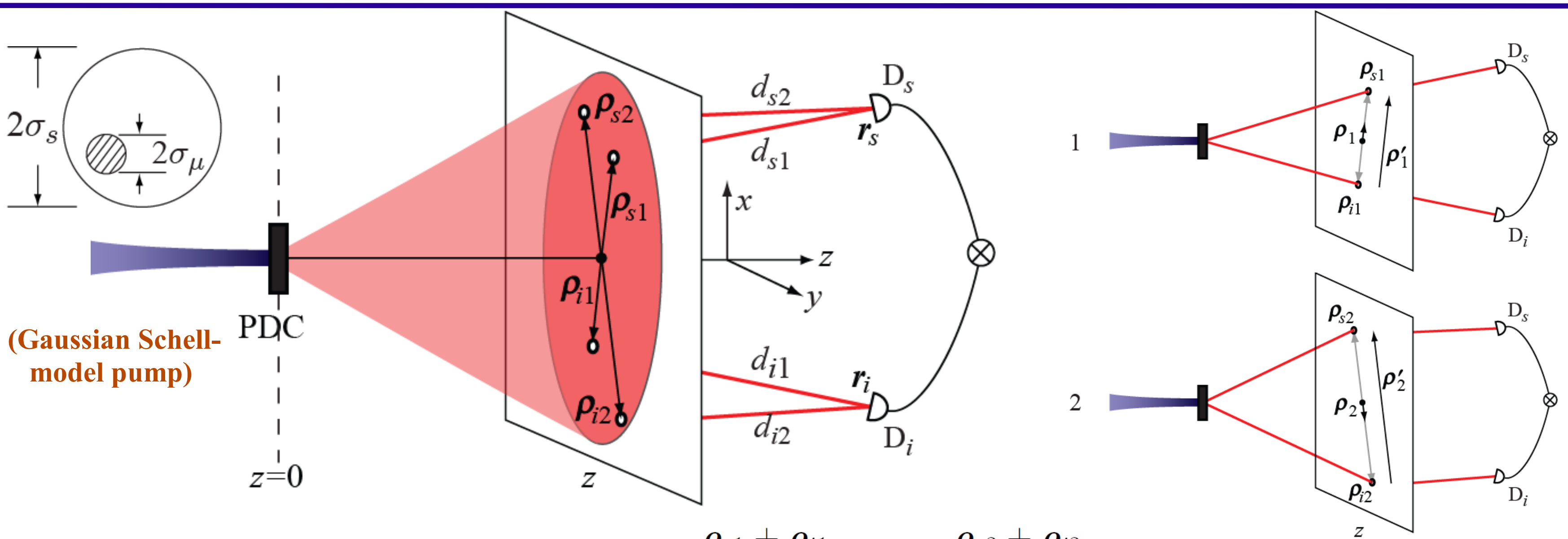
$$R_{si}(\boldsymbol{r}_s, \boldsymbol{r}_i) \propto S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z) + W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)e^{ik_0[(d_{s1}+d_{i1})/2-(d_{s2}+d_{i2})/2]} + \text{c.c.}$$

Two-photon cross-spectral density:

$$|W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)| = \sqrt{S^{(2)}(\boldsymbol{\rho}_1, z)S^{(2)}(\boldsymbol{\rho}_2, z)}\mu^{(2)}(\Delta\boldsymbol{\rho}, z)$$

Jha and Boyd, Accepted in PRA

Spatial Two-photon Interference



Two-photon transverse position vector : $\boldsymbol{\rho}_1 \equiv \frac{\boldsymbol{\rho}_{s1}+\boldsymbol{\rho}_{i1}}{2}, \quad \boldsymbol{\rho}_2 \equiv \frac{\boldsymbol{\rho}_{s2}+\boldsymbol{\rho}_{i2}}{2}; \quad \Delta\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$

Two-photon position-asymmetry vector : $\boldsymbol{\rho}'_1 \equiv \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i1}, \quad \boldsymbol{\rho}'_2 \equiv \boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i2}; \quad \Delta\boldsymbol{\rho}' = \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2$

Coincidence count rate:

$$R_{si}(\boldsymbol{r}_s, \boldsymbol{r}_i) \propto S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z) + W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) e^{ik_0[(d_{s1}+d_{i1})/2-(d_{s2}+d_{i2})/2]} + \text{c.c.}$$

Two-photon cross-spectral density:

$$|W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)| = \sqrt{S^{(2)}(\boldsymbol{\rho}_1, z) S^{(2)}(\boldsymbol{\rho}_2, z)}\, \mu^{(2)}(\Delta\boldsymbol{\rho}, z)$$

Two-photon spectral density:

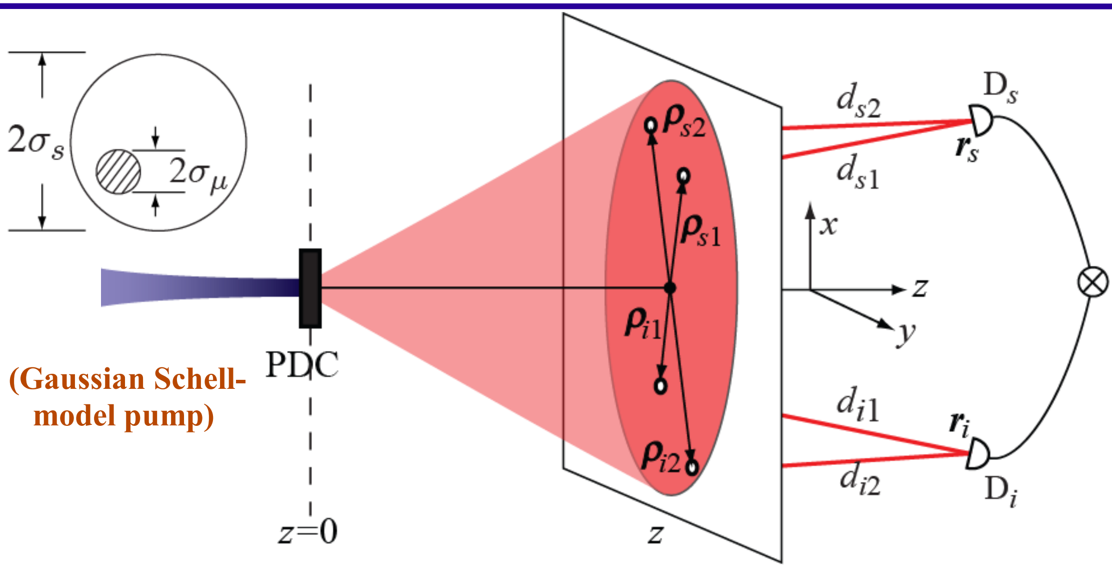
$$S^{(2)}(\boldsymbol{\rho}_1, z) = C \exp\left\{-(1/2)\left[\boldsymbol{\rho}_1/\sigma_s^{(2)}(z)\right]^2\right\} \qquad \sigma_s^{(2)}(z) = z\sqrt{\sigma_\mu^2 + 4\sigma_s^2}/2k_0\sigma_s\sigma_\mu$$

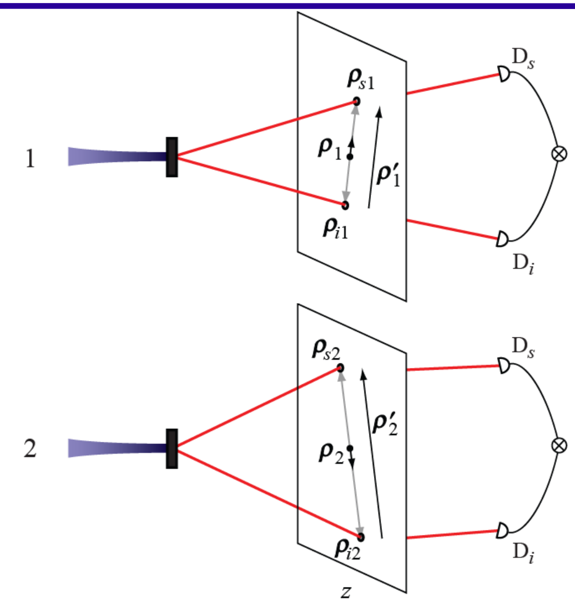
Degree of spatial-two-photon coherence:

$$\mu^{(2)}(\Delta\boldsymbol{\rho}, z) = \exp\left\{-(1/2)\left[\Delta\boldsymbol{\rho}/\sigma_\mu^{(2)}(z)\right]^2\right\} \qquad \sigma_\mu^{(2)}(z) = z\sqrt{\sigma_\mu^2 + 4\sigma_s^2}/2k_0\sigma_s^2$$

Jha and Boyd, Accepted in PRA

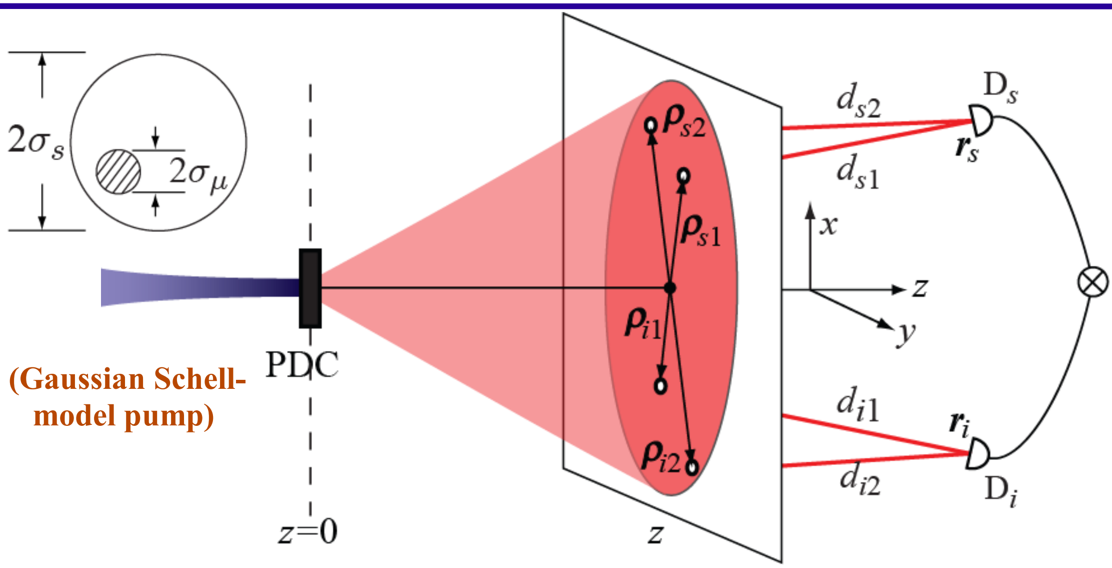
Two-Photon Coherence and Entanglement



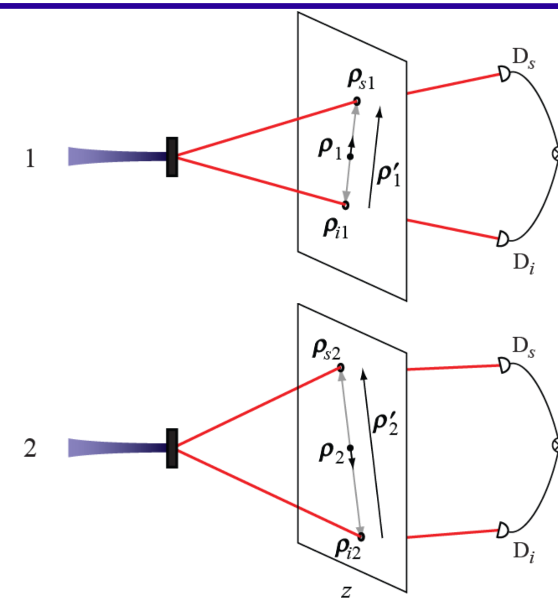


Jha and Boyd, Accepted in PRA

Two-Photon Coherence and Entanglement



(Gaussian Schell-model pump)



Entangled two-qubit state

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}$$

$$a = \eta S^{(2)}(\boldsymbol{\rho}_1, z)$$

$$b = \eta S^{(2)}(\boldsymbol{\rho}_2, z)$$

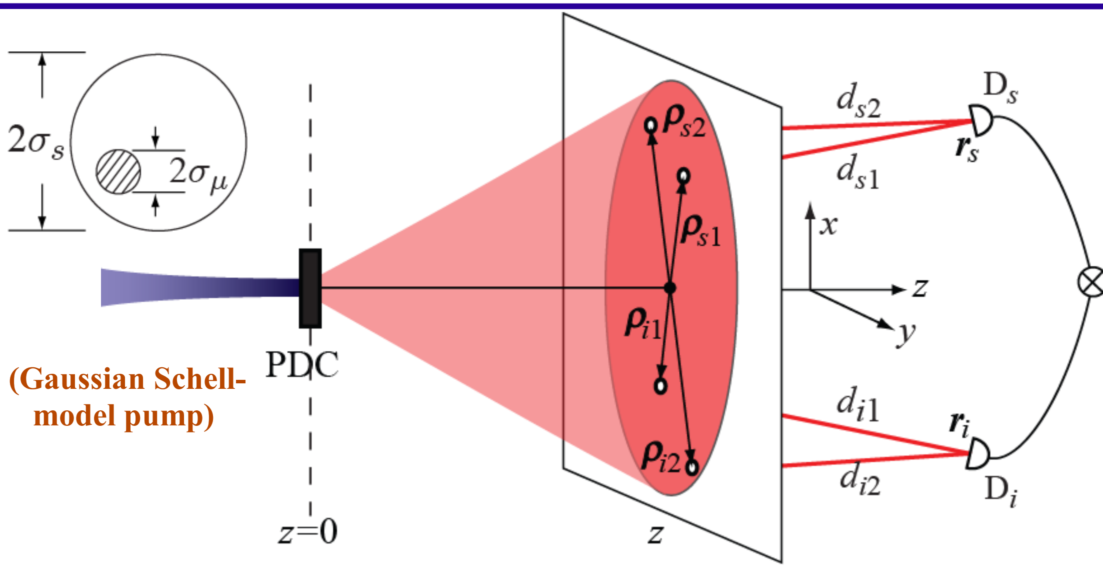
$$c = d^* = \eta W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$$

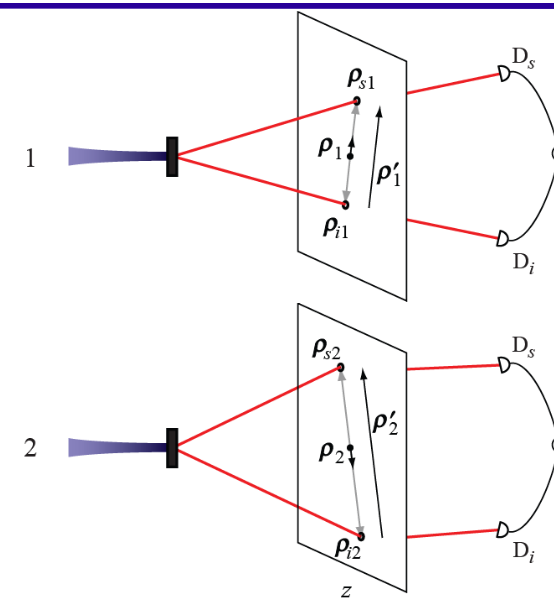
$$\eta = 1/[S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z)]$$

O'Sullivan et al., PRL **94**, 220501 (2005)
Neves et al., PRA **76**, 032314 (2007)
Walborn et al., PRA **76**, 062305 (2007)
Taguchi et al., PRA **78**, 012307 (2008)

Jha and Boyd, Accepted in PRA

Two-Photon Coherence and Entanglement





Entangled two-qubit state

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}$$

$$a = \eta S^{(2)}(\boldsymbol{\rho}_1, z)$$

$$b = \eta S^{(2)}(\boldsymbol{\rho}_2, z)$$

$$c = d^* = \eta W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$$

$$\eta = 1/[S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z)]$$

O'Sullivan et al., PRL **94**, 220501 (2005)
Neves et al., PRA **76**, 032314 (2007)
Walborn et al., PRA **76**, 062305 (2007)
Taguchi et al., PRA **78**, 012307 (2008)

Entanglement of the state (Concurrence) :

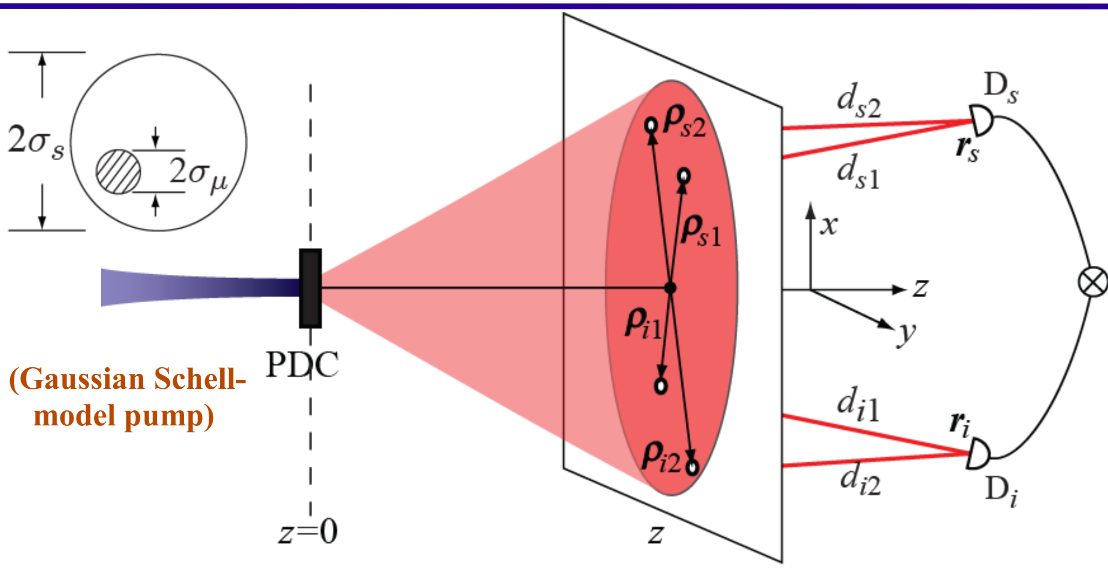
Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

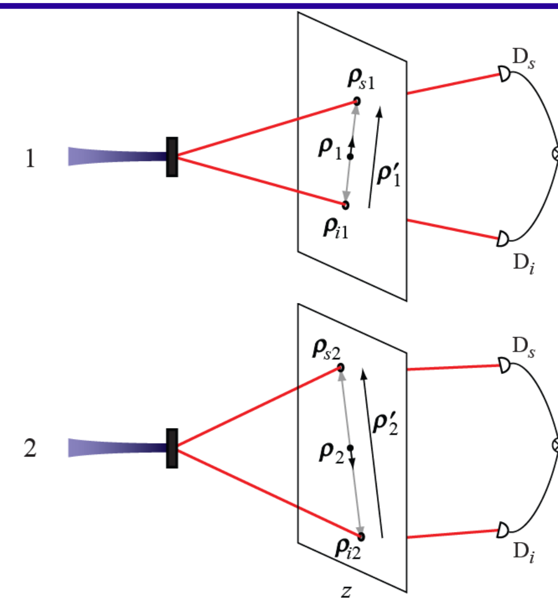
$$\zeta = \rho_{\text{qubit}}(\sigma_y \otimes \sigma_y)\rho^*_{\text{qubit}}(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\text{qubit}}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

Jha and Boyd, Accepted in PRA

Two-Photon Coherence and Entanglement





Entangled two-qubit state

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}$$

$$a = \eta S^{(2)}(\boldsymbol{\rho}_1, z)$$

$$b = \eta S^{(2)}(\boldsymbol{\rho}_2, z)$$

$$c = d^* = \eta W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$$

$$\eta = 1/[S^{(2)}(\boldsymbol{\rho}_1, z) + S^{(2)}(\boldsymbol{\rho}_2, z)]$$

O'Sullivan et al., PRL **94**, 220501 (2005)
Neves et al., PRA **76**, 032314 (2007)
Walborn et al., PRA **76**, 062305 (2007)
Taguchi et al., PRA **78**, 012307 (2008)

Entanglement of the state (Concurrence) :

$$C(\rho_{\text{qubit}}) = 2|c| = 2\eta|W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)|$$

$$C(\rho_{\text{qubit}}) = \mu^{(2)}(\Delta\boldsymbol{\rho}, z)$$ **(with** $a = b$ **)**

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho_{\text{qubit}}(\sigma_y \otimes \sigma_y)\rho^*_{\text{qubit}}(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\text{qubit}}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

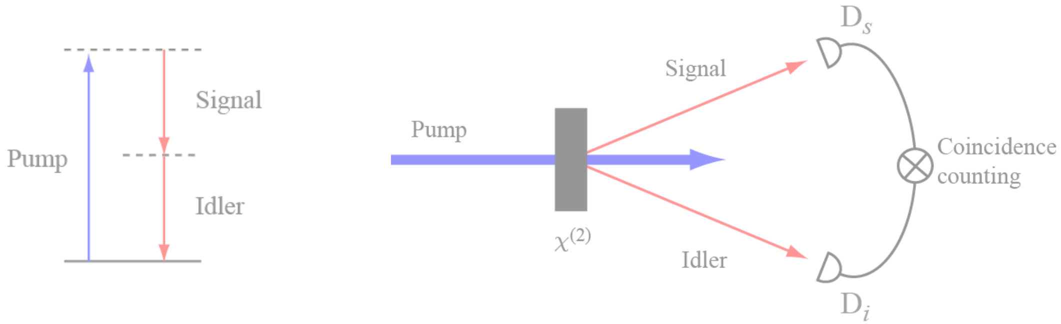
Jha and Boyd, Accepted in PRA

Introduction and Outline

Quantum Entanglement

- EPR paradox and non-locality, Hidden variable theories, Bell inequalities …
- Quantum cryptography, Quantum dense coding, Quantum lithography…

Parametric down-conversion provides a source of entangled photons



$\omega_p = \omega_s + \omega_i$ **Entanglement in time and energy**

"Temporal" two-photon coherence

$q_p = q_s + q_i$ **Entanglement in position and momentum**

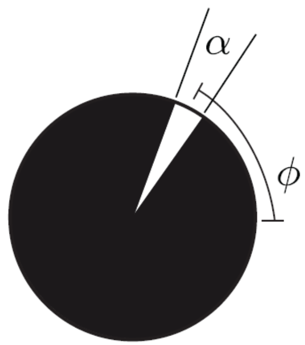
"Spatial" two-photon coherence

$l_p = l_s + l_i$ **Entanglement in angular position and orbital angular momentum**

"Angular" two-photon coherence

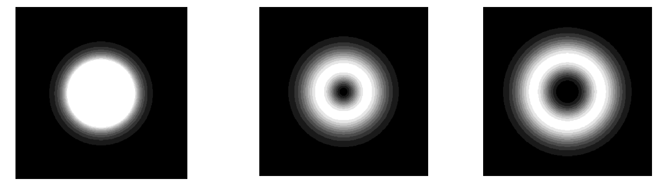
Angular Fourier Relationship

Angular position



Laguerre-Gauss basis

LG_p^l with p=0



***l*=0** ***l*=1** ***l*=2**

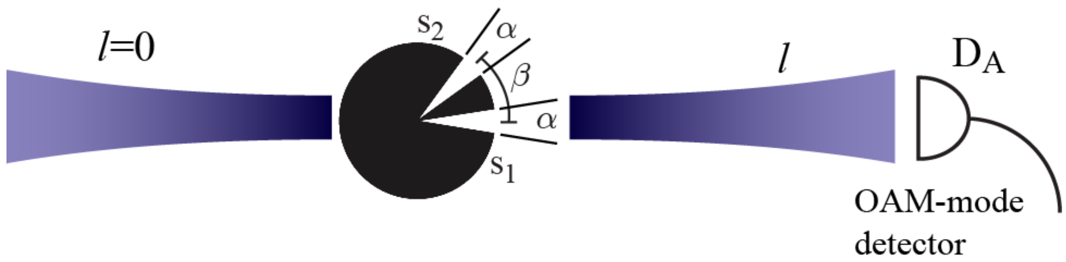
Allen et al., PRA **45**, 8185 (1992)

$$A_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi(\phi) \exp(-il\phi)$$

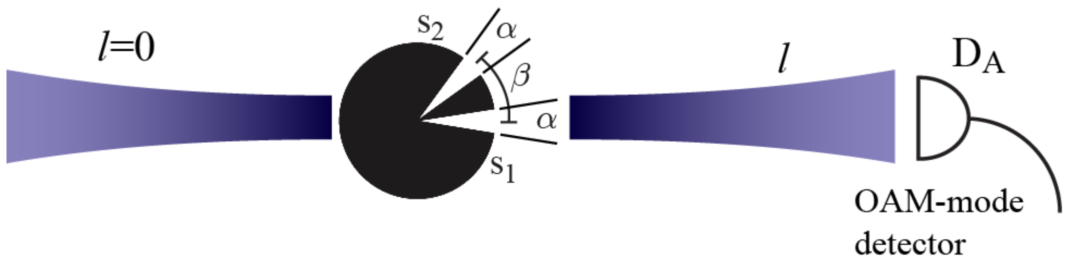
$$\Psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} A_l \exp(il\phi)$$

Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

Angular One-Photon Interference



Angular One-Photon Interference

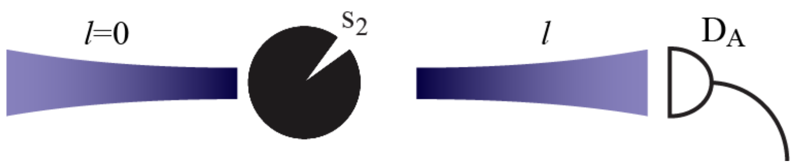


1

$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$

$$= \frac{\alpha}{\sqrt{2\pi}} \mathrm{sinc}\left(\frac{l\alpha}{2}\right)$$

2

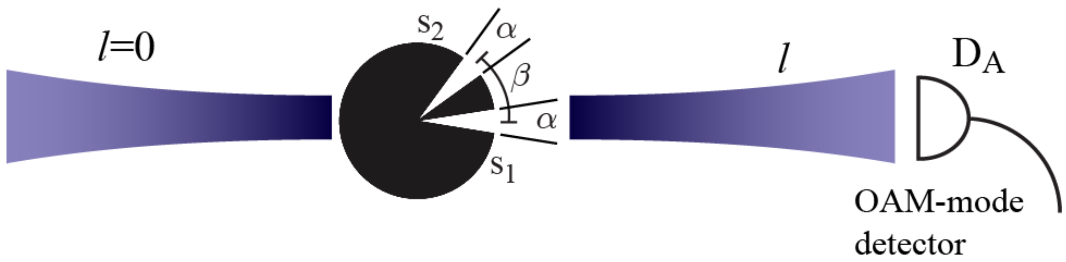


$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \mathrm{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

Yao et al., Opt. Express **14**, 13089 (2006)

Jha, Jack, Yao, Leach, Boyd, Buller, Barnett, Franke-Arnold, Padgett, PRA **78**, 043810 (2008)

Angular One-Photon Interference

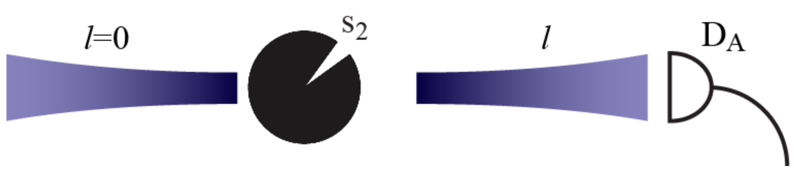


1

$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$

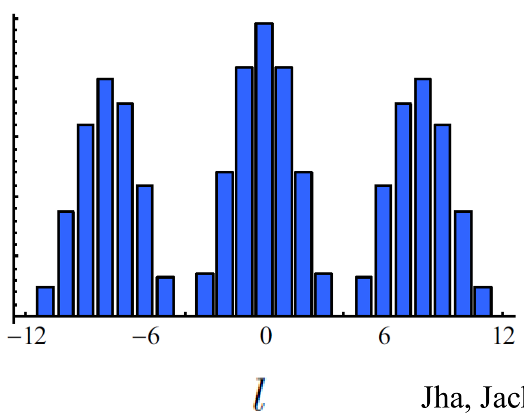
$$= \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right)$$

2



$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

$\alpha = \pi/10$
$\beta = \pi/4$



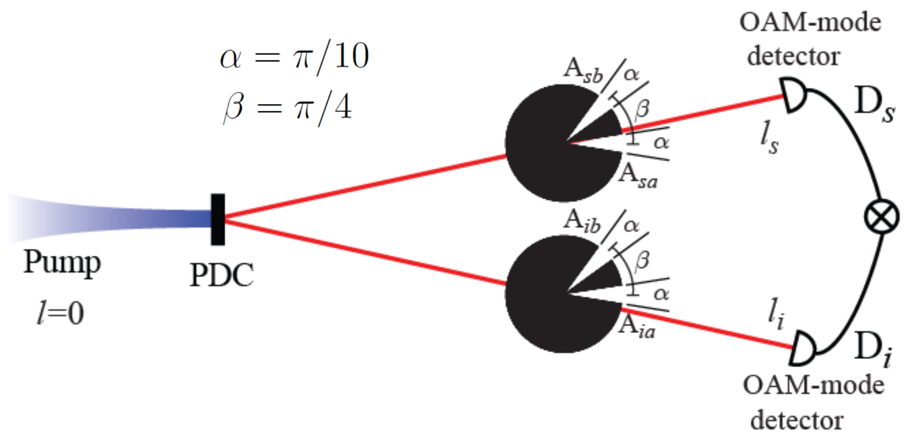
OAM-mode distribution:

$$I_A = C\frac{\alpha^2}{\pi}\text{sinc}^2\left(\frac{l\alpha}{2}\right)[1 + \cos(l\beta)]$$

Yao et al., Opt. Express **14**, 13089 (2006)

Jha, Jack, Yao, Leach, Boyd, Buller, Barnett, Franke-Arnold, Padgett, PRA **78**, 043810 (2008)

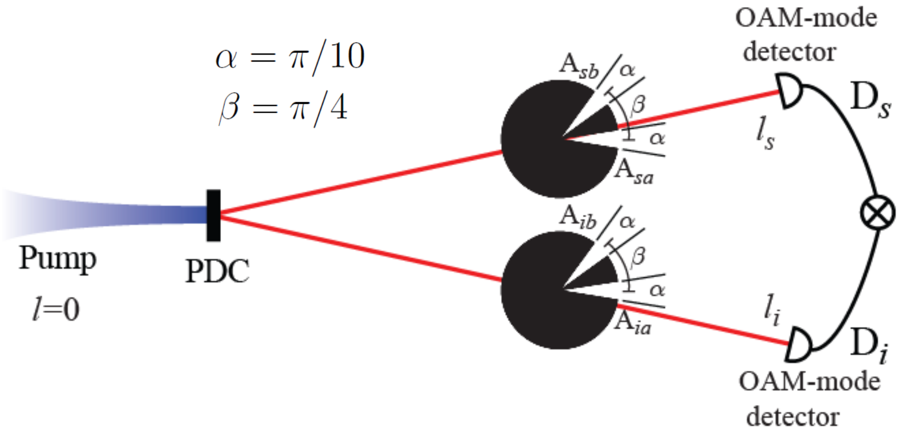
Angular Two-Photon Interference

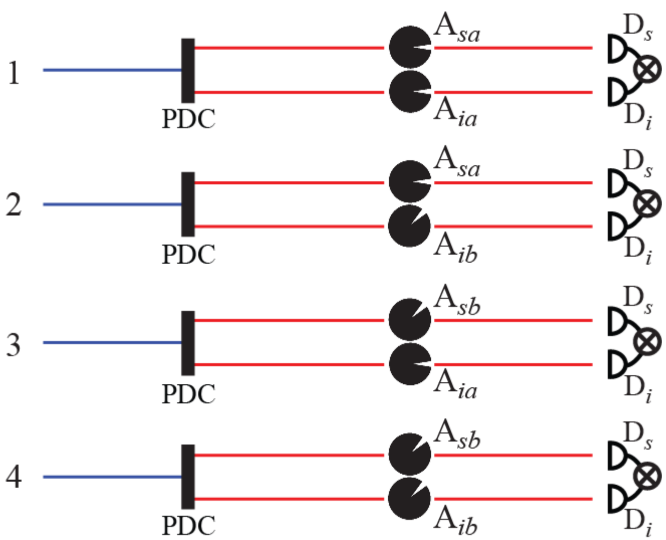


State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\mathrm{tp}}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

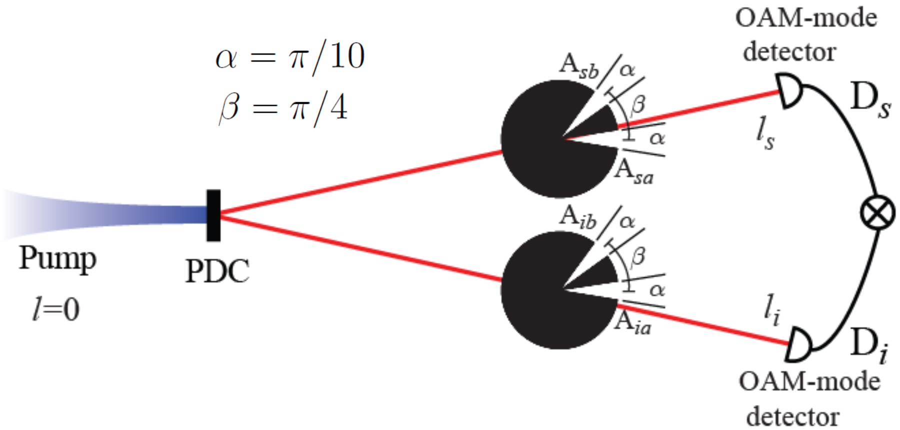
State of the two photons after the aperture:

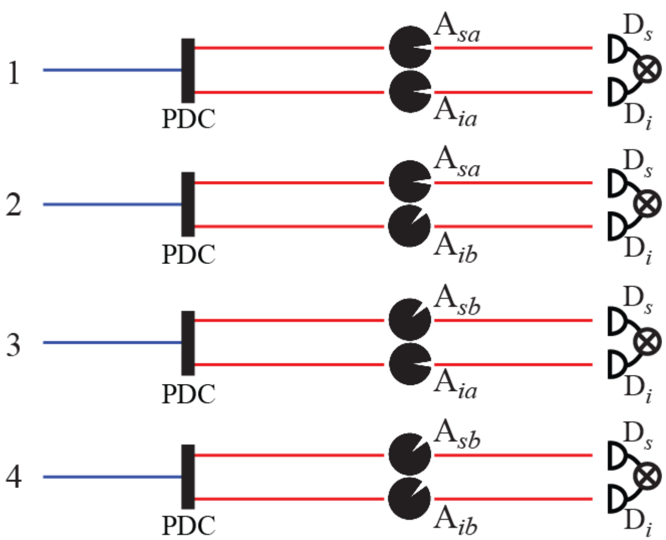
$$\rho_{\mathrm{qubit}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^{*} = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\text{tp}}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

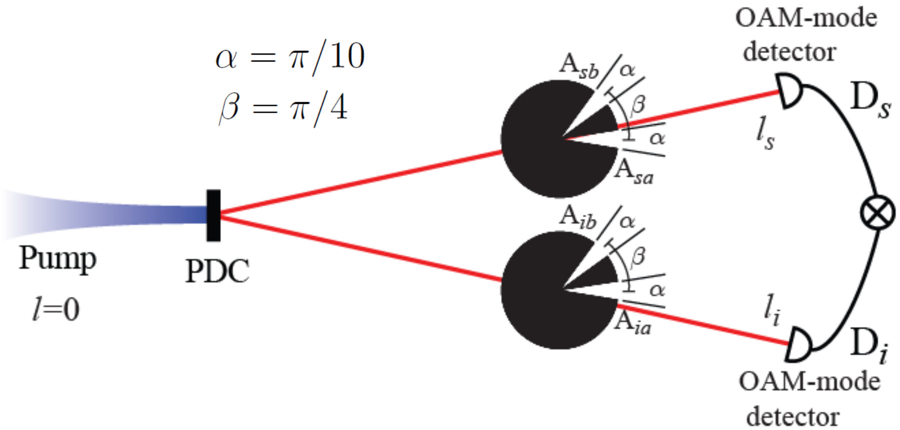
$$\rho_{\text{qubit}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^{*} \\ &= \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta} \\ \rho_{11} + \rho_{44} &= 1 \end{aligned}$$

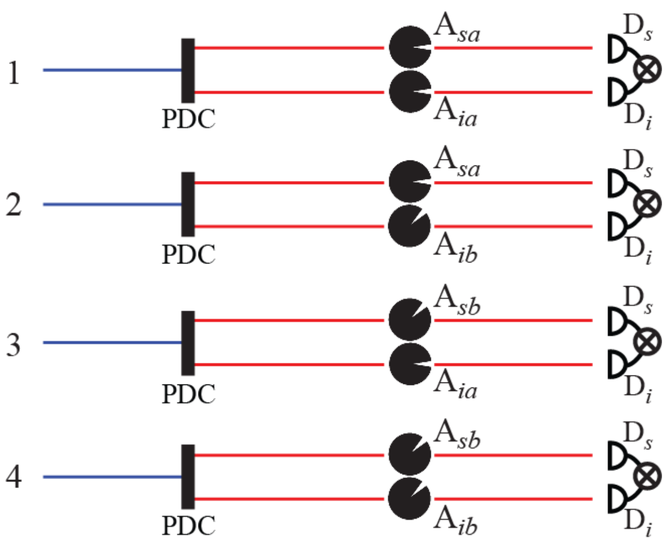
Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \operatorname{sinc}\left[(l_s - l)\frac{\alpha}{2}\right]\operatorname{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2 \times \left\{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu\cos\left[(l_s + l_i)\beta + \theta\right]\right\}$$

Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta} \end{aligned}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$

$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

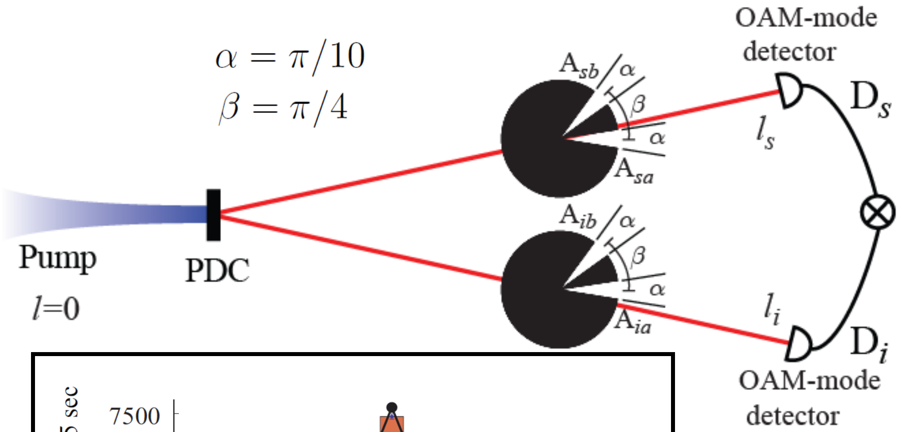
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

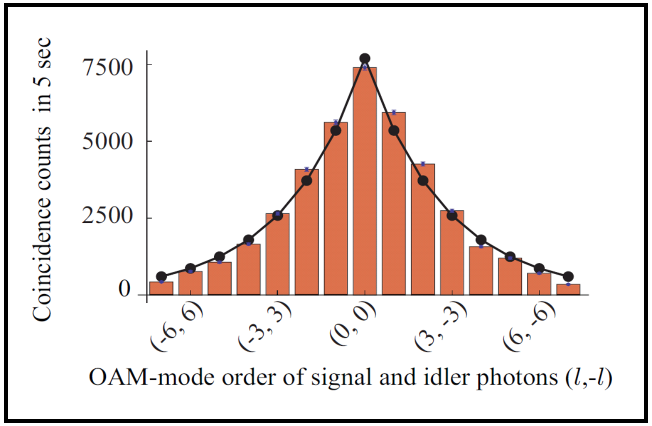
Concurrence of the two-qubit state:

$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu \ = V$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\text{tp}}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\text{qubit}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^* = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \operatorname{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \operatorname{sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2 \times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

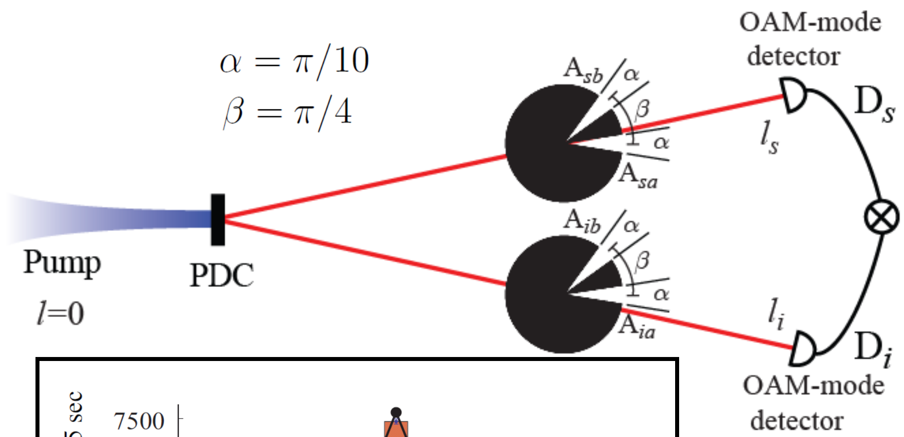
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

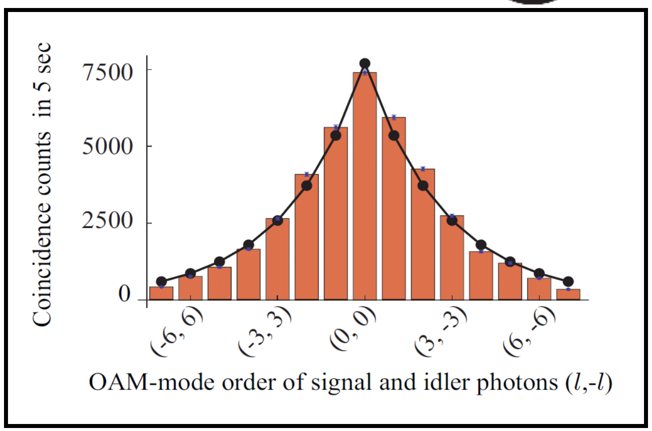
Concurrence of the two-qubit state:

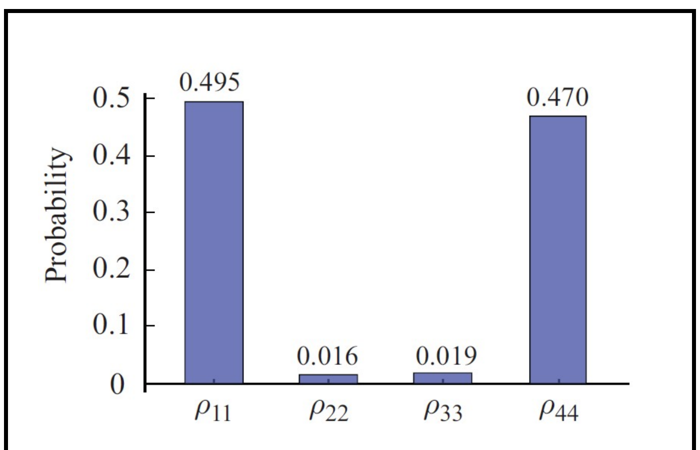
$$C(\rho_{\text{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu = V$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Angular Two-Photon Interference







State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta} \\ \rho_{11} + \rho_{44} &= 1 \end{aligned}$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2$$

$$\times \left\{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos\left[(l_s + l_i)\beta + \theta\right]\right\}$$

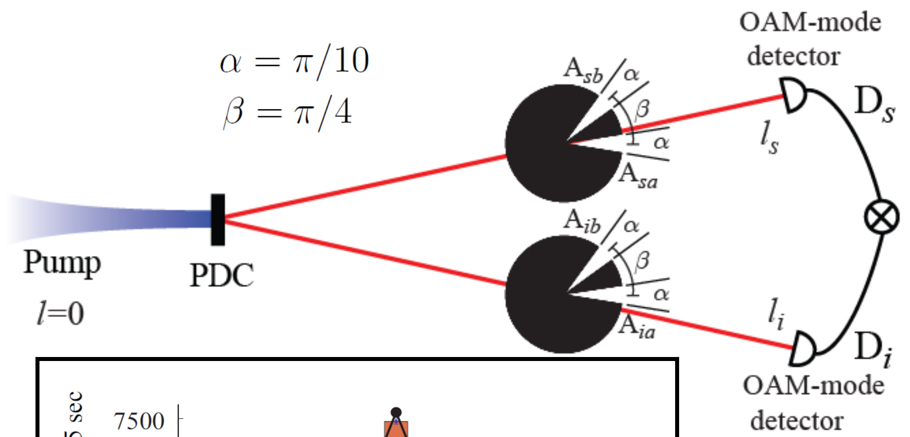
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

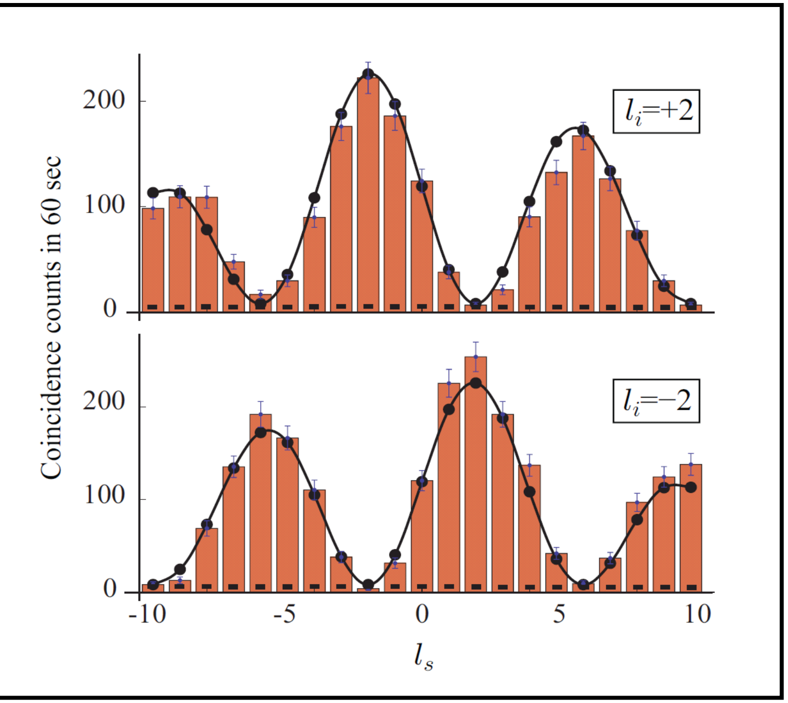
Concurrence of the two-qubit state:

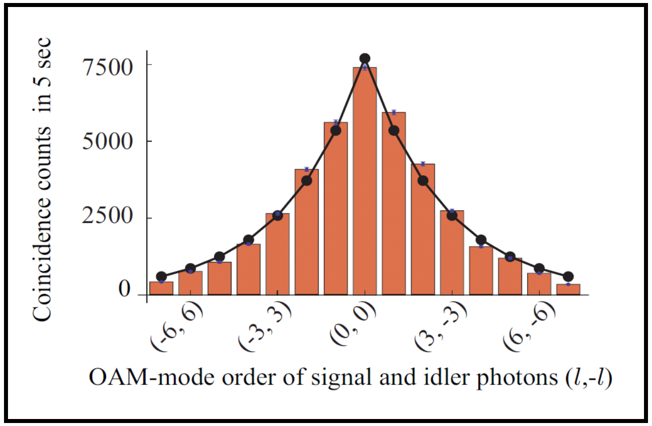
$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu = V$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Angular Two-Photon Interference



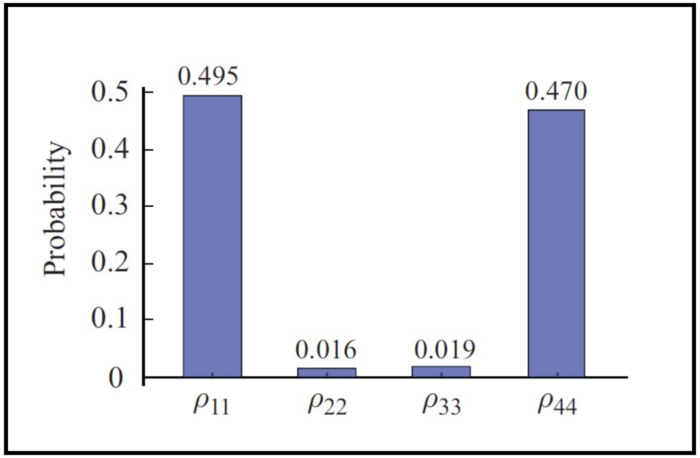




Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$

$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\,\mu \cos[(l_s + l_i)\beta + \theta]\}$$



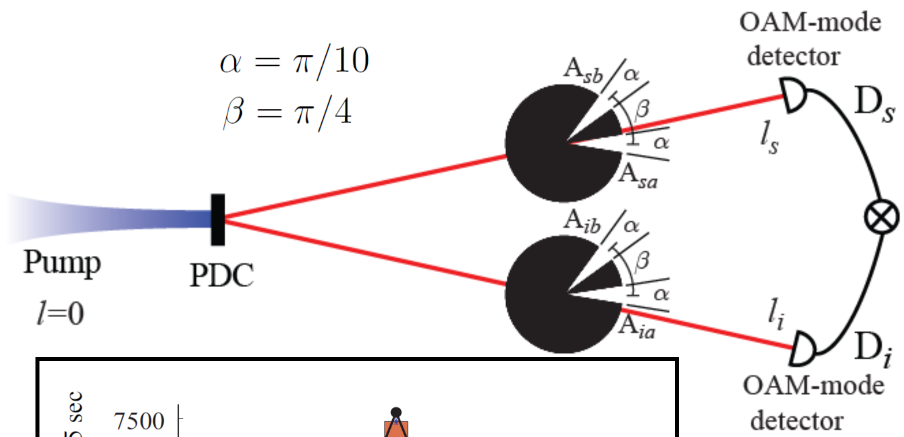
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

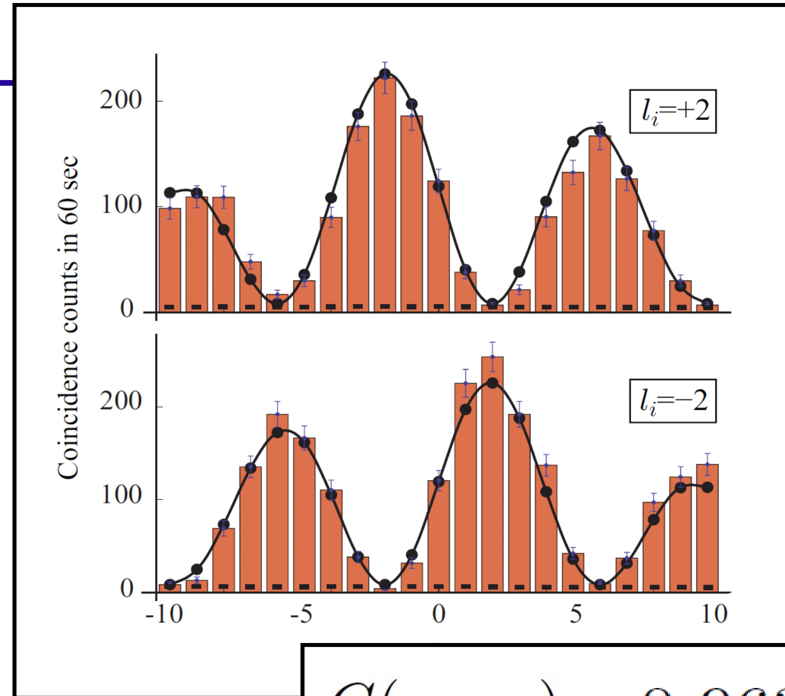
Concurrence of the two-qubit state:

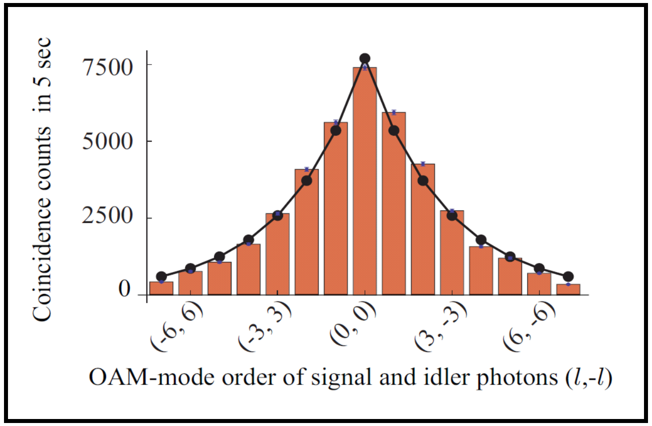
$$C(\rho_{\mathrm{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu = V$$

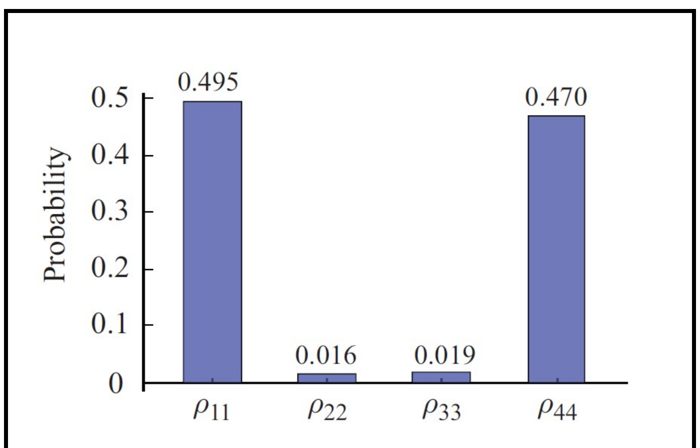
Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Angular Two-Photon Interference









$$C(\rho_{\text{qubit}}) = 0.963$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\text{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \text{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$

$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\,\mu \cos[(l_s + l_i)\beta + \theta]\}$$

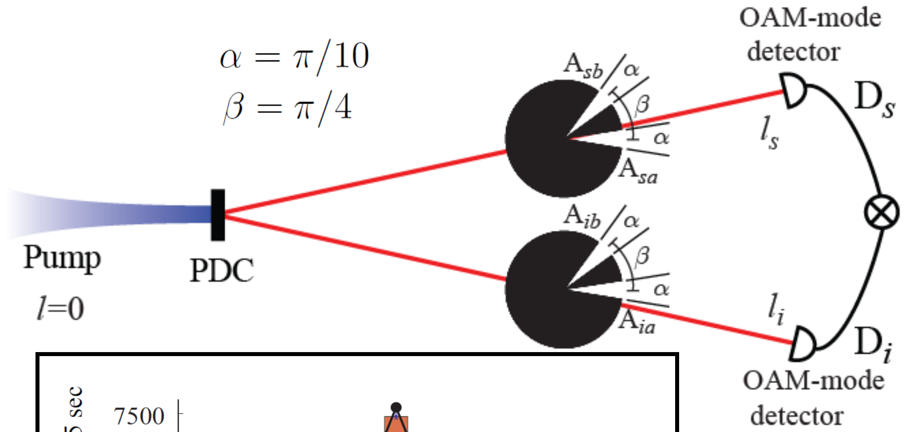
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

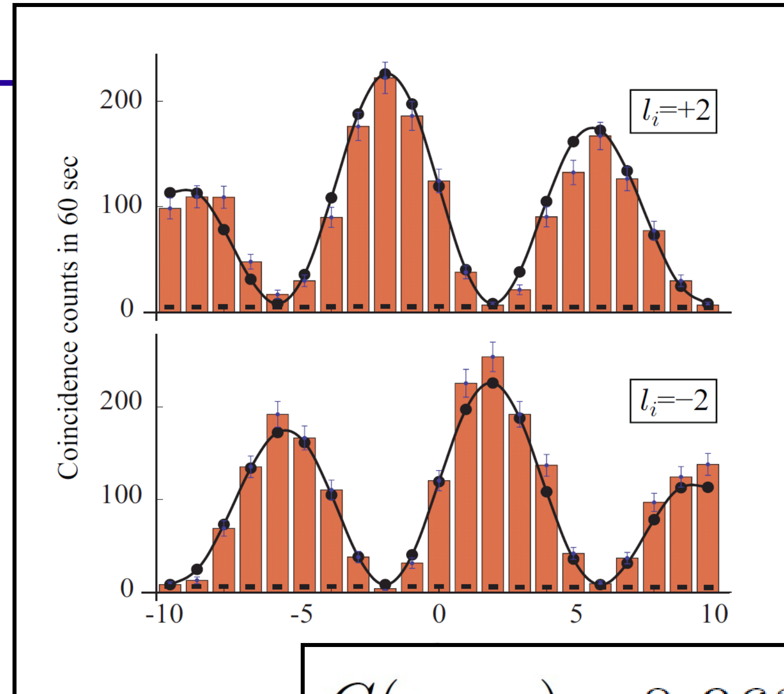
Concurrence of the two-qubit state:

$$C(\rho_{\text{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu = V$$

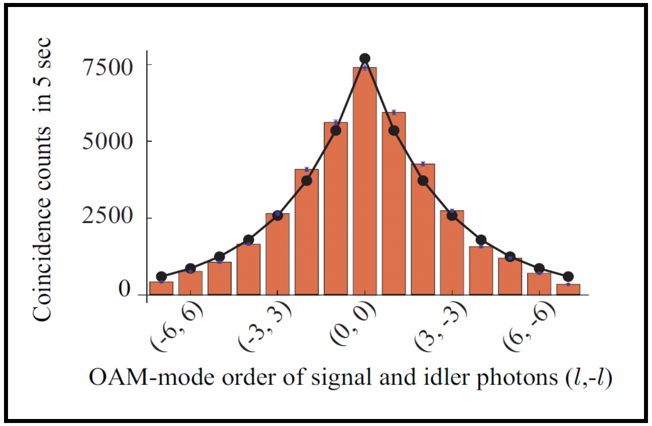
Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

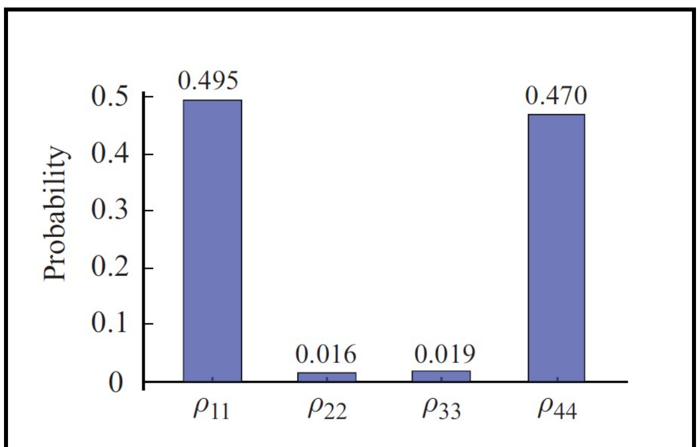
Angular Two-Photon Interference





$$C(\rho_{\text{qubit}}) = 0.963$$





$$\rho^{(c)}_{\text{qubit}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho^*_{41} = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

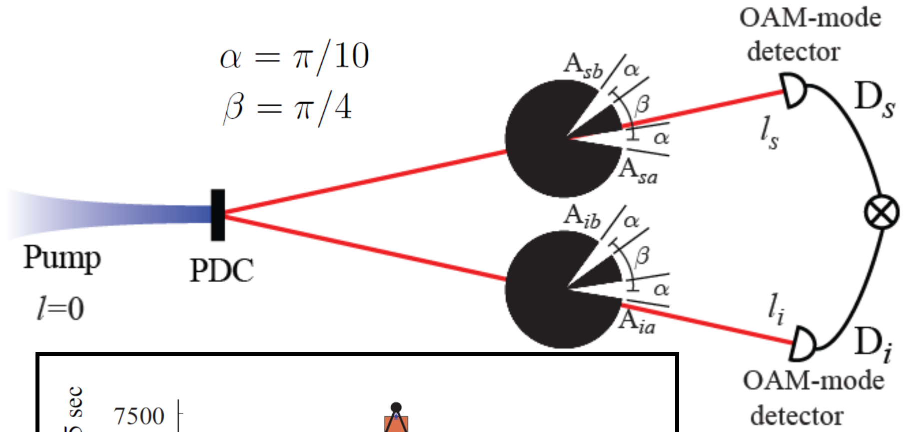
$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$$

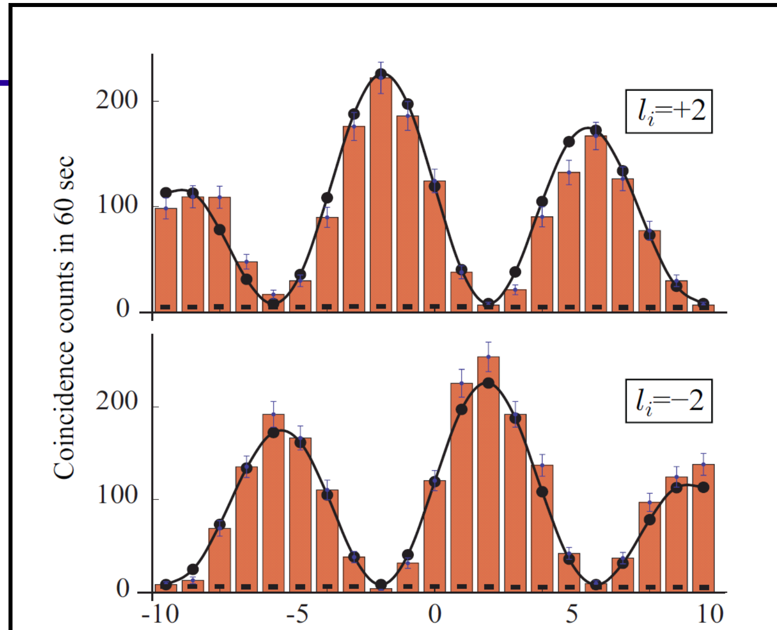
Concurrence of the two-qubit state:

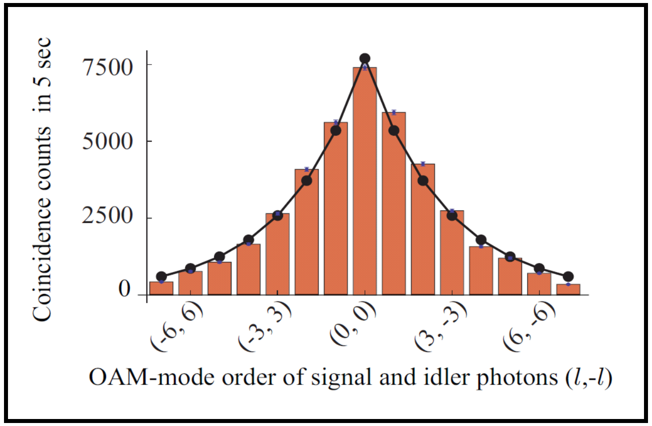
$$C^{(c)}(\rho_{\text{qubit}}) = V^{(c)} - \sqrt{\rho_{22}\rho_{33}}$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Angular Two-Photon Interference

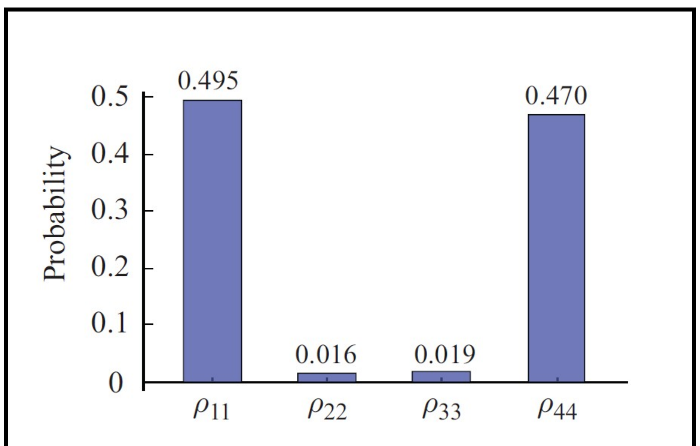






$$C^{(c)}(\rho_{\text{qubit}}) = 0.929$$

$$C(\rho_{\text{qubit}}) = 0.963$$



$$\rho_{\text{qubit}}^{(c)} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^{*} = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$$

Concurrence of the two-qubit state:

$$C^{(c)}(\rho_{\text{qubit}}) = V^{(c)} - \sqrt{\rho_{22}\rho_{33}}$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

Summary and Conclusions

1. Temporal two-photon interference

(i) Presented a description of temporal two-photon coherence in terms of ΔL and $\Delta L'$

(ii) Showed that time-energy entanglement can also be explored using geometric phase

2. Spatial two-photon interference

(i) The spatial coherence properties of the pump beam get entirely transferred to the spatial coherence properties of the entangled two-photon field.

(ii) The entanglement of spatial two-qubit state is equal to the degree of spatial two-photon coherence

3. Angular two-photon interference

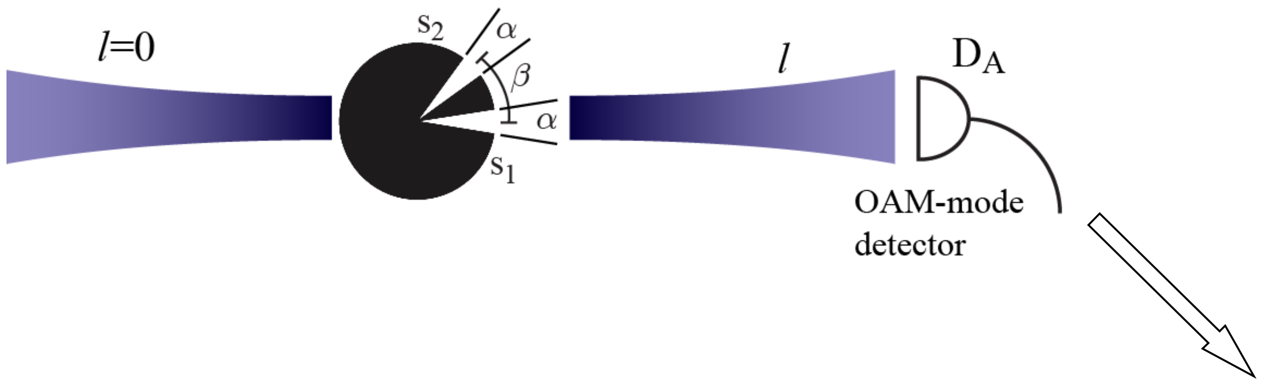
(i) Verified angular Fourier relationship using entangled photons

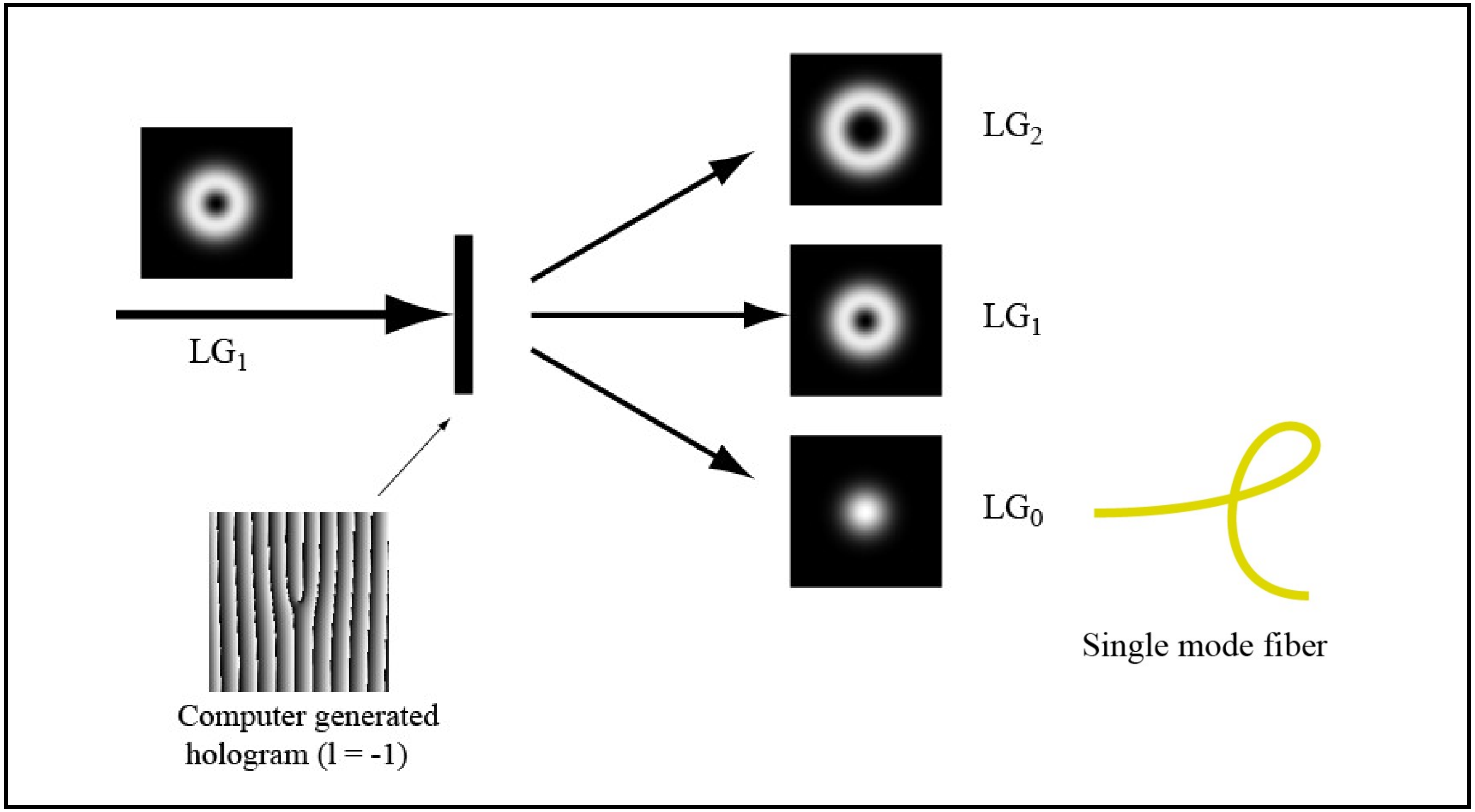
(ii) Studied angular two-photon interference effects and demonstrated an angular two-qubit state

Acknowledgments

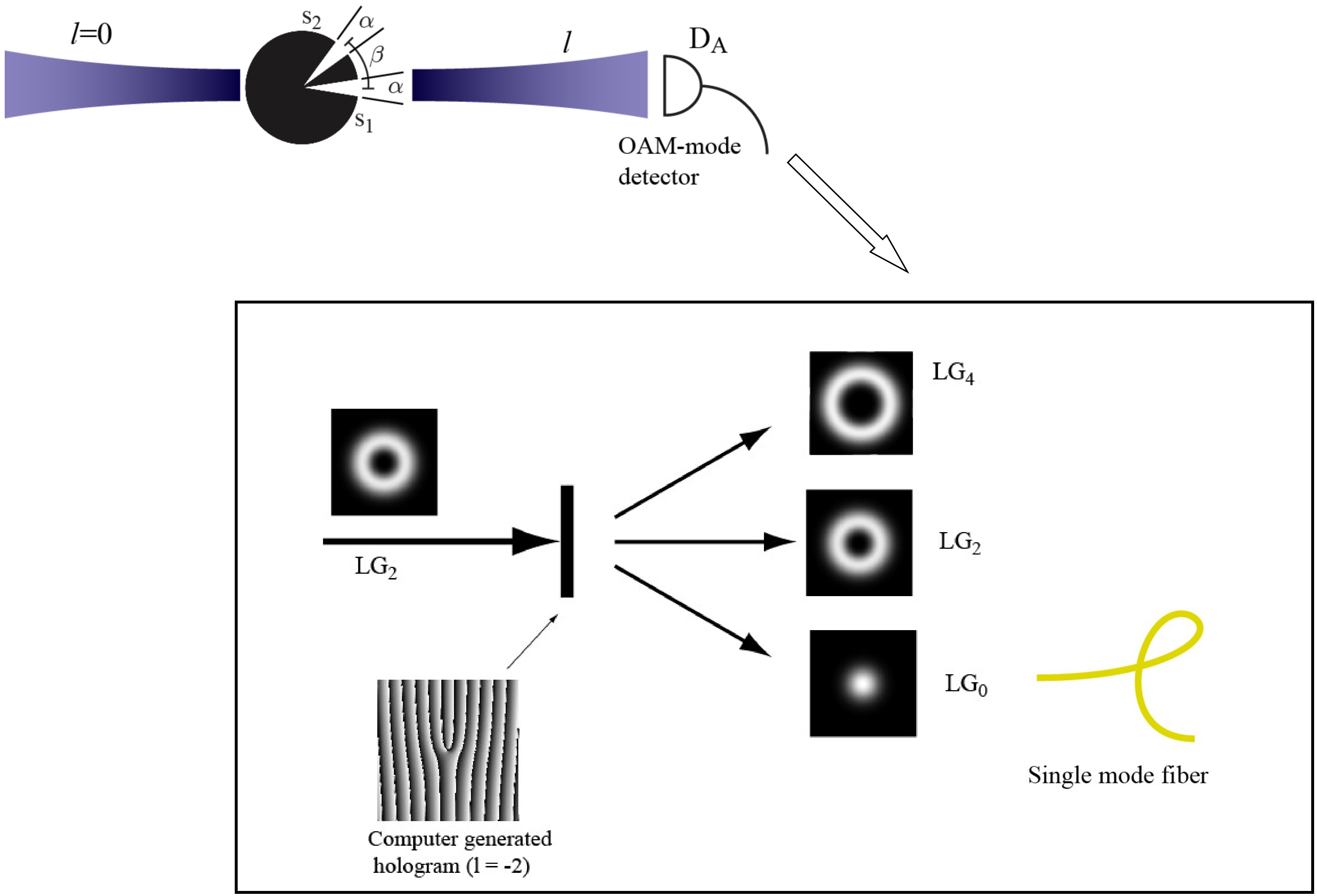
- Prof. Robert Boyd
- Prof. Carlos Stroud
- Prof. Emil Wolf
- Prof. Miles Padgett and his research group (Jonathan Leach, Barry Jack, Sonja Franke-Arnold)
- Prof. Steve Barnett
- Dr. Cliff Chan, Malcolm O’Sullivan, Mehul Malik
- The US Army Research Office, The US Air Force Office

Angular One-Photon Interference

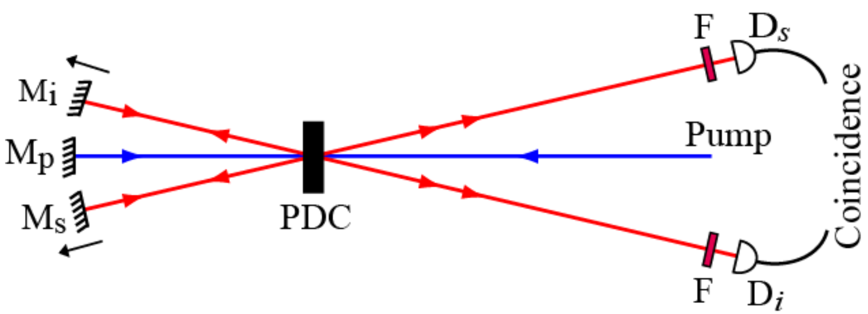


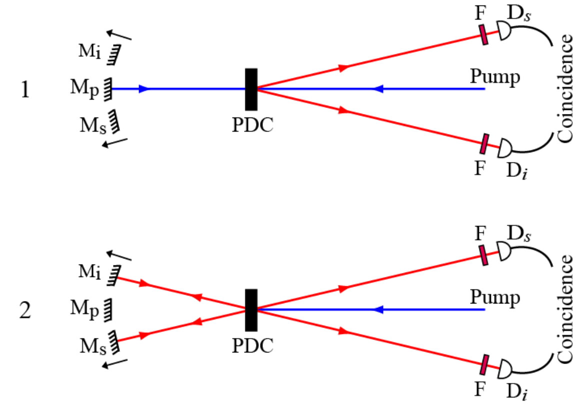


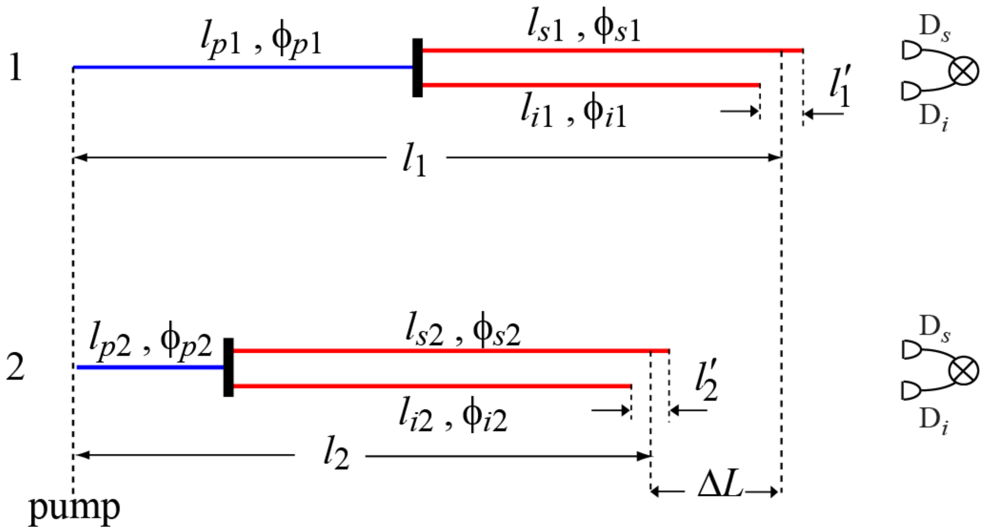
Angular One-Photon Interference



Two-Photon Interference: A two-photon interferes with itself







$$\Delta L \equiv l_1 - l_2$$

Two-photon path-length

$$\Delta L' \equiv l_1' - l_2'$$

Two-photon path-asymmetry length

$$\Delta\phi \equiv (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L + k_d \Delta L' + \Delta\phi)]$$

$$k_d \equiv (k_{s0} - k_{i0})/2$$

Necessary conditions for two-photon interference:

$$\Delta L < l_{\rm coh}^{p} \sim 10 \text{ cm}$$

$$\Delta L' < l_{\rm coh} = \frac{c}{\Delta\omega} \sim 100\ \mu\text{m}$$

Jha, O'Sullivan, Chan, and Boyd, Phys. Rev. A. **77**, 021801(R) (2008)

Introduction

Quantum Entanglement

Led to many foundational work in Quantum Mechanics

- EPR paradox and non-locality — A. Einstein et al., Phys. Rev. **47**, 777 (1935)
- Hidden variable theories — J. S. Bell, Physics **1**, 195 (1964)
- Bell inequalities — D. Bohm, Phys. Rev. **85**, 166 (1952)
-
-

Has applications in Quantum Computation and Quantum Information

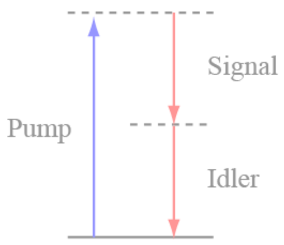
- Quantum cryptography — A. K. Ekert, PRL **67**, 661 (1991)
- Quantum dense coding — C. H. Bennett et al., PRL **69**, 2881 (1992)
- Quantum lithography — A. N. Boto et al., PRL **85**, 2733 (2000)
-
-

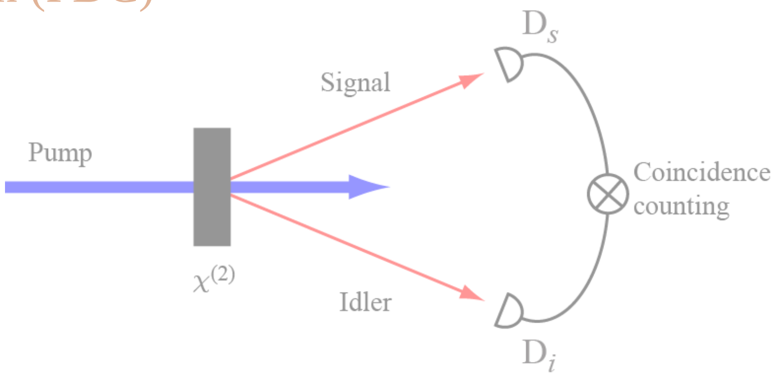
Entanglement can exist between Photons, Atoms, Ions,…

Parametric down-conversion provides a source of entangled photons

Outline

Parametric down-conversion (PDC)





Burnham and Weinberg, PRL **25**, 85 (1970)

Robert W. Boyd, *Nonlinear Optics,* 2nd ed.

$$\omega_p = \omega_s + \omega_i$$

Entanglement in time and energy

"Temporal" two-photon coherence

$$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$$

Entanglement in position and momentum

"Spatial" two-photon coherence

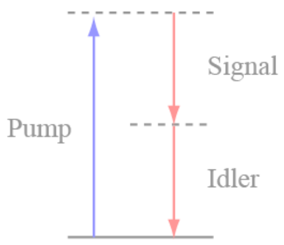
$$l_p = l_s + l_i$$

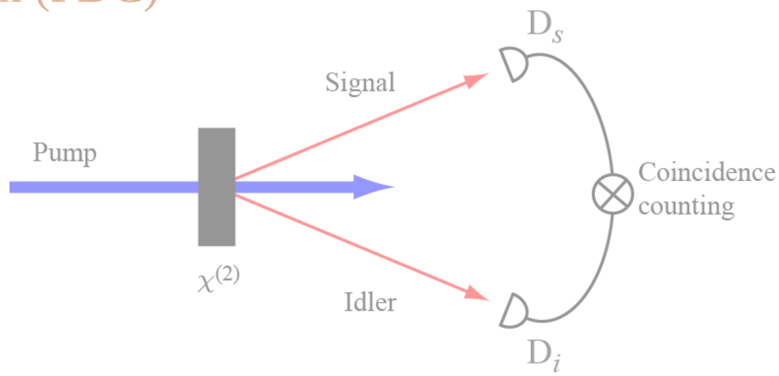
Entanglement in angular position and angular momentum

"Angular" two-photon coherence

Outline

Parametric down-conversion (PDC)





Burnham and Weinberg,
PRL **25**, 85 (1970)

Robert W. Boyd,
Nonlinear Optics, 2nd ed.

$$\omega_p = \omega_s + \omega_i$$

Entanglement in time and energy

“Temporal” two-photon coherence

$$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$$

Entanglement in position and momentum

“Spatial” two-photon coherence

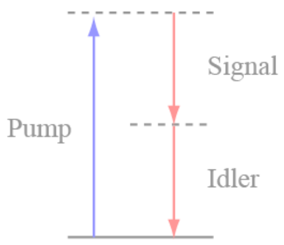
$$l_p = l_s + l_i$$

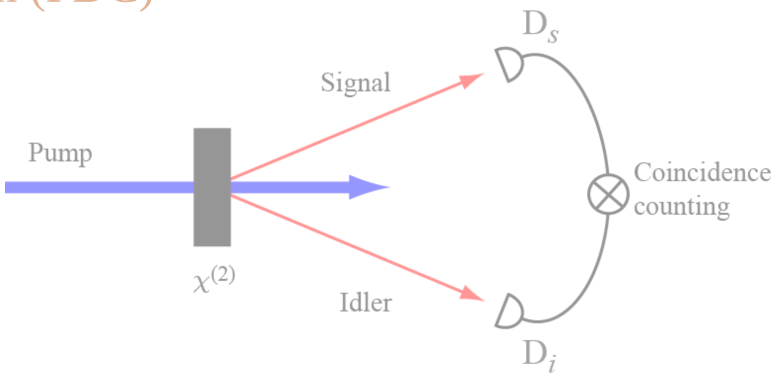
Entanglement in angular position and angular momentum

“Angular” two-photon coherence

Outline

Parametric down-conversion (PDC)





Burnham and Weinberg, PRL **25**, 85 (1970)

Robert W. Boyd, *Nonlinear Optics,* 2nd ed.

$\omega_p = \omega_s + \omega_i$

Entanglement in time and energy

"Temporal" two-photon coherence

$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$

Entanglement in position and momentum

"Spatial" two-photon coherence

$l_p = l_s + l_i$

Entanglement in angular position and angular momentum

"Angular" two-photon coherence