Quenched scramblings and symmetry

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Abstract

We show that under the validity of the eigenstate thermalization hypothesis, the post-quench, out-of-time-ordered correlator of suitably chosen operators, at late times, remembers whether the original state breaks the symmetry or not upto entropic corrections.

Let us consider an Hamiltonian H_0 which is suddenly changed to H at time $t = 0$. Let us further assume that at $t = 0$ the quantum state of the system is $|\psi_0\rangle$, which maybe taken to be some eigenstate of H_0. We are interested in the post-quench scrambling, for which the relevant quantity is the in-in out of time ordered correlator:

$$F(t) = \langle\psi_0|O_1 O_2(t) O_3 O_4(t)|\psi_0\rangle. \tag{1}$$

As the Hamiltonian has changed, operator evolutions are now dictated by: $O(t) = e^{iHt}Oe^{-iHt}$. We are interested here in the late time-averaged quantity:

$$F := \lim_{T\to\infty} \frac{1}{T}\int_0^T dt F(t). \tag{2}$$

Using completeness of the energy eigenbasis of the final Hamiltonian, we decompose $|\psi_0\rangle = \sum_l c_l|l\rangle$ and inserting identities are able to write:

$$F(t) = \sum_{m,n,j,k,l} c_l^* c_m (O_1)_{lk}(O_2)_{kj}(O_3)_{jn}(O_4)_{nm} \exp\left[it(E_n - E_m + E_k - E_j)\right]. \tag{3}$$

In the above equation we have also used the notation: $O_{lk} = \langle l|O|k\rangle$. Next, if we do the long time averaging, the leading non-zero contributions can come from two different cases : *(1)* $E_n = E_m, E_k = E_j$ and *(2)* $E_n = E_j, E_m = E_k$. We may use these cases to sum over two sets of indices. In the first case we obtain:

$$F_1(t) = \sum_{n,k,l} c_l^* c_n (O_1)_{lk}(O_2)_{kk}(O_3)_{kn}(O_4)_{nn}. \tag{4}$$

For the second case we get:

$$F_2(t) = \sum_{n,k,l} c_l^* c_k (O_1)_{lk}(O_2)_{kn}(O_3)_{nn}(O_4)_{nk}. \tag{5}$$

At this stage we invoke the eigenstate thermalization hypothesis (ETH), which is succinctly captured in the following statement about matrix elements of local operator O [1]:

$$\langle E_a|O|E_b\rangle = O_{ab} = \delta_{ab} O_{th}(E) + e^{-S(E)/2} f(E,\omega) R_{ab}. \tag{6}$$

The diagonal part is a smooth function of the average energy $E = (E_a + E_b)/2$ while the entropically suppressed off-diagonal part is proportional to a random matrix R_{ab} and another smooth function f which also depends on the energy difference, $\omega = E_a - E_b$. It is straightforward to see that F_2 is entropically suppressed in comparison to F_1 owing to presence of more off-diagonal entries. By the ETH, the diagonal entries are also independent of the state indices ($O_{aa} \sim \mathcal{O}(1)$), and hence to a good approximation,

$$F \simeq \sum_{n,k,l} c_l^* c_n (O_1)_{lk}(O_3)_{kn} = \langle\psi_0|O_1 O_3|\psi_0\rangle. \tag{7}$$

For usual OTOC measurements O_1 and O_3 are taken to be the same operator O, but generically in different locations. If it is possible to define an order parameter via $\langle O\rangle$ in the original Hamiltonian, then we see that the late time averaged OTOC clearly distinguishes between the ordered and disordered nature of $|\psi_0\rangle$. In the symmetry broken phase we thus expect a non-zero F whereas in the disordered phase $F \sim 0$ upto entropic corrections of the ETH. This is consistent with the findings of [2].

References

[1] M. Srednicki, "The approach to thermal equilibrium in quantized chaotic systems," *Journal of Physics A: Mathematical and General* **32** (Jan, 1999) 1163–1175, http://dx.doi.org/10.1088/0305-4470/32/7/007.

[2] M. Heyl, F. Pollmann, and B. Dóra, "Detecting Equilibrium and Dynamical Quantum Phase Transitions in Ising Chains via Out-of-Time-Ordered Correlators," *Physical Review Letters* **121** (Jul, 2018) http://dx.doi.org/10.1103/PhysRevLett.121.016801.