Problem Set - 01

08/01/2020

1. Show that the group of integers under addition modulo n is isomorphic to $\mathbb{Z}_n$.

2. The set of the functions form a group under product rule,

$$(f_i, f_j) \mapsto f_i \circ f_j.$$

,

$$\begin{aligned} f_1(z) &= z \qquad f_2(z) = \frac{1}{1-z} \qquad f_3(z) = \frac{z-1}{z}, \\ f_4(z) &= \frac{1}{z} \qquad f_5(z) = 1-z \qquad f_6(z) = \frac{z}{z-1}. \end{aligned}$$

   Fill in the multiplication table.

3. Show that for a group with even order, there is at least one element other than identity which squares to identity.

4. A Coxeter group is defined by, $a_i^2 = 1$ and $(a_i a_j)^{n_{ij}} = 1$ for $n_{ij} \geq 2$ $\forall i, j$. Show that for $n_{ij} = 2$, $a_i a_j = a_j a_i$.

5. Show that in a abelian group, every element forms its own conjugacy / equivalence class.