On the Spatial Performance of Users in Indoor VLC Networks with Multiple Reflections

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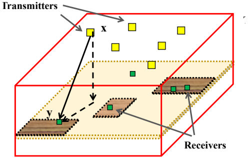
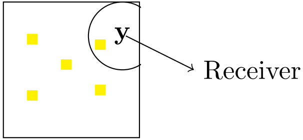
Introduction

- Indoor VLC network for high data rates.
- Past literature: transmitter process is assumed to be stationary. This may work for
  - Users at the center of the room
  - Or infinite networks.



- In indoor case with non-centered user, performance depends on its location
  - Non-stationarity of transmitter process
  - Reflections via wall

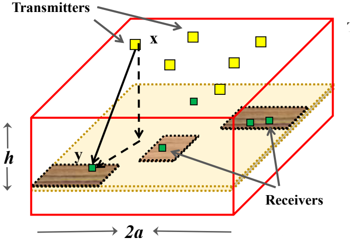


Contributions

- Stochastic geometric based framework for downlink indoor VLC network for a typical arbitrarily located user
- SINR and rate distribution
- Modeling reflections of kth order
- Characterize the impact of location on the user's performance

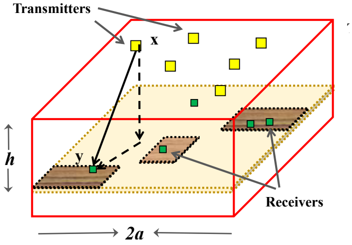
System Model

- Downlink VLC network with optical attocells (OAs) in a square room with height h and length $2a$.
- 2D floor denoted by set $\mathcal{S}(0, a)$
  $\mathcal{S}(\mathbf{x}, a)$: a square at center $\mathbf{x}$ with sides $2a$.
- OAs:
  - Homogeneous Poisson point process (PPP) at roof.
  - $\Phi = \{\mathbf{x}_i \in \mathbb{R}^2\}$ with a density of λ
  - ith OA's location is given as $(\mathbf{x}_i, h)$
- OAs with LEDs oriented vertically downwards.



System Model (Contd.)

- UEs
  - Homogeneous PPP at the desktop level with density λ_{u} across the room.
  - A typical user is located uniformly in the room: $(\mathbf{y}, 0)$.
- Here, $\mathbf{y} = (y_1, y_2) \in \mathbb{R}^2$ such that $y_1, y_2 \sim \mathrm{U}(-a, a)$.

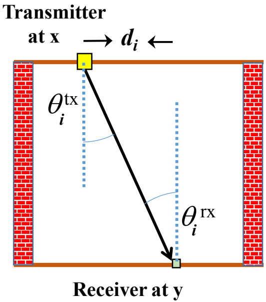


System Model (Contd.)

- Lambertian emission radiation profile.
- Channel gain between the user and ith OA

$$G_i = (m+1)\xi \frac{A_{pd}}{2\pi d_i^2} \cos^m(\theta_i^{\mathrm{tx}}) \cos(\theta_i^{\mathrm{rx}}) G_{\mathrm{cf}}(\theta_i^{\mathrm{rx}})$$

d_i	distance of the user to ith OA.
m	the order of Lambertian emission
θ_i^{tx}	ith OA's irradiance angle wrt the user.
θ_i^{rx}	ith OA's incidence angle wrt the user.
A_{pd}	the detection area of the photodetector (PD)
$G_{\mathrm{cf}}(\theta_i^{\mathrm{rx}})$	gain of rx optical filter and concentrator (constant).
ξ	the average responsivity of PD.



$$\cos(\theta_i^{\mathrm{tx}}) = \cos(\theta_i^{\mathrm{rx}}) = h/\sqrt{h^2 + d_i^2}$$

System Model (Contd.)

Receiver power from the ith transmitter is

$$P_i = P_{\text{tx}} \alpha^2 \ell(\mathbf{x}, \mathbf{y})$$

Here,

- Pathloss: $\ell(\mathbf{x}, \mathbf{y}) = \left(|\mathbf{x} - \mathbf{y}|^2 + h^2\right)^{-\beta}$.
- Transmit power: P_{tx}
- Gain: $\alpha = (m+1) A_{pd} \xi G_{\text{cf}} h^{m+1} / (2\pi)$
- $\beta = m + 3$

SINR from the ith OA located at $\mathbf{x}_i$ at the user

$$\text{SINR}_i(\mathbf{y}) = \frac{\ell(\mathbf{x}_i, \mathbf{y})}{I(\Phi \setminus \mathbf{x}_i) + \sigma^2}$$

Interference: $I(\Phi \setminus \mathbf{x}_i) \quad = \sum_{\mathbf{x}_j \in \Phi \setminus \mathbf{x}_i} \ell(\mathbf{x}_j, \mathbf{y})$.

Effective noise: $\sigma^2 = N_0 B_f / (\alpha^2 P_{\text{tx}})$

Association based on maximum received power

$$\mathbf{x}_0 = \arg\max_{\mathbf{x}_i} \ell(\mathbf{x}_i, \mathbf{y}).$$

System Model (Contd.)

For user located at $\mathbf{y}$: SINR coverage probability is

$$\mathrm{P_c}(\tau, \mathbf{y}) = \mathbb{P}\left[\mathrm{SINR}_0(\mathbf{y}) > \tau\right].$$

For typical user: SINR coverage probability is

$$\begin{aligned}\mathrm{P_c}(\tau) &= \mathbb{P}\left[\mathrm{SINR}_0 > \tau\right] \\ &= \frac{\int_{\mathcal{S}(0,a)} \mathrm{P_c}(\tau, \mathbf{y})\mathrm{d}\mathbf{y}}{|\mathcal{S}(0,a)|} \\ &= \frac{1}{4a^2} \int_{\mathcal{S}(0,a)} \mathrm{P_c}(\tau, \mathbf{y})\mathrm{d}\mathbf{y}\end{aligned}$$

Approach

Compute the distribution of r (distance of serving BS from UE)

Compute the Laplace transform of interference conditioned on r

Compute distribution of I and then $\mathrm{P_c}$

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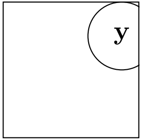

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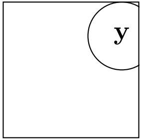

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$\mathbb{1}\left(\mathrm{SINR}_0 > \tau\right) = \sum_i \mathbb{1}\left(\mathrm{SINR}_i > \tau\right)$ for $\tau > 1$

Campbell-Mecke Theorem:
$\mathbb{E}\left[\sum_{\mathbf{x}_i \in \Phi} f(\mathbf{x}_i, \Phi \setminus \mathbf{x}_i)\right] = \int \lambda \mathbb{E}\left[f(\mathbf{x}), \Phi\right] \mathrm{d}\mathbf{x}$

Doesn't require conditional interference. Compute LT of I. Compute $\mathrm{P_c}$.

Coverage Probability

$$\begin{aligned} \mathrm{P_c}(\tau, \mathbf{y}) &= \lambda \int_{\mathcal{S}(0,a)} \mathbb{P}\left[\frac{\ell(\mathbf{x},\mathbf{y})}{I(\Phi)+\sigma^2} > \tau\right] \mathrm{d}\mathbf{x} \\ &= \lambda \int_{\mathsf{A}_{\mathbf{y}}} \mathbb{P}\left[I < \tau^{-1}\left(|\mathbf{x}|^2 + h^2\right)^{-\beta} - \sigma^2\right] \mathrm{d}\mathbf{x}. \end{aligned} \tag{1}$$

$$\mathsf{A}_{\mathbf{y}} = \mathcal{S}(-\mathbf{y}, a) \cap \mathcal{B}(0, a_{\mathrm{D}})$$

$$\downarrow$$

represents that APs can only be located within a circular range
this is due to the condition SNR$> \tau$ due to absence of fading

Interference Distribution

Gill Pelaez inversion Lemma:

$$\mathbb{P}\left[I < s\right] = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}\mathsf{Im}\left[e^{-jts}\mathcal{L}_I(-jt)\right]\mathrm{d}t$$

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Laplace transform of the sum interference I

$$\mathcal{L}_I\left(s\right) = \exp\left(-\lambda\int_{\mathcal{S}(-\mathbf{y},a)}\left(1-\exp\left[\frac{-s}{\left(|z|^2+h^2\right)^{\beta}}\right]\right)\mathrm{d}z\right). \tag{2}$$

Coverage Probability

Theorem: The probability of SINR coverage of a receiver located at location $\mathbf{y}$ in a indoor VLC network is given as

$$\mathrm{P_c}(\tau, \mathbf{y}) = \frac{\lambda|\mathsf{A}_{\mathbf{y}}|}{2} - \frac{\lambda}{\pi}\int_0^{\infty} \frac{1}{t}\mathrm{Im}\left[e^{jt\sigma^2}\mathcal{F}(jt\tau^{-1}, \mathsf{A}_{\mathbf{y}})\mathcal{K}(jt, \mathcal{S}(-\mathbf{y}, a))\right]\mathrm{d}t$$

where $\mathcal{F}(.)$ and $\mathcal{K}(.)$ are given as

$$\mathcal{F}(s, \mathsf{A}) = \int_{\mathsf{A}} e^{-s\left(|\mathbf{x}|^2+h^2\right)^{-\beta}}\mathrm{d}\mathbf{x}$$

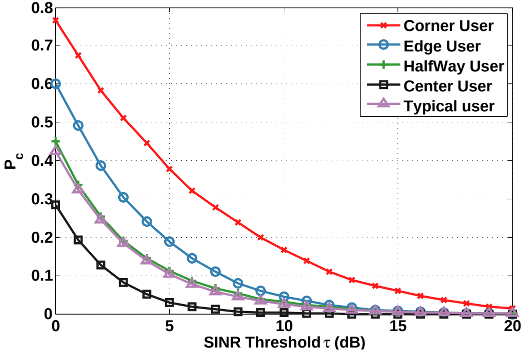
$$\mathcal{K}(s, \mathsf{A}) = \exp\left(-\lambda\int_{\mathsf{A}}\left(1 - \exp\left[\frac{s}{(|z|^2 + h^2)^{\beta}}\right]\right)\mathrm{dz}\right).$$

Location of a User Plays an Important Role!

Params	Value	Params	Value
Ψ_{HALF}	60^{o}	m	1
$N_0 B_f$	-117 dBm	A_{pd}	.01 m^2
ξ	0.4 A/W	G_{cf}	2.25
P_{tx}	30 dBm	a	9 m
h	3.5 m	$(\lambda, \lambda_{\text{u}})$	$(.1,\ .5)/m^2$

Four user locations

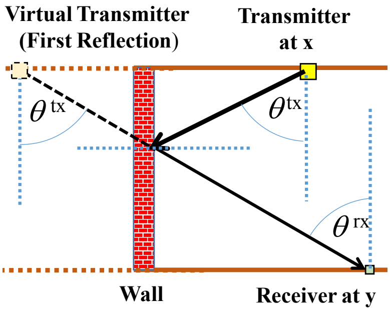
Corner ($L_1 = [a, a]$)
Edge ($L_2 = [a, 0]$)
Halfway ($L_3 = [\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}]$)
Center ($L_4 = [0, 0]$)

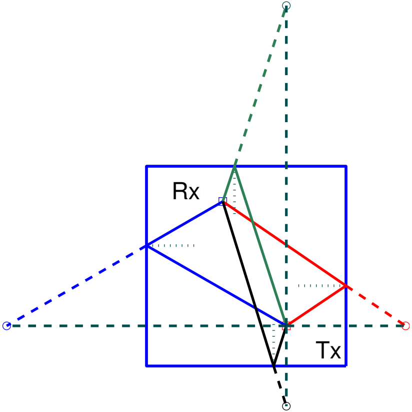


Probability of SINR coverage

Modeling Reflections

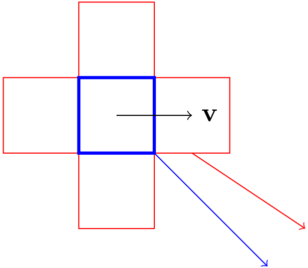
First Order Reflections





Four reflections due to four walls

First Order Reflections



Space of image
$\mathcal{S}(0, a) \oplus \mathbf{v}$

Room $\mathcal{S}(0, a)$

$$\mathsf{F}_{\mathrm{I}} = \mathcal{S}(0, a) \oplus 2a\mathsf{G}_1$$

$$\mathsf{G}_1 = \{(\pm 1, 0), (0, \pm 1)\}$$

First reflection images of transmitters form a PPP Φ_1 with density λ in the area F_{I}

Received power at $\mathbf{y}$ from a tx image $\mathbf{x}$:

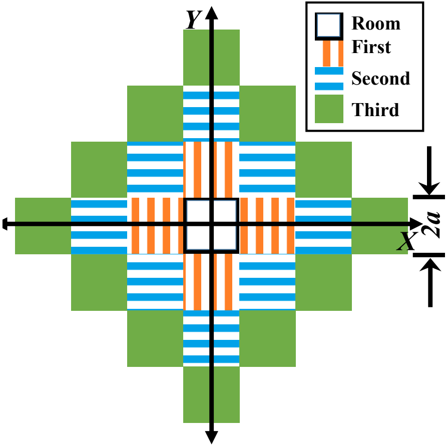
$$\ell(\mathbf{x}, \mathbf{y}) = \eta(|\mathbf{x} - \mathbf{y}|^2 + h^2)^{-\beta}, \;\; \mathbf{x} \in \mathsf{F}_{\mathrm{I}}$$

η: reflection loss coefficient

Combining the path-loss for real and image transmitters:

$$\ell(\mathbf{x}, \mathbf{y}) = \begin{cases} (|\mathbf{x} - \mathbf{y}|^2 + h^2)^{-\beta}, & \mathbf{x} \in \mathcal{S}(0, a) \\ \eta(|\mathbf{x} - \mathbf{y}|^2 + h^2)^{-\beta}, & \mathbf{x} \in \mathsf{F}_{\mathrm{I}} \end{cases} .$$

kth Order Reflections



kth reflection images of transmitters form a PPP Φ_k in the area F_k with η^k reflection loss.

$$\mathsf{F}_\mathrm{k} = \mathcal{S}(0,a) \oplus 2ka\mathsf{G}_k$$

$$\mathsf{G}_k = \left\{\left(\pm\tfrac{k_1}{k}, \pm\tfrac{k_2}{k}\right) : k_1 + k_2 = k, k_1 \in \mathbb{N}\right\}$$

$$\ell(\mathbf{x},\mathbf{y}) =$$
$$\begin{cases} \left(|\mathbf{x}-\mathbf{y}|^2 + h^2\right)^{-\beta}, & \text{if } \mathbf{x} \in \mathcal{S}(0,a) \\ \eta^k\left(|\mathbf{x}-\mathbf{y}|^2 + h^2\right)^{-\beta}, & \text{if } \mathbf{x} \in \mathcal{S}(0,a) \oplus 2ak\mathsf{G}_k. \end{cases}$$

Note that all sets F_k are mutually disjoint and cover whole space $\mathbb{R}^2$.

Coverage Probability with Reflections

SINR coverage with maximum Kth order reflections

$$\mathrm{P_c}(\tau, \mathbf{y}) = \frac{\lambda |\mathsf{B}_{\mathbf{y}}|}{2} - \frac{\lambda}{\pi} \int_0^{\infty} \frac{1}{t} \mathrm{Im} \left[e^{jt\sigma^2} \mathcal{F}_M(jt/\tau, \mathbf{y}) \mathcal{K}_M(jt, \mathbf{y}) \right] \mathrm{d}t$$

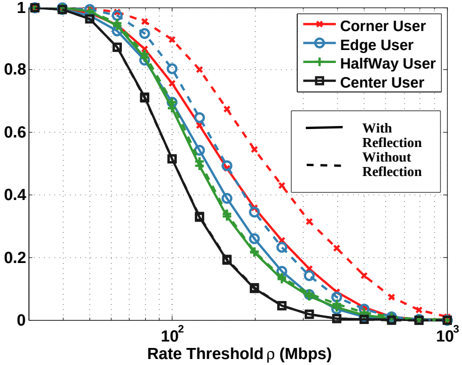
$$\text{where } \mathcal{F}_M(s, \mathbf{z}) = \sum_{k=0}^{K} \int_{\mathsf{B}_{k\mathbf{z}}} e^{-s\eta^k \left(|\mathbf{x}|^2 + h^2\right)^{-\beta}} \mathrm{d}x$$

$$\mathcal{K}_M(s, \mathbf{z}) = \exp\left(-\lambda \sum_{k=0}^{K} \int_{\mathsf{F}_{\mathrm{k}}(-\mathbf{z}, a)} \left(1 - e^{\frac{s\eta^k}{(|\mathbf{z}|^2 + h^2)^{\beta}}} \right) \mathrm{d}\mathbf{z} \right)$$

$$\text{with } \mathsf{B}_{\mathbf{y}} = \cup_{k=0}^{\infty} \left(\mathsf{F}_{\mathrm{k}}(-\mathbf{y}, a) \cap \mathcal{B}\left(0, b_{\mathrm{D}}^{k}\right) \right).$$

Reflections Impacts Corner and Edge the Most!

- Median rate of the corner user decreased by 25% due to presence of reflections.
- Images rarely act as serving OAs, but they add significantly to interference.
- Center and halfway users are not impacted significantly from reflections.



Probability of Rate coverage

Conclusions

- Developed a framework to analyze an indoor VLC network
    - for a typical user located arbitrarily, and
    - considering multiple reflections from the walls.
- SINR and rate coverage of a user significantly depend on its location.
- Reflections can be an important factor in determining performance for corner and edge users.