

# The small-scale structure in interstellar H I: a resolvable puzzle

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## ABSTRACT

During the past decade or so, measurements of Galactic H I absorption using VLBI against extragalactic sources, as well as multi-epoch observations in pulsar directions, have detected small-scale transverse variations corresponding to tens of au at the distance of the absorbing matter. Hitherto these measurements have been interpreted as small-scale structure in the H I distribution with densities  $n_{\text{H I}} \sim 10^4\text{--}10^5 \text{ cm}^{-3}$ , orders of magnitude greater than those of the pc-scale structure. Naturally, it is difficult to imagine how such structures could exist in equilibrium with other components of the ISM.

In this paper we show that structure on all scales contributes to the differences on neighbouring lines of sight, and that the observed differences can be accounted for by a natural extension of the distribution of irregularities in the distribution of H I opacities at larger scales, using a single power law. This, in our opinion, should *put an end to the decades-long puzzle of the so-called small-scale structure* in H I and other species in the Galaxy.

**Key words:** pulsars: general – ISM: clouds – ISM: molecules – ISM: structure – radio lines: ISM.

## 1 INTRODUCTION

The warm component of the Galactic neutral atomic hydrogen (H I), studied extensively through its 21-cm line emission using single-dish measurements, shows largely uniform distribution and has revealed the large-scale structure of our Galaxy. The cold atomic component has also been probed using the 21-cm absorption observable in the spectra of *bright* continuum sources in the background. The earliest interferometric study towards the bright supernova remnant Cas A (Clark, Radhakrishnan & Wilson 1962; Clark 1965) revealed the presence of structure finer than known earlier in cold H I. Later, many aperture-synthesis observations of the Perseus-arm features detected structures down to the resolution limit (an arcminute) in these observations (Greisen 1973; Bregman et al. 1983; Schwarz et al. 1986). These and various other indications had suggested that the H I gas in our Galaxy is organized on, and maintains, a hierarchy of scales from 1 kpc to at least 1 pc. Contribution from scales much smaller than the parsec-scale was expected to be a tiny fraction of the total (see, e.g., Dickey & Lockman 1990). There were no serious difficulties in understanding this picture.

However, subsequent H I absorption studies using VLBI observations of extragalactic sources (as background sources) triggered what has remained as a puzzle for a few decades. Dieter, Welch & Romney (1976) were the first to note variations in the H I

opacity on a scale smaller than 0.16 arcsec (in the direction of 3C 147). This they interpreted as implying a cold-H I cloud size smaller than 70 au and a volume density in the cloud of  $\sim 10^5 \text{ cm}^{-3}$ . More than a decade later, Diamond et al. (1989) reported more VLBI observations on 3C 147 as well as two other extragalactic sources, supporting the conclusions of Dieter et al. They interpreted their findings as suggesting linear diameters of the absorbing clouds to be as small as  $\sim 25$  au, and correspondingly high densities. More elaborate, recent VLBI observations (Davis, Diamond & Goss 1996; Faison et al. 1998; Faison 1999) confirm some of the earlier reports of opacity variations on small transverse scales, while in some other cases find no detectable variation across different components of the background sources.

A new technique employing multi-epoch H I-absorption measurements in pulsar directions was suggested by Frail et al. (1991) as well as by Deshpande et al. (1992) independently. Based on an extensive multi-epoch study of seven pulsars (sampling various directions in the Galaxy), Frail et al. (1994) reported optical-depth changes of  $\leq 0.1$  on the small spatial scales of 5 to 100 au, and concluded that a significant fraction (10–15 per cent) of the cold H I gas is in such small-scale structure.

Naturally, there are serious difficulties about such a structure being in pressure equilibrium with the other components of the medium, given that the estimated volume density is so high, and hence about what processes would generate and help maintain such apparently commonly encountered structure. In a recent paper, Heiles (1997) has summarized the main results from these

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observations of the so-called small-scale structure in the interstellar H I (as well as from optical observations of interstellar absorption lines of Na I and Ca II). The paper also points out that existence of such a tiny-scale atomic structure would imply, under conventional interpretation, an overabundance of H<sub>2</sub> leading to very large extinction. To ease such difficulties, Heiles has proposed geometric solutions invoking structures consisting of cold, dense curved filaments or sheets (that line up along sight-lines) to explain the observed variations in the H I opacity, but with moderate values of the implied volume densities. On the other hand, Dickey & Lockman (1990), based on many arguments, conclude that while small-scale structure does exist, it is only a tiny fraction of the total H I column density along any sight-lines.

The two important interrelated questions raised by the apparently puzzling detections of the opacity variations over small spatial scales, and often asked, are the following. (a) Does the atomic medium resemble the diffuse ionized component? That is, does it have a power-law distribution of sizes? If so, is the behaviour similar over the whole range of scales probed? (b) Is the au-sized structure only peripherally related to the parsec or larger scale structure in H I? That is, does it represent a physically distinct population of structures?

In this paper we show that the observations have been misinterpreted, and that the observed small-scale structure is not at all unexpected. The observed opacity differences are consistent with a single power-law description of the distribution of H I opacities in the interstellar medium, and almost all scales contribute significantly to the observed differences. It was incorrect to assume that the observed structure must be due to high-density clouds whose longitudinal dimension is the same as the separation between the lines of sight at distances comparable with the distances of the absorbing matter.

## 2 WHAT DO WE ACTUALLY MEASURE IN THE VLBI AND MULTI-EPOCH PULSAR OBSERVATIONS?

### 2.1 Effects of the source structure and telescope filter function

The situation in the H I-opacity-variation measurements using VLBI on extragalactic sources and the multi-epoch pulsar observations<sup>1</sup> can be analysed by identifying three important ingredients that dictate what actually we would measure, namely, (1) structure of the background source, (2) structure in the absorbing gas, and (3) the spatial frequency filter function of the telescope. As for their frequency dependence (over the observing bandwidths that are usually small compared to the centre frequency), the first quantity can be assumed to be constant, the third one will vary only slightly but predictably, while the second quantity can vary considerably from channel-to-channel in frequency (or velocity).

<sup>1</sup> For further discussion, we will treat, without loss of any details, the multi-epoch pulsar measurements as equivalent to those made against a background source consisting of as many number of incoherent component sources as the number of epochs, and each component location defined by the apparent pulsar direction at the corresponding epoch. Due to the interstellar scattering, the size of the component sources may appear larger compared to that intrinsic to the pulsar radiation. Although the pulsar observations are made usually with a single dish, for the above equivalence to be complete, we consider the angular resolution to match the scatter broadened size.

In the continuum channels of the observed band (trivial case of zero-opacity), the source structure apparent to the observer is, of course, the ‘true’ structure of the background source. Whereas in the line channels, which are the ones of interest, the apparent source structure is modified by the opacity structure. The apparent structure is always a product of the two ‘true’ structures, making it, in general, appear finer (and consequently extending its visibility range to higher spatial frequencies) compared to the individual ones. Two instructive cases are (1) uniform finite opacity and VLBI-scale structure in the background source, and (2) small-scale structure in opacity and uniform brightness background source. Let us view these two situations, say, using one VLBI baseline. In the first case, even though the opacity is uniform (i.e., has no structure), the absorption will be *visible* as long as the ‘true’ source structure (which it mimics) has finite visibility at a given baseline. Thus the absorption ‘visibility’ on a given baseline does not necessarily suggest a structure in opacity on the corresponding angular (or the related spatial) scale. In the second case, the continuum visibility would be zero, but there may be finite visibility in the line channels. So any observed ‘visibility’ should be directly attributed to a structure in opacity. In fact, it provides a reasonably pure measure of the power at the corresponding spatial scale in the spectrum characterizing the distribution of opacity (particularly at small optical depths<sup>2</sup>). Green (1993) has indeed made such interferometric measurements of the 21-cm emission line to sample the associated power spectrum directly at discrete spatial frequencies, and Lazarian (1995) has presented a technique to study the underlying 3D characteristics using such measurements. We will return to this case again later.

In reality, the situation is somewhere in between the two relatively simple extremes and hence needs even more care in interpreting the measurements. An essential step then involves proper imaging of the apparent structures in continuum- and in line-channels and using the comparison to estimate opacity in the usual way. The opacity can be estimated only over the extent of the background source imaged. The opacity estimates in closely spaced directions can be compared, as is done in the studies using pulsar probes and in more recent VLBI studies (Davis et al. 1996; Faison et al. 1998; Faison 1999) that have carefully imaged the opacity distributions.

### 2.2 Expected differences in the opacity and contributing scales

Now, we ask and try to answer two crucial questions: (a) *what is the magnitude of opacity differences that we would expect to observe between a given pair of sight-lines?* and (b) *what scale(s) from the opacity distribution should be considered as contributing to the observed opacity differences between two given sight-lines?*

Let  $\tau_z(x, y)$  represent a two-dimensional distribution of opacity in the transverse coordinates  $(x, y)$  for a given velocity channel. For simplicity, let us choose the transverse spatial separation corresponding to the angular separation between a given pair of

<sup>2</sup> Although we have used absorption and opacity as analogous to each other, what the measurements respond to is the modified source structure. Also, the fractional difference in the apparent structures in two spectral channels gives the structure in fractional absorption from which the opacity is to be computed. In this context, the particular non-linear correspondence between the depth of absorption and the opacity (optical depth) should be borne in mind, particularly at large opacities.

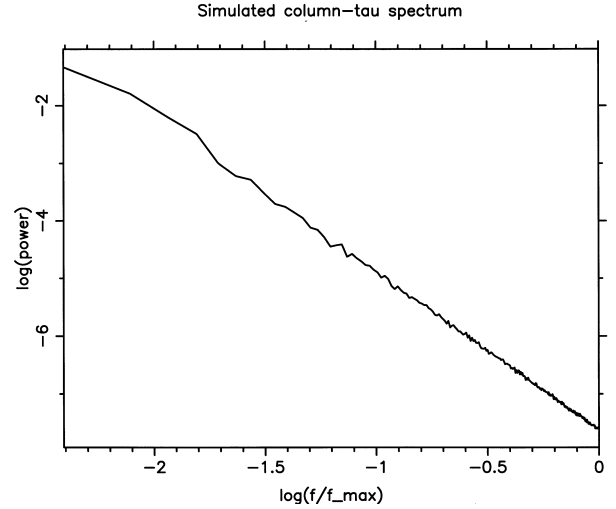
thin sight-lines at the H I-screen distance to be along the  $x$ -axis and denoted by  $x_o$ . The rms value of the opacity difference ( $\Delta\tau_v(x_o)$ ) expected to be observed is given simply by the square-root of the structure function of  $\tau_v(x,y)$  at a spatial scale of  $x_o$ . Analytically, this can be expressed as

$$[\Delta\tau_v(x_o)]^2 = S_{\tau_v}(x_o) = \langle [\tau_v(x,y) - \tau_v(x-x_o,y)]^2 \rangle$$

where  $\langle \rangle$  denotes ensemble average of the quantity over all  $(x,y)$ .

To examine the contributing scales to this opacity difference at separation  $x_o$ , we consider the power spectrum  $[P_{\tau}(f_x, f_y)]$  as a function of the spatial frequencies  $(f_x, f_y)$  (corresponding to spatial scales  $1/f_x, 1/f_y$ ) associated with the distribution  $\tau_v(x,y)$ , such that  $P_{\tau}$  is the Fourier transform of the autocorrelation function of  $\tau_v(x,y)$ . If the average power spectrum can be described as a power law, i.e.,  $\langle P_{\tau}(f_s) \rangle = P_{\tau}^o f_s^{-\alpha}$  (where  $f_s = \sqrt{f_x^2 + f_y^2}$  and such that  $\alpha$  is positive), then for  $2 < \alpha < 4$ , the structure function can also be described as a power law such that  $S_{\tau_v}(x_o) = S_{\tau_v}^o x_o^{\alpha-2}$  (Lee & Jokipii 1975). Then, from the above equation, it follows that  $\Delta\tau_v(x_o) = \Delta\tau_v^o x_o^{\frac{\alpha-2}{2}}$ .

The important thing to note here is that the quantity  $\Delta\tau_v(x_o)$ , which relates statistically to the opacity differences observers have measured, has a much slower dependence on the transverse separation ( $x_o$ ) between the sight-lines than the *amplitude* at the spatial frequency ( $1/x_o$ ) would have in the spectrum. Now, to make some quantitative estimates, we need to know the details of the power spectrum. Fortunately, such details are now available from a recent power spectrum analysis of opacity in the direction of Cas A (Deshpande, Dwarakanath & Goss 2000, hereafter DDG) using the opacity images measured by Bieging, Goss & Wilcots (1991, hereafter BGW). DDG report that the power spectrum is of a power-law nature over scales ranging from  $\sim 0.02$  to  $\sim 4$  pc, and the value of  $\alpha$  to be close to 2.75, significantly different from the Kolmogorov value of  $11/3$ . When viewed over  $0.5 \text{ km s}^{-1}$  wide velocity channels (similar to that used in the small-scale structure studies being discussed here), the rms variation in opacity across these images is about unity, making the structure function at  $x_o \sim 4$  pc (corresponding to the angular size of Cas A and the location of the absorbing cold H I) equal to about 2. From this *calibration* and the value of  $\alpha$  as suggested by the Cas A data, it follows that optical depth differences of typically 0.2 (rms) should be, in fact, expected at transverse separations of  $\sim 1000$  au. Given the power-law index of the structure function, at 100 au separation, the expected (rms) differences would drop by a factor of  $\sim 2$ . To assess further the expected opacity differences, we should examine ideally the (probability density) distribution of the expected magnitudes of differences for each value of  $x_o$ , the structure function itself representing the second moments of such distributions as a function of  $x_o$ . Unfortunately, the observations of BGW have 7-arcsec smoothing, which limits the range of scales (at the smaller scale end) that we would like to examine. Hence simulations avoiding such smoothing were considered (see Fig. 1). A more detailed discussion on this and related issues is given by DDG. A complex Hermitian symmetric spectrum was simulated with the contributions (the real and imaginary parts) at different spatial frequency  $f_s$  represented by uncorrelated random numbers following zero-mean Gaussian statistics having variance matched to the  $f_s^{-\alpha}$  power law. Such a spectrum over the  $512 \times 512$  matrix (in the 2D case, and a  $2^{18}$  point array in the 1D case) was Fourier-transformed to obtain a simulated  $\tau$ -distribution, and suitably scaled to have an rms of unity. The spatial extent of the distribution is assumed to correspond to  $\sim 4$  pc, consistent with the

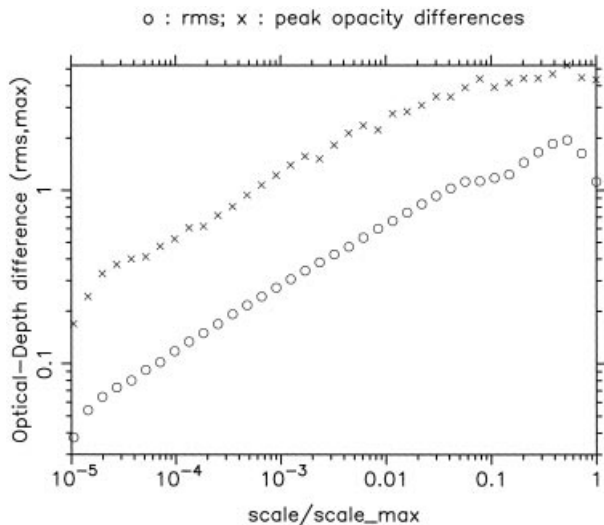


**Figure 1.** An azimuthally averaged power spectral version of a simulated 2D spectrum of opacity distribution with  $\alpha = 2.75$ , as suggested by the data in the Cas A direction (DDG). The y-axis is calibrated to indicate the power contributed by each spatial frequency to the  $\tau$  distribution.

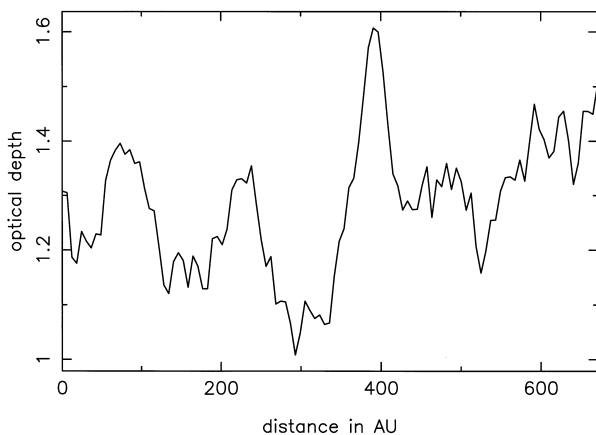
data in the Cas A direction. Using the simulated distribution of opacity (similar to that observed by BGW in the Cas A direction), we have estimated, as a function of the spatial separation, two quantities indicative of the related (one-sided) probability distribution of the absolute differences. These are the rms value (i.e., the square-root of the structure function) and the maximum value of the magnitudes of opacity differences. The result of this estimation is shown in Fig. 2, and it is clear that occasionally the opacity difference can be nearly an order of magnitude higher than the rms values (related to the structure function). This makes the *detected* differences hardly surprising, and therefore they should be treated as only consistent with a single power-law spectral description of the opacity distribution (e.g., as derived in the Cas A direction). Fig. 3 shows a small section of the simulated distribution as an example of the expected variation on small transverse scales. A detailed examination of several realizations of such distributions shows that the optical-depth variations of 0.2–0.4 across the relatively small transverse separation (50–100 au) as seen in Fig. 3 are not very rare, but would be expected with  $\sim 10$  per cent probability.

We would like to re-emphasize here that in opacity difference measurements with a transverse separation  $x_o$ , one is *probing* a point of the structure function and *not* a point in the spectrum of opacity distribution. In general, all scales in the spectrum contribute to such a measurement, except, of course, the uniform component and, ironically, the scales  $x_o$  and its integral submultiples that contribute nothing. In detail, the structure function value at  $x_o$  is a result of a sum of the contributions at *all* spatial frequencies in the power spectrum after modulation by  $1 - \cos(2\pi f_x x_o)$ , and considering an ensemble average of such sums<sup>3</sup> corresponding to all possible orientations and locations of vectors of length  $x_o$  (along with  $f_x, f_y$  axes). This modulation is simply a result of the two-point difference measurement. The modulating function has its first peak at  $f_x = 1/(2x_o)$  and at odd multiples of it from then on. When the power spectrum is red (i.e.,

<sup>3</sup>More formal analytical expressions involving Bessel functions are commonly used (see, e.g., Cordes, Weisberg & Boriakoff (1985).



**Figure 2.** Two quantities (related to the magnitude of differences expected as a function of the transverse scale probed) estimated from 1D simulations of optical-depth ( $\tau$ ) distribution are shown. The trends with symbols  $\circ, \times$  indicate respectively the rms (i.e., square-root of the structure function) and the typical peak difference magnitudes. The 1D distribution of opacity over 5 orders of magnitude in scales ( $1:2^{18}$ ) was obtained from a simulated spectrum in 1D with a power-law index of 1.75 (corresponding to a 2D equivalent index of 2.75). Due to the computational requirements being too high, the 2D case was not attempted on the same scale-range. However, the 2D simulation referred to in Fig. 1 was used to confirm the consistency of the results over a range of longer scales spanning two orders of magnitude. The y-axis values here are calibrated such that the maximum scale may be equated to 4 pc (see text for details). The  $\tau$  distribution used was obtained by Fourier-transforming a simulated spectrum in the spatial frequency domain. The repetitive nature implicit in the Fourier transforms can affect estimation on scales close to the transform length; hence the above estimates are made only for scales that are less than half of the transform length.



**Figure 3.** A sample section from our (1D) simulated opacity distribution is shown. The axis scales are calibrated consistent with the observations in the Cas A direction (corresponding to the scale range at longer-scales).

$\alpha$  is positive) and steep, even the highly attenuated contributions (due to the modulation) from the low spatial frequencies (long scales) can, and do indeed, dominate in the net contribution, making an equivalent scale being probed much larger than  $x_o$ . One may estimate the equivalent scale-size by considering a weighted average over all scales, where weights are determined by the

associated values of the modulated power spectrum. In a simple estimation, for example, considering a 2D spectrum over  $|f_s| \leq 1/x_o$ , we find the equivalent scale to be nearly one order of magnitude longer than  $x_o$ . A proper estimation should include the full spectrum. However, since such an equivalent scale has little physical meaning, we have not pursued such an estimation further. In summary, the earlier interpretations of the opacity difference observed at a transverse separation  $x_o$  as being a result of a cloudlet of the size same as the separation  $x_o$  appear, to us, erroneous.

### 2.3 The over-dense (and over-pressured) H I cloudlets?

One of the major mysteries which owes its origin to the above-mentioned misinterpretation is that of the observed opacity changes combined with an *assumed longitudinal scale* implying highly over-dense ( $n_{\text{H}} \sim 10^4\text{--}10^5 \text{ cm}^{-3}$ ) and, consequently, over-pressured cloudlets. Even if the observed opacity changes were to be accepted at their face value, their implication needs to be reinterpreted, since (a) as emphasized in the previous section, the observed variation is contributed by the whole range of scales and not by just one particular scale same as the transverse separation probed, and (b) the measured value can not be directly associated with a particular longitudinal scale. These considerations are applicable not just to the ‘two-point comparisons’ (i.e., variations expected across a 1D cut), but also to the features observed in the 2D images of the opacity. The variations like those apparent in our 1D simulation (Fig. 3) would be equally probable in an equivalent 2D image where they would appear as one-dimensional features. The *edges* or the elongated features apparent in *only some* of the 2D opacity images (Davis et al. 1996; Faison et al. 1998; Faison 1999) are therefore not at all surprising, whereas an opacity variation feature that is narrow in both dimensions should be considered relatively rare.

If one wants to estimate, using the data, the properties of the small scales in the H I distribution, then the following is one correct way to proceed. One treats the measured value of opacity difference or of the associated H I column-density change as just an estimate of the associated structure function at the probed transverse separation. From this, and with some knowledge of the spectrum (or the structure function itself), it is possible to estimate the implied power in the same scale as the transverse separation probed. Now, this power (from the power spectrum) or the amplitude of the *ripple* corresponding to that scale, can justifiably be interpreted in terms of the associated *fluctuating* optical depth or density. More formally, with the measurement giving an estimate of (square-root of) the structure function  $S_{\tau_o}(x_o)$ , the optical depth variation associated with the scale  $x_o$  is to be estimated as the (square-root of) power spectral contribution  $\langle P_{\tau_o}(f_s = \frac{1}{x_o}) \rangle$ , a value significantly smaller than the former. For example, using the relevant values observed in the Cas A direction, a observed change of 0.2 in the optical depth between two sight-lines with a transverse separation of about 1000 au would imply a  $\tau$  fluctuation on the scale of  $\sim 1000$  au to be about  $10^{-5}$ . Assuming a velocity-channel width of  $0.5 \text{ km s}^{-1}$  and a spin temperature of about 100 K, the contribution (or deficit) in the volume density from that scale would be less than  $0.1 \text{ cm}^{-3}$ , very much smaller than what the earlier interpretations would suggest! This value would be even smaller when the possible statistical enhancement due to the finite *thickness* of the medium is accounted for. For example, if the contribution is from more

than one, say,  $N$  layers along the sight-line, then the corresponding contribution to volume density would be  $\sqrt{N}$  times smaller. It follows that the well-understood parsec scale would contribute an H I volume density of  $\sim 1 \text{ cm}^{-3}$  or smaller, in good agreement with relevant observations. Of course, the actual density at a given spatial location would be a sum total of such contributions also from a hierarchy of scales longer and shorter than  $x_o$ . It is easy to show that such contributions to the volume density at a given spatial point would follow a power law as a function of the scale-size. For example, in the Cas A direction, it may be expressible as  $\Delta n_H(x_o) \sim A x_o^{\frac{\alpha-\beta}{2}}$ , where  $x_o$  is expressed in parsec,  $A$  is a constant close to unity, and  $\beta$  is between 1 to 2 depending on whether our sight-line encounters contributions many (of the order of  $1/x_o$ , with  $x_o$  in pc) or just one layer of scale  $x_o$ , respectively. Even in the worst case, i.e., when  $\beta = 2$ , the rms fluctuations in the volume density, estimated on the parsec scales, would be below  $100 \text{ cm}^{-3}$ . The more likely value of  $\beta$  in the sub-parsec regime of scales is closer to unity, and then the volume-density fluctuations would be moderate, with an rms of  $\sim 10 \text{ cm}^{-3}$ . The detailed quantitative picture may differ between different directions in the Galaxy by a factor of 10 or less, and would hopefully be revealed by future suitable observations and a careful interpretation.

To conclude, there appears to be no compelling observational evidence for the so called ‘highly over-dense small-scale structure’, and even the observations probing small transverse scales are not at all inconsistent with what we would expect by extrapolating from the better studied range of large and moderate spatial scales in the cold neutral medium.

## 2.4 The reported measurements versus uncertainties

So far, we have taken the reported observations (estimations) of the opacity differences at their face value. However, certain uncertainties inherent to the measurements are worth noting. As we have discussed earlier, any observed line-visibility in VLBI observation results from structure in both the background source and the opacity as seen by a given interferometer baseline. The interpretation can become more complicated when only a limited number of baselines are used, and can even be misleading if any changes in the relative orientation of the baselines (in addition to its projected length) are not accounted for. As was already pointed out (Radhakrishnan & Deshpande 1990, unpublished), the uniqueness of the interpretation of Diamond et al. (1989) becomes debatable on these grounds. Later similar observations, however, have resolved the possible ambiguities by actually mapping the opacity distribution across the extragalactic sources, and hence the estimated opacity differences can be considered reliable.

One general but important aspect, addressed earlier by Deshpande et al. (1992), is that of the contribution from the H I emission to the measurement uncertainty (relevant to both the interferometric and pulsar probes). Even if a VLBI baseline resolves out the large-scale H I emission, each of the elements of the interferometer does respond to the H I emission contribution and the equivalent system temperature of the interferometer can be significantly higher in the corresponding spectral channels compared to that for only continuum contribution. As for the single-dish observation, the H I emission contribution to the estimation uncertainty is rather obvious. The measurements in the pulsar directions (Deshpande et al. 1992; Frail et al. 1994), as well as more recent interferometric measurements (Davis et al.

1996; Faison et al. 1998; Faison 1999), do explicitly take into account the H I emission contribution. Interestingly, the system temperatures in the line channels also depend, in principle, on the optical depths at the corresponding frequencies. This can be a significant effect when the background sources make dominant contribution to the system temperature (in continuum channels). In such cases, higher optical depths result in significantly smaller system temperature in the corresponding spectral channels.

## 3 DISCUSSION

In this paper we have addressed some aspects related to the observation, analysis and interpretation of opacity differences across small (sub-parsec) transverse scales. Although the considerations we raise are simple-minded, they appear to have serious implications that argue against certain interpretations, such as those suggesting the so called small-scale structure of highly over-dense cold H I cloudlets as being responsible for the observed opacity differences.

We have emphasized the need for recognizing the nature of the actual quantity one measures through the probes that have been employed, and the fact that almost all scales contribute to the measured opacity differences. However, so far, the existing studies have misinterpreted the observed opacity difference between two sight-lines of the associated transverse separation as due to opacity structure on *that scale in three dimensions*, and therefore it is not surprising that the implied volume densities appear extraordinarily large. As illustrated in the earlier sections of this paper, the observations appear consistent (both qualitatively and quantitatively) with a single-power law description of the H I distribution over the entire relevant range of scales, and do not imply any mysterious structure. In our simple picture, the spectral behaviour studied up to moderate scales (e.g.,  $\sim 0.1$ -pc scale as by DDG) is assumed to extend with the same power-law index to the 10-au scale. We are aware of the study by Croviser, Dickey & Kazes (1985) that claimed a cut-off in structures below 0.2 pc. Considering (1) that their method in which signatures of small-scale structure were expected to show up close to the zero-velocity, (2) that the major absorption-line features (differences in which were probed) were well away from zero-velocity, and (3) that the velocity resolution was coarse, we think that their study suffers from serious selection against structures smaller than about 0.5 pc.

The recent suggestion by Heiles (1997) did for the first time distinguish between the transverse scale probed and the longitudinal scale for estimating the volume density, but by invoking ‘thin’ structures such as filaments and sheets that should preferentially align along sight-lines. As we have discussed, the distinction between the transverse scale probed and the *corresponding equivalent* longitudinal scale appears to be rather simple, and more inherent to the basic measurement than any geometrical shapes would imply. While any anisotropy in structures is likely to increase (statistically) the expected magnitude of the opacity differences (and elongation factors up to  $\sim 2$  may be common), it does not appear necessary for understanding the available observations. In any case, the structures suggested by Heiles may be difficult to produce and maintain, particularly in the certain alignment that they need to have with *our* sight-lines.

The velocity spread associated with turbulence can produce additional corrugations in the opacity distribution when viewed

over velocity channels narrow compared to the spread due to turbulence (as is normally the case). It would be instructive to assess such an effect quantitatively.

It may be important to note that the radio/optical observations of opacity changes in other species [e.g., (a)  $\text{H}_2\text{CO}$  and OH by Moore & Marscher 1995; (b) Na I and Ca II by Mayer & Blades 1996 and Watson & Meyer 1996] should be interpreted in much the same way as we have discussed in the context of HI, and then a similar puzzle these observations appeared to have raised should stand resolved. The discussion in this paper is relevant also to the dispersion measure changes detected in pulsar directions, and in general to any situation involving a similar probe.

Although in the course of our discussion we have drawn upon the HI data in the Cas A direction as a useful example, we do recognize the possibility that the atomic medium in other directions may have quite different column densities as well as power-spectral descriptions from that in the Cas A direction. However, the main issues we have addressed are of a more general nature, and are not crucially based on the quantitative estimates from the Cas A data. The HI emission-line study by Green (1993) indicates that the power spectra (and structure functions) may be less steep in some directions than that for cold HI in Cas A direction. A recent study of HI in the Small Magellanic Cloud suggests a relatively steeper power spectrum (Stanimirovic et al. 1999). If the power spectra of scale distribution in cold HI also have a similar variation, then in some directions we should expect even larger opacity differences on small transverse separations (much more than even those seen in recent VLBI studies). For example, a change of 0.1 in  $\alpha$  would increase the *expected* opacity changes on the au scales by a factor of  $\sim 2$ .

Further investigations should benefit from using the available data for a systematic estimation of the structure function associated with the opacity distribution over the relevant range of transverse scales. HI absorption measurements on moderate size, bright background sources should help extending the direct power spectral analysis (e.g., in the case of Cas A) to intermediate and small transverse scales.

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