Problem Set - 06

10/05/2020

The problems for this assignment are from Mathews and Walker (MW).

1. **MW:8-1** Find the lowest frequency of oscillation of acoustic waves in a (3-spatial dimensional) hollow sphere of radius R. The PDE obeyed by the wave is,
$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}.$$
The boundary condition is
$$\left.\frac{\partial\psi}{\partial r}\right|_{r=R} = 0.$$

2. **MW:8-29** Find the lowest three values of k^2 admitting non-trivial solution to the Helmholtz PDE in 2 dimensions:
$$\nabla^2\phi + k^2\phi = 0.$$
The geometry is in Fig. 1.

3. **MW:8-3** A solid sphere of radius R is kept initially at temperature $T = 0$. At time $t = 0$ it is immersed in a liquid which is at temperature T_0. Find the distribution of temperature $T(r, t)$ inside the sphere at a later time. Assume κ to be the thermal conductivity.

4. **MW:8-4** Find the lowest three eigenvalues of the time-independent Schrődinger's equation inside a cylindrical box (radius a and height h, with $h \simeq q$). The wavefunction ψ is zero at the walls of the cylinder.

5. **MW:8-15** The temperature in a homogenous sphere of radius a obeys,

$$\nabla^2 T = \frac{1}{\kappa}\frac{\partial T}{\partial t}.$$

The boundary condition is on the surface of the sphere is in Fig. 2. Find $T(t)$ at the centre of the sphere.

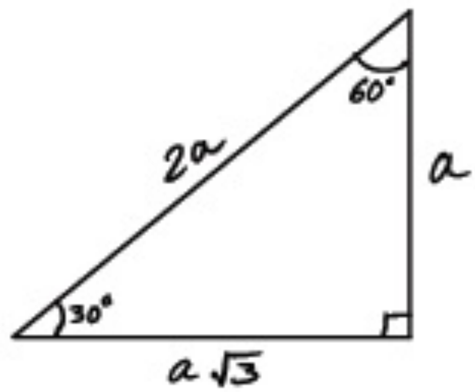

Figure 1: The boundary condition is $\phi = 0$ on the perimeter of the triangle.

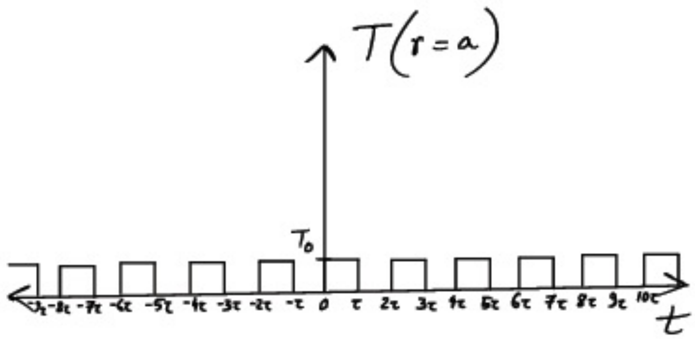

Figure 2: The boundary condition is the above temperature distribution on the surface of the sphere.