Assignment 1: State Space Modeling and Linearization

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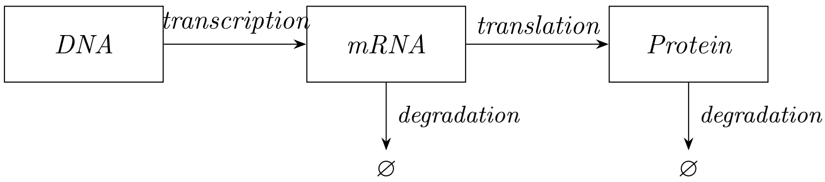
Instructions

- Submit your solutions as a single PDF file.
- Use this link below to upload the PDF.

  https://forms.gle/tbcWQuh7xTHH5x1f6
- Clearly state the assumptions, if you are using any other governing physical laws from given in the box, and variables used in developing each mathematical model.
- Show the main steps of the derivation. Answers containing only the final equations will not receive full credit.
- For state-space models, clearly define the state vector, input, and, where applicable, the output.
- Any figures, free-body diagrams, or schematic representations used in the solution should be clearly labeled.
- If external references, software, or AI-assisted tools are used, their use should be appropriately acknowledged. The chat/prompt history should also be submitted as an Appendix to the solution.
- Ensure that the submitted PDF is legible and that the problems are answered in the same order as they appear in the assignment.
- Name the submitted file `RollNumber_Name_Assignment1.pdf`. We will use an automated system for evaluation and score entry. Incorrectly named files may not be processed by the system, and you will be responsible for any resulting missing score.

Problem 1 (Gene Expression)**.** *The central dogma of molecular biology describes how a cell uses genetic information stored in DNA is finally converted to protein, through two sequential steps. In **transcription**, cellular machinery copy sequence from DNA into a messenger RNA (mRNA) molecule. In **translation** another set of cellular machinery then reads the mRNA sequence to assemble a protein. Both the mRNA and the protein are also continuously degraded.*



> ***Law of Mass Action.*** *The rate of an elementary reaction is proportional to the product of the concentrations of the reacting species, each raised to the power of its stoichiometric coefficient.*
> *For $A \to B$, the rate is $k[A]$. For $A + B \to C$, the rate is $k[A][B]$.*

Consider a gene present at fixed DNA concentration D (the gene is not created or destroyed; only its expression is being modeled). Transcription produces mRNA at a rate governed by the law of mass action applied to the reaction $\mathrm{DNA} \to \mathrm{DNA} + \mathrm{mRNA}$*; translation produces protein via* $\mathrm{mRNA} \to \mathrm{mRNA} + \mathrm{Protein}$*; mRNA and protein are each removed by first-order degradation, with rate constants δ_m and δ_p respectively, and production rate constants k_m (transcription) and k_p (translation).*

(a) *Using the law of mass action for each of the four reactions (transcription, translation, mRNA degradation, protein degradation),*

$$G \xrightarrow{k_m} G + m \qquad \textit{(transcription)}, \tag{1}$$

$$m \xrightarrow{k_p} m + p \qquad \textit{(translation)}, \tag{2}$$

$$m \xrightarrow{\gamma_m} \varnothing \qquad \textit{(mRNA degradation)}, \tag{3}$$

$$p \xrightarrow{\gamma_p} \varnothing \qquad \textit{(protein degradation)}. \tag{4}$$

derive the two ODEs governing the mRNA concentration $m(t)$ and protein concentration $p(t)$.

(b) *Write this system in state space form $\dot{x} = Ax + b$, with $x = (m, p)^T$, and identify A and b.*

(c) *Find the equilibrium (m^*, p^*).*

(d) *Is this system linear or nonlinear? Justify your answer, and state whether the Jacobian you would compute for a linearization differs in any way from A itself.*

Now suppose transcription is not constitutive, but requires the gene's promoter to be bound by an activator supplied externally at concentration $u(t)$, the input. At any instant, each copy of the gene's promoter is in one of two states: $\boldsymbol{DNA}_{\mathrm{off}}$*, unbound and transcriptionally silent, or* $\boldsymbol{DNA}_{\mathrm{on}}$*, bound by the activator and actively transcribing. Binding and unbinding are themselves reactions governed by the law of mass action:*

$$\mathrm{G} + u \underset{k_{\mathrm{off}}}{\overset{k_{\mathrm{on}}}{\rightleftharpoons}} \mathrm{G}_{\mathrm{on}},$$

and only $\mathrm{G_{on}}$ *transcribes, at the same rate constant* k_m *as before. Assume the total amount of this gene's promoter is conserved, neither created nor destroyed on the timescale of this model:*

$$G(t) + G_{\text{on}}(t) = G_{\text{tot}}, \quad \textit{a constant.}$$

(e) Using the law of mass action for the binding and unbinding reactions, and the conservation assumption above to eliminate G *in favor of* $G_{\text{tot}} - G_{\text{on}}$*, derive the ODE governing* $D_{\text{on}}(t)$*.*

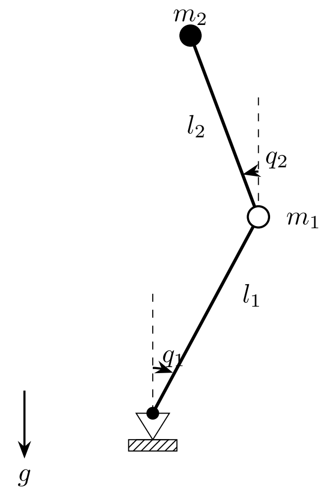
(f) Using the same transcription, translation, and degradation laws as in part (a), but with $G_{\text{on}}(t)$ *in place of the constant* D*, write the full state equation for* $x = (G_{\text{on}}, m, p)$ *with input* u*.*

(g) Is this system linear or nonlinear? Identify, specifically, which term is responsible for your answer.

(h) Find the equilibrium $(G_{\text{on}}^*, m^*, p^*)$ *for a constant input* $u(t) \equiv u_0$*.*

(i) Compute the Jacobian $A(G_{\text{on}}, m, p)$ *in general, then evaluate it at the equilibrium from part (h). You should find it is triangular.*

Problem 2 (Two-Link Manipulator)**.** *A robot arm consists of two rigid links connected by revolute joints, with masses* m_1, m_2 *concentrated at the end of each link, lengths* l_1, l_2*, joint angles* q_1 *(from vertical) and* q_2 *(of the second link relative to the first), and joint torques* τ_1, τ_2 *as inputs.*



(a) Using $x = (q_1, q_2, \dot{q}_1, \dot{q}_2)$*, write the state equation* $\dot{x} = f(x, \tau)$*.*

(b) Find all equilibria for $\tau = 0$*.*

(c) Linearise the dynamics around the equilibrium point.

Problem 3 (Pendulum with a Stretchable Link)**.** *Consider a point mass* m *connected to a fixed pivot through a stretchable link modeled as a linear spring of stiffness* k *and natural length* l_0*. Let* $r(t)$ *denote the instantaneous length of the link and let* $\theta(t)$ *denote the angular displacement measured from the downward vertical. A torque input* $u(t)$ *is applied at the pivot. Gravity acts vertically downward.*

Euler–Lagrange Equation. *For a mechanical system described by generalized coordinates q_i, the equations of motion are obtained from*

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i,$$

where $L = T - V$ is the Lagrangian, T and V are the kinetic and potential energies, respectively, and Q_i is the generalized non-conservative force associated with q_i.

(a) Develop a mathematical model of the system using the Euler–Lagrange formulation. Obtain the coupled nonlinear differential equations governing $r(t)$ and $\theta(t)$.

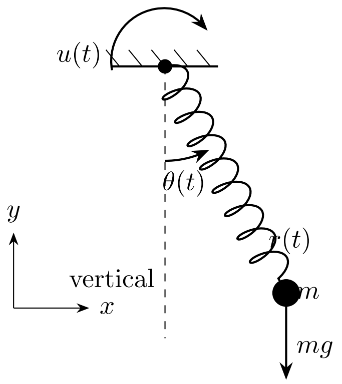
(b) Re-develop a mathematical model of the system using the Newton's second law (resolving forces along radial and tangential directions). Obtain the coupled nonlinear differential equations governing $r(t)$ and $\theta(t)$.

(c) For $u = 0$, determine the equilibrium configurations of the system.

(d) Express the model in first-order state-space form using

$$x = \begin{bmatrix} r & \theta & \dot{r} & \dot{\theta} \end{bmatrix}^T.$$

(e) Linearize the nonlinear model about each equilibrium and obtain the corresponding linear state-space model.



Problem 4 (Bistable Opinion Dynamics: One Voter, Then Two)**.** *Opinion change does not follow a physical conservation law; it is modeled here phenomenologically. Suppose an individual's comfort with holding opinion $x \in \mathbb{R}$ is captured by a potential $V(x)$ with two minima, at $x = \pm 1$, and a local maximum at neutral, $x = 0$:*

$$V(x) = -\frac{x^2}{2} + \frac{x^4}{4}.$$

Principle of Steepest-Descent Dynamics. *A quantity governed by a potential $V(x)$, in the absence of other forces, evolves in the direction that decreases V as quickly as possible: $\dot{x} = -dV/dx$.*

An external input u represents targeted messaging that directly shifts opinion.

(a) Compute dV/dx and hence derive the ODE governing $x(t)$.

(b) Find all equilibria of this system for $u = 0$, and relate them to the critical points of V.

(c) *Compute $A = \partial f/\partial x$ at a general x, then evaluate it at each equilibrium found in part (b). State the sign of A at each one, and relate this sign to whether V has a local minimum or a local maximum there.*

> ***Law of Diffusive Coupling.*** *Two interacting quantities are pulled toward each other at a rate proportional to their difference: the coupling term contributed to $\dot{x}_1$ by x_2 is $k(x_2 - x_1)$, for coupling strength $k \geq 0$.*

Now consider two individuals, or clusters of a social network, each governed by the potential-descent law from part (a) for their own opinion, but also coupled through a recommendation algorithm following the law above.

(d) *Using the individual law from part (a) for each opinion, plus the coupling law just stated, write the pair of ODEs governing $x_1(t)$ and $x_2(t)$, with independent inputs u_1, u_2.*

(e) *Find all equilibria of the form $x_1 = x_2 = x^*$ for $u_1 = u_2 = 0$, and compare with part (b).*

(f) *Compute the Jacobian and evaluate it at a symmetric equilibrium $x_1 = x_2 = x^*$.*

(g) *Find the two eigenvalues by testing $(1,1)^T$ and $(1,-1)^T$. Express both in terms of x^* and k, and compare the eigenvalue along $(1,1)^T$ with your answer to part (c).*

(h) *At $x^* = 0$, find the value k_c at which the eigenvalue along $(1,-1)^T$ changes sign.*

(i) *At $x^* = \pm 1$, state the sign of the eigenvalue along $(1,-1)^T$ for every $k \geq 0$.*

Problem 5 (Optimization as a Dynamical System)**.** *Optimization problems arise when one seeks a value of a decision variable $x \in \mathbb{R}^n$ that minimizes a cost function*

$$J : \mathbb{R}^n \to \mathbb{R}.$$

Consider the quadratic cost

$$J(x) = \frac{1}{2}x^T Q x + b^T x + c,$$

where $Q = Q^T > 0$, $b \in \mathbb{R}^n$, and $c \in \mathbb{R}$.

Since Q is positive definite, the cost function has a unique minimum. A necessary condition for the minimizer $x^\star$ is

$$\nabla J(x^\star) = 0.$$

One way to search for this minimum is to continuously move in the direction opposite to the gradient of the cost. This gives the gradient-flow *dynamics*

$$\dot{x} = -\nabla J(x).$$

In numerical optimization, the variable is instead updated at discrete instants. The corresponding gradient-descent algorithm is

$$x_{k+1} = x_k - \alpha \nabla J(x_k),$$

where $\alpha > 0$ is called the step size or learning rate.

> ***Gradient of a Quadratic Function.*** *For*
>
> $$J(x) = \frac{1}{2}x^T Q x + b^T x + c,$$
>
> *with $Q = Q^T$, its gradient is*
>
> $$\nabla J(x) = Qx + b.$$
>
> *At a stationary point,*
>
> $$\nabla J(x^\star) = 0.$$

(a) Determine the minimizer $x^\star$ of the cost function.

(b) Treat the continuous-time gradient-flow algorithm as a dynamical system. Write it in state-space form and determine its equilibrium.

(c) Show that the equilibrium of the gradient-flow dynamics coincides with the minimizer $x^\star$.

(d) Linearize the gradient-flow dynamics about $x^\star$.

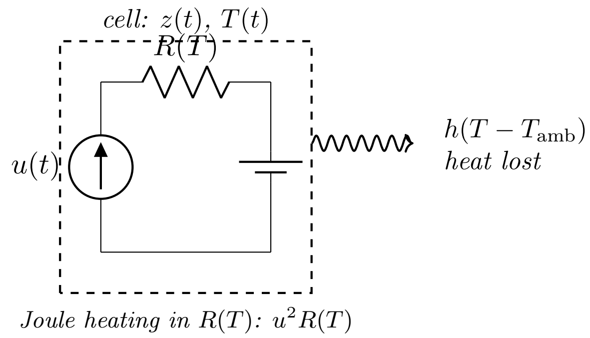
(e) Treat the gradient-descent iteration as a discrete-time dynamical system and write it in state-space form.

(f) Determine the equilibrium of the discrete-time system and show that it also coincides with $x^\star$.

(g) Determine the eigenvalues of the discrete-time state matrix. Obtain a condition on the step size α for which the gradient-descent iteration converges to $x^\star$.

(h) Compare the continuous-time gradient flow and the discrete-time gradient-descent algorithm. In particular, explain the role played by the step size α in the discrete-time system.

Problem 6 (EV Battery: From Isothermal Charging to Coupled Thermal-Electrical Dynamics)**.** *A battery cell is charged at current $u(t)$. Two quantities are tracked: the state of charge $z(t)$ (the fraction of the cell's capacity Q, measured in ampere-hours, remaining) and the cell temperature $T(t)$.*



> ***Governing Physical Laws. Coulomb counting:*** *charge removed equals the time integral of current.* ***Ohm's law / Joule heating:*** *power dissipated in a resistor is I^2R.* ***Newton's law of cooling:*** *heat is lost to the surroundings at rate $h(T - T_{\text{amb}})$.* ***First law of thermodynamics*** *(lumped thermal mass): rate of change of stored thermal energy, $C_{\text{th}}\dot{T}$, equals rate of heat generated minus rate of heat lost.*

(a) Using Coulomb counting, converting seconds to hours by dividing by 3600 and expressing the result as a fraction of capacity Q, derive the ODE governing $z(t)$ in terms of u and Q.

(b) Assuming constant resistance R_0 and constant charging current u_0, use the first law of thermodynamics together with Joule heating and Newton's law of cooling to derive the ODE governing $T(t)$, against a time-varying ambient temperature $T_{\text{amb}}(t)$.

(c) Classify the equation from part (b) as time-invariant or time-varying, and explain why.

(d) For constant $T_{\text{amb}}(t) \equiv T_{\text{amb},0}$, find the equilibrium temperature T_0 in closed form.

(e) Compute $A = \partial \dot{T}/\partial T$ at this equilibrium, and state its sign.

> ***Arrhenius Law for Electrolyte Conductivity.*** *The internal resistance of a battery cell falls exponentially with temperature: $R(T) = R_0 e^{-\alpha(T - T_{\text{ref}})}$, $\alpha > 0$, a standard approximation for how ionic conductivity of the electrolyte increases with temperature.*

(f) Using the same energy-balance law as part (b), but with $R(T)$ in place of R_0, and the same Coulomb-counting law as part (a), write the coupled ODEs governing $z(t)$ and $T(t)$ for a general (possibly time-varying) current $u(t)$.

(g) Classify this system: linear or nonlinear, time-invariant or time-varying. Justify both in one line each.

(h) For a constant charging current $u(t) \equiv u_0 \neq 0$, explain why $z(t)$ has no equilibrium value, even though $T(t)$ may settle into a quasi-steady value.

(i) Suppose $T_{\text{amb}}(t)$ varies slowly enough that, at each instant, $T(t)$ tracks a quasi-steady value $T_0(t)$ satisfying

$$u_0^2 R(T_0(t)) = h\big(T_0(t) - T_{\text{amb}}(t)\big)$$

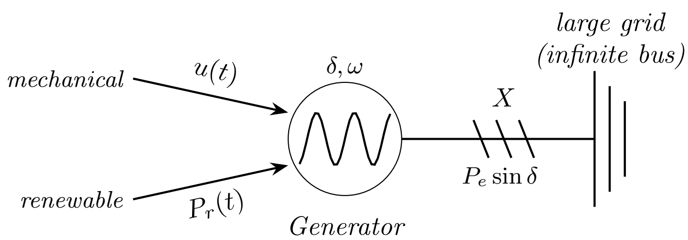
exactly, and let $z_0(t)$ be the corresponding ramp in state of charge. Treat $(z_0(t), T_0(t), u_0)$ as a nominal trajectory. Using the general trajectory-linearization formula, derive $A(t)$ and $B(t)$, expressing your answer in terms of $R(T_0(t))$, $R'(T_0(t))$, h, C_{th}, u_0, and Q. You are not required to solve for $T_0(t)$ explicitly.

(j) One eigenvalue of $A(t)$ is exactly zero for all t. Identify which state this corresponds to, and explain why this eigenvalue must be zero regardless of the operating point.

(k) Express the remaining eigenvalue of $A(t)$ in terms of $R(T_0(t))$, and determine its sign given $\alpha, R_0, h, C_{\text{th}} > 0$.

Problem 7 (Power Grid Frequency Dynamics)**.** *A synchronous generator is connected to a large power grid through a transmission line of reactance X. Let $\delta(t)$ denote the rotor angle of the generator relative to the grid reference, and let $\omega(t) = \dot{\delta}(t)$ denote the corresponding frequency deviation.*

The generator receives a controllable mechanical power input $u(t)$. An additional power injection $P_r(t)$ represents the net contribution from a renewable source connected to the system.



> ***Swing Equation.*** *The rotational dynamics of a synchronous generator are described by Newton's second law for rotation. In power-system form, the rate of change of rotor speed is proportional to the imbalance between the supplied power and the electrical power delivered to the grid.*
> *For a generator connected to an infinite bus through a predominantly reactive transmission line, the electrical power transferred is proportional to $\sin\delta$.*

Let $M > 0$ denote the inertia constant, $D > 0$ the damping coefficient, and $P_e > 0$ the maximum electrical power that can be transferred through the line.

(a) *Develop a mathematical model of the generator-grid system using the swing equation. With*

$$x = \begin{bmatrix} \delta & \omega \end{bmatrix}^T, \qquad \omega = \dot{\delta},$$

express the resulting model in first-order state-space form $\dot{x} = f(x, u, P_r)$.

(b) *Suppose*

$$u(t) = u_0, \qquad P_r(t) = P_{r0},$$

where

$$0 < u_0 + P_{r0} < P_e.$$

Determine all equilibrium points (δ_e, ω_e) in the interval $\delta_e \in (-\pi, \pi]$.

(c) *Linearize the nonlinear state-space model about each equilibrium point and obtain the corresponding matrices A and B.*

(d) *Compare the linearized models obtained at the two equilibria. Identify the term whose sign changes between the two operating points.*

(e) *Now suppose that the renewable power $P_r(t)$ varies with time. Explain briefly why a fixed equilibrium point may no longer be an appropriate operating condition and what quantity should be used instead when linearizing the system.*

Problem 8 (Hall 10 Canteen Queue: A Discrete-Time Model)**.** *During peak hours, students arriving at the Hall 10 canteen may form a queue at the food counter. To model the queue, divide time into intervals of five minutes and let q_k denote the number of students waiting at the beginning of the kth interval.*

During interval k, let a_k students arrive at the counter. Depending on the number of serving staff and the service rate, the counter can serve at most u_k students during the interval. Clearly, the counter cannot serve more students than are actually available in the queue.

> ***Discrete Balance Law.*** *For a quantity observed at successive discrete instants,*
>
> $$\text{amount at next instant} = \text{amount at current instant} + \text{inflow} - \text{outflow}.$$
>
> *A discrete-time state-space model therefore has the general form*
>
> $$x_{k+1} = f(x_k, u_k).$$

(a) *Develop a discrete-time mathematical model for the number of students in the queue. Your model should account for the fact that the queue length cannot become negative.*

(b) *Suppose that the number of arriving students and the service capacity remain constant from one interval to the next:*

$$a_k = a_0, \qquad u_k = u_0.$$

Determine the possible equilibrium queue lengths.

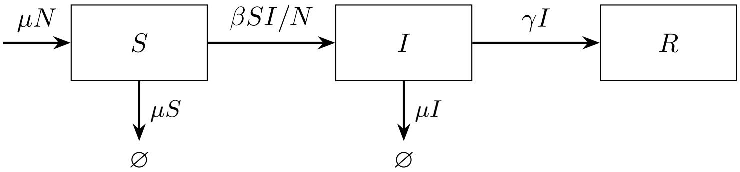
(c) *Using your model, explain what happens to the queue in each of the following situations:*

$$a_0 < u_0, \qquad a_0 = u_0, \qquad a_0 > u_0.$$

(d) *Treat the service capacity as the control input and the student arrivals as an external disturbance. Identify the state, input, and disturbance, and write the model in the form*

$$x_{k+1} = f(x_k, u_k, w_k).$$

Problem 9 (Epidemic Spread: Eigenvalues and the Reproduction Number)**.** *A population of constant size N is divided into susceptible $S(t)$, infected $I(t)$, and recovered $R(t)$ individuals.*



Law of Mass Action. *As in Problem 1, the rate of an elementary reaction is proportional to the product of the concentrations, here populations, of the reacting species. New infections arise from encounters between susceptible and infected individuals, at rate proportional to the product SI, scaled by $1/N$ since the transmission rate β is defined per capita. Recovery and demographic turnover are each first-order processes, at constant per-capita rates γ and μ.*

Since R does not influence S or I (no one re-enters the susceptible or infected class from R in this model), only S and I need be tracked as states.

(a) *Using conservation of individuals, rate of change of each compartment equals inflow minus outflow, derive the pair of ODEs governing $S(t)$ and $I(t)$ in terms of μ, N, β, γ.*

(b) *Verify that $(S_e, I_e) = (N, 0)$, the disease-free state, is an equilibrium of your equation for any constant β.*

(c) *Linearize about this equilibrium and show that the Jacobian is upper triangular, with eigenvalues $-\mu$ and $\beta - (\mu + \gamma)$.*

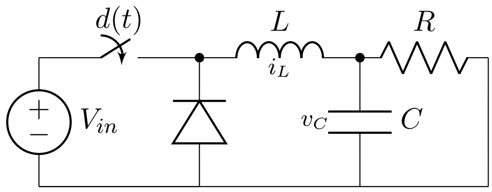
(d) *The basic reproduction number is defined as $R_0 = \beta/(\mu + \gamma)$. Show that both eigenvalues found in part (c) are negative if and only if $R_0 < 1$.*

(e) *Now suppose transmission is seasonal, $\beta(t) = \beta_0 (1 + \varepsilon \cos(2\pi t/365))$. Explain why $(N, 0)$ remains an equilibrium for every t, even though the system is now time-varying, and write down $A(t)$.*

(f) *The scalar equation for $I(t)$ near this equilibrium is $\dot{I} = (\beta(t) - (\mu + \gamma))I$, which can be solved exactly using an integrating factor. Using this exact solution, find the condition on β_0, γ, μ under which $I(t) \to 0$ as $t \to \infty$, and explain why checking the sign of $\beta(t) - (\mu + \gamma)$ at one fixed instant t does not by itself answer this question.*

Problem 10 (DC-DC Converters: From Buck to Boost by Switched-Circuit Averaging)**.** *A switch-mode converter connects an input voltage source V_{in} to a load through an inductor L, a capacitor C, and a switch operated periodically with duty cycle $d(t) \in [0, 1]$, the fraction of each cycle the switch spends in the ON position. State variables are the inductor current $i_L(t)$ and capacitor voltage $v_C(t)$.*

Kirchhoff's Laws and State-Space Averaging. *Within each switch position, the circuit is linear, and Kirchhoff's voltage and current laws give one set of state equations for the ON interval and another for the OFF interval. Since switching is fast compared to the circuit's natural response, the* ***state-space averaged model*** *replaces the two sets of equations with a single weighted average, weighted by d and $(1-d)$ respectively: if $\dot{x} = f_{\text{on}}(x)$ during the ON interval and $\dot{x} = f_{\text{off}}(x)$ during the OFF interval, the averaged model is $\dot{x} = d\, f_{\text{on}}(x) + (1-d) f_{\text{off}}(x)$.*



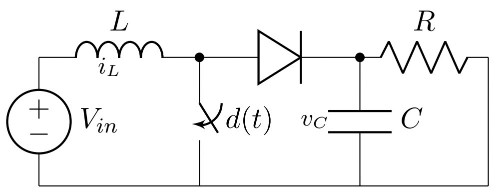
First consider a buck converter (step-down),

(a) *Write dynamics of the circuit for the ON interval and for the OFF interval, and apply state-space averaging to derive the averaged ODEs governing $i_L(t)$ and $v_C(t)$, with $d(t)$ as the input.*

(b) *Is this averaged system linear or nonlinear in (i_L, v_C, d) jointly? Justify your answer by identifying whether d multiplies a state variable anywhere.*

(c) *For constant duty cycle $d(t) \equiv D_0$, find the equilibrium (i_L^*, v_C^*), and confirm that $v_C^* = D_0 V_{in}$, the standard buck converter voltage relation.*

(d) *Compute the Jacobian A.*



Now consider a boost converter (step-up),

(e) *Using the same state-space averaging law, derive the averaged ODEs governing $i_L(t)$ and $v_C(t)$ for the boost converter, with $d(t)$ as the input.*

(f) *Is this averaged system linear or nonlinear? Identify, specifically, which term is responsible for your answer.*

(g) *For constant duty cycle $d(t) \equiv D_0$, find the equilibrium (i_L^*, v_C^*), and confirm that $v_C^* = V_{in}/(1-D_0)$, the standard boost converter voltage gain formula.*

(h) *Compute the Jacobian A at this equilibrium.*

(i) *Using $\operatorname{trace}(A) =$ sum of eigenvalues and $\det(A) =$ product of eigenvalues (from the linear algebra notes), show that $\operatorname{trace}(A) < 0$ and $\det(A) > 0$ for every $D_0 \in (0,1)$ and $L, C, R > 0$, and explain why these two facts together imply both eigenvalues of A have negative real part.*

Problem 11 (Explore and Model a Dynamical System of Your Interest)**.** *Choose a real dynamical system from an area of your interest. Examples may come from anywhere such as robotics, electrical or electronic systems, power and energy systems, biological systems, transportation, aerospace, mechanical systems, environmental systems, economics, social dynamics, or another suitable domain BUT should not be covered from above.*

Your objective is to identify and mathematically capture an interesting dynamical behavior *of the chosen system using a state-space model. The behavior should involve genuine dynamics rather than only a static input–output relationship.*

(a) *Briefly describe the physical or conceptual system and clearly identify the particular dynamical phenomenon that you wish to model. Explain why this behavior is interesting.*

(b) *Draw a schematic, block diagram, free-body diagram, reaction diagram, circuit, or another suitable representation of the system. Clearly state the assumptions used in developing the model.*

(c) *Identify suitable state variables, inputs, and outputs. Explain briefly why your chosen variables form an appropriate state description.*

(d) *Starting from appropriate physical laws, conservation principles, constitutive relations, empirical laws, or other justified modeling assumptions, derive a mathematical model of the system.*

(e) *Express the model in state-space form*

$$\dot{x} = f(x, u), \qquad y = h(x, u), \tag{5}$$

or, if the system is naturally discrete,

$$x_{k+1} = f(x_k, u_k), \qquad y_k = h(x_k, u_k). \tag{6}$$

(f) *Determine whether the resulting model is linear or nonlinear, and time-invariant or time-varying. Identify the terms or assumptions responsible for this classification.*

(g) *Identify at least one equilibrium, operating point, or nominal trajectory of interest. Where appropriate, obtain a linearized state-space model about this operating condition.*

(h) *[Optional] Demonstrate that your model captures the dynamical behavior identified in part (a). You may use analytical reasoning, a numerical simulation, or a suitable plot. Briefly interpret the observed behavior in terms of the states and parameters of your model.*

(i) *State at least one important feature of the real system that your model does* not *capture, and suggest how the model could be extended to include it.*

The emphasis of this problem is on modeling decisions and reasoning rather than model complexity. A simple model that clearly captures an interesting dynamical phenomenon is preferable to a complicated model whose assumptions or states are not well justified. Any external source used for physical laws, parameter values, or model structure should be properly cited.