Why The Barker Sequence Bit Length Does Not Exceed Thirteen ?

It is shown that binary Barker sequences, which are polarity reversed coded rectangular pulses of N bits with an autocorrelation function of peak value N and side lobe level unity or less, do not exist for N greater than 13.

THE BINARY Barker (phase) code sequence (X_o, X_1, ..., X_{N-1}) is one whose autocorrelation $R\left(\frac{T}{N}K\right)$ has a maximum $R(0) = T$, where $K = 0, 1$, ..., $N-1$ with side lobes $R(K \neq 0) \leqslant T/N$, and where

$$R\left(\frac{T}{N}K\right) = \sum_{i=0}^{N-1} X_i\, X_{i+K}$$

Barker sequences have large effective bandwidth necessary for range resolution [1] and are hence optimum for pulse compression radars [2]. Although the phase characteristic of a Barker code approximately follows that of a frequency modulated pulse, the effective bandwidth achievable using Barker codes is more than that with frequency modulation for the same phase change [1]. This can be explained by the nonlinear nature of Barker codes.

Binary Barker sequences of lengths larger than 13 are not known. It has been shown [3] that such odd length (N mod 2 = 1) sequences do not exist for $N > 13$. A simple but general derivation for integral N (even or odd) is given here to show that binary Barker sequences do not exist for $N > 13$, subject to sufficiently general conditions.

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The authors are not aware of the existence of such a derivation in the published literature for integral N and hence this note.

DERIVATION THAT $N \leqslant 13$

Consider the sequence $\mu(t)$ defined by a distribution of rectangular (phase reversed) pulses with levels between +1 and −1 as shown in Fig 1, and such that

$$N = F + 1N_1 + 2N_2 + 3N_3 + \ldots\ldots + xN_x \qquad (1)$$

where N_i = number of pulses with width (iT/N) each

F = initial flat portion of width (FT/N) and further $0 < F \leqslant i_{max}$ where $N_{i\,max+1} = N_{i\,max+2} = \ldots\ldots = N_x = 0$ and $N_i \geqslant 0$.

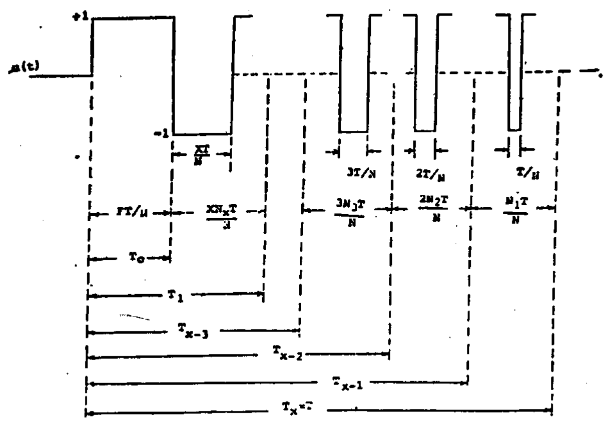

Fig 1 N Bit Barker Sequence.

This waveform or sequence should satisfy

$R\mu\,(T/N) = 0 \text{ or } \pm T/N$

Now

$$R_\mu(T/N) = \int_0^T \mu(t)\,\mu(t-T/N)\,dt$$
$$= \int_0^{T_o} \mu'(t)\,\mu(t-T/N)\,dt + \int_{T_{x-1}}^{T_x} \mu(t)\mu'(t-T/N)dt$$
$$+ \sum_{i=0}^{x-2} \int_{T_i}^{T_{i+1}} \mu(t)\,\mu\,(t-T/N)\,dt$$

where

$$T_o = \frac{FT}{N}\,;\ T_x = T \text{ and}$$

$$T_i\,(0 < i \leqslant x-1) = T_{i-1} + (x-i+1)\,N_{(x-i+1)}\,T/N$$

The first integral would give $(F-1)\ T/N$. The second integral corresponds to pulses of width T/N of $\mu(t)$ and they are multiplied by the opposite polarity pulses of $\mu(t-T/N)$ to give $-N_1\ T/N$. Each of the integrals inside the summation sign would result in $(x-i-2)\ N_{x=i}\ T/N$, because for each pulse of width $(x-i)\ T/N$ or $\mu(t)$ there exists one +ve to −ve or −ve or −ve to +ve transition from $\mu(t-T/N)$. This reduces the contribution to the integral from each pulse of $\mu(t)$ by $\frac{2T}{N}$.

Now if $R_\mu\ (T/N) = \pm T/N$ or 0 and if $T/N = 1$ for simplicity, by expanding *ie* writing down the terms of the summation one obtains,

$$(F-1)-N_1+N_3+2N_4+3N_5+\ldots+(x-2)N_x = 0 \text{ or } \pm 1 \quad (2)$$

The sequence should also satisfy, another condition given by:

$$R_\mu\,(3T/N) = 0 \text{ or } \pm T/N = 0 \text{ or } \pm 1, \text{ if } T/N = 1 \text{ (assumed)} \quad (3)$$

Now let us consider the following three cases:

Case 'A': If $i_{max} \geqslant 3$ and $N_1 \geqslant 2$, the condition in (3) implies

$$F-1-N_1-3N_3-2N_4-N_5+N_7+2N_8+\ldots+(x-6)N_x = 0 \text{ or } \pm 1 \quad (4)$$

By subtracting (4) from (2), we get

$$4N_3+4N_4+4N_5+\ldots\ldots+4N_x = 0 \text{ or } \pm 1 \text{ or } \pm 2$$

which implies $N_3 = N_4 = N_5 = \ldots = N_x = 0$

Case 'B': If $i_{max} < 3$, the definition of i_{max} suggests that

$$N_3 = N_4 = N_5 = \ldots = N_x = 0$$

Case 'C': If $N_1 < 2$, equation (2) implies

$$F + N_3 + 2N_4 + 3N_5 + \ldots + (x-2)\,N_x \leqslant 3 \quad (5)$$

Therefore i_{max} should be $\leqslant 3$.

The condition that $i_{max} < 3$, as we have already seen, results in:

$$N_3 = N_4 = N_5 = \ldots = N_x = 0.$$

If $i_{max} = 3$, it can be seen that the inequality in (5) also results in $N_3 = N_4 = N_5 = \ldots = N_x = 0$.

The final results in all the above cases are the same, so that they can be used in (2) to give

$$R_\mu\,(T/N) = F-1-N_1 = 0 \text{ or } +1 \text{ or } -1 \quad (6)$$

Further $R_\mu\,(4T/N)$ can now be written as:

$$R_\mu\,(4T/N) = S(F-4)+S(N_1-4)+2S(N_2-2) = 0,\ +1,\ -1 \quad (7)$$

where $S(K) = K$ if $K \geqslant 0$, and $= 0$ otherwise.

From equations (6) and (7) it is seen that

(*i*) $1 \leqslant F \leqslant 5$

(*ii*) $0 \leqslant N_1 \leqslant 5$

(*iii*) $0 \leqslant N_2 \leqslant 2$

so that $(N_2)_{max} = 2$. Now to find the maximum possible value of N, *ie* N_{max}, let us consider the following cases:

(*i*) If $F = 5$ then to satisfy (7), $(N_1)_{max}$ should be equal to 4. Therefore, $N_{max} = (N_1)_{max} + F_{max} + 2(N_2)_{max} = 4 + 5 + (2\times 2) = 13$.

(*ii*) Similarly if $F \leqslant 4$, then $(N_1)_{max} = 5$ and therefore $N_{max} = 5 + 4 + (2\times 2) = 13$.

Thus all distributions in F, N_1, N_3, ..., N_x have been exhausted so that the maximum integral number of bits possible in the Barker code is 13. A necessary, but not a sufficient, set of conditions that a Barker code should satisfy has thus been derived.

REFERENCES

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