Appendix - Supplementary File

Investigation of Approximation Accuracy in the Hitting Probability in a 3-D Molecular Communication System with Multiple Fully Absorbing Receivers

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In this document, we evaluate the derived hitting probability expression in [1] and compute approximation error under various configurations.

I. Absolute Error Plots of Hitting Probability of an MC System

In this section, we show the variation of the absolute error (=|Analytical value − Simulated value|) of the hitting probability at FAR_3 for a $3-$ FAR system. The step size Δt taken for the particle-based simulation simulation is 10^{-4} s. The two FARs are placed in fixed locations (centers represented by small blue circles), and the third FAR is moved in the $y-z$ plane. The small red circle represents the location of the transmitter. The areas in which the FAR overlap with other FARs and the transmitter are shown in white color.

A. Configuration 1: Transmitter and the center of FARs in the UCA are not in the same plane

Here, a SIMO system with a point transmitter at the origin and multiple FARs arranged as UCA (as seen in Fig. S- 1) is consider. The UCA center is located at $x-$ axis at $[w, 0, 0]$ and UCA is in $y-z$ plane. Let d be the radius of the circle of UCA. The location of FAR_i is at $x_i = [w,\ d\cos(2\pi n/N),\ d\sin(2\pi n/N)],\ n \in \{1, 2, \cdots, N\}$.

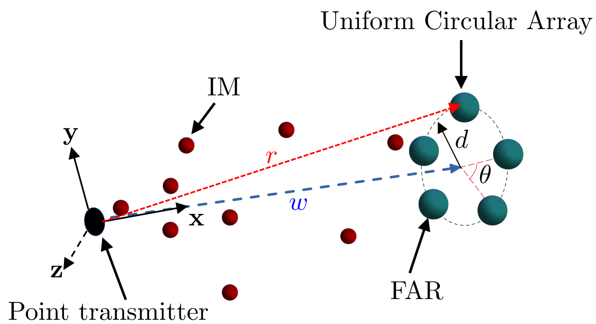
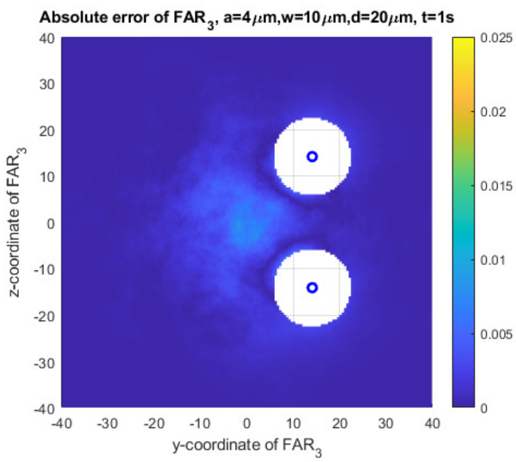


Fig. S- 1: UCA of FARs



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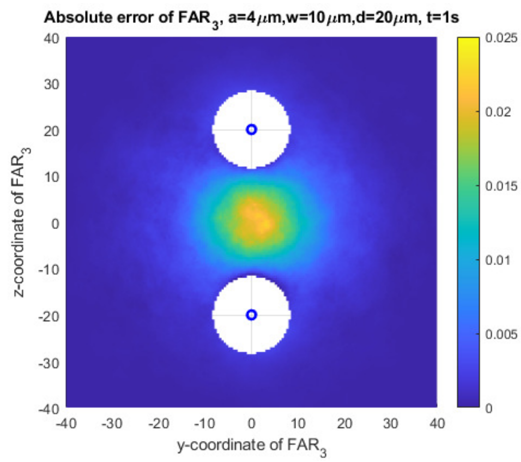

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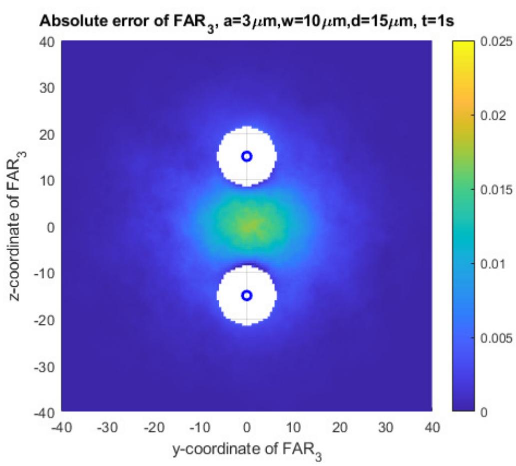

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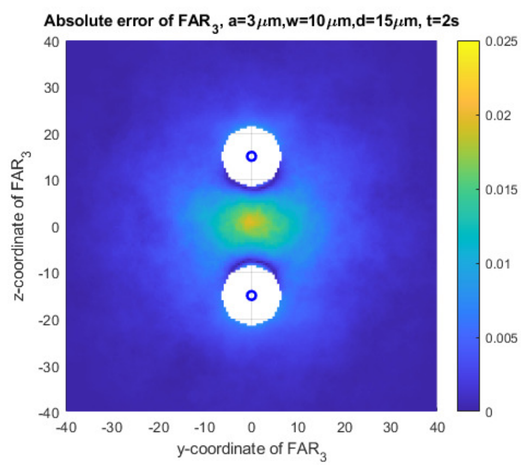
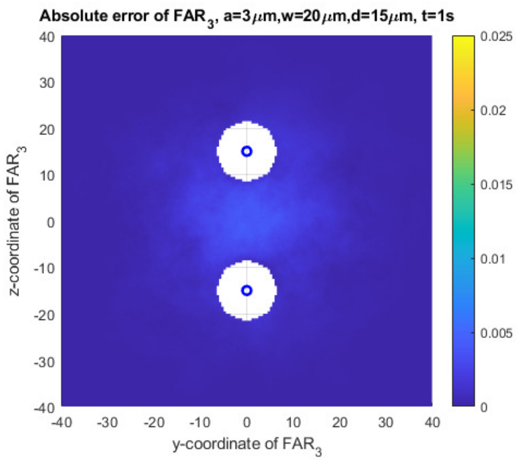


(d)

Fig. S- 2: Variation of the absolute error of the hitting probability of FAR$_3$ in a 3- FAR system. The small blue circles represents the center of fixed FARs.



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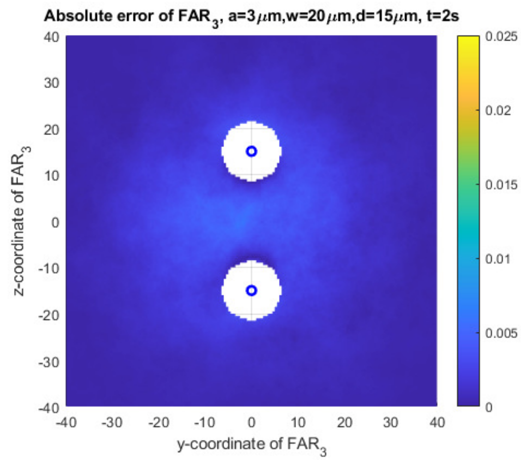

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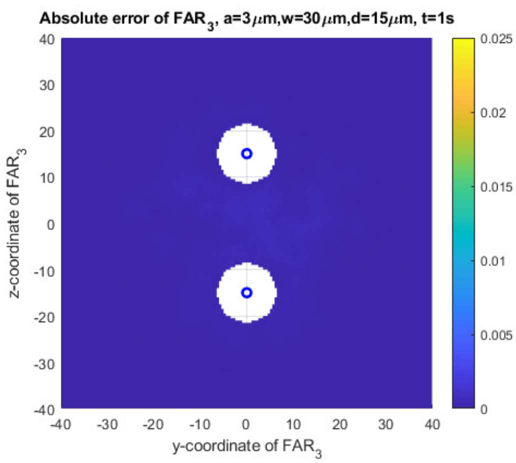

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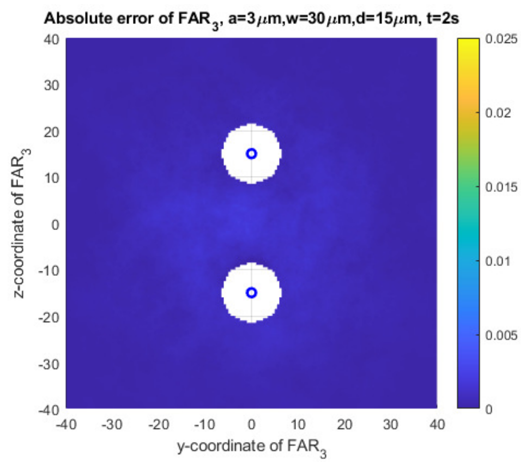


(d)

Fig. S- 3: Variation of the absolute error of the hitting probability of FAR_3 in a 3- FAR system. The small blue circles represents the center of fixed FARs.

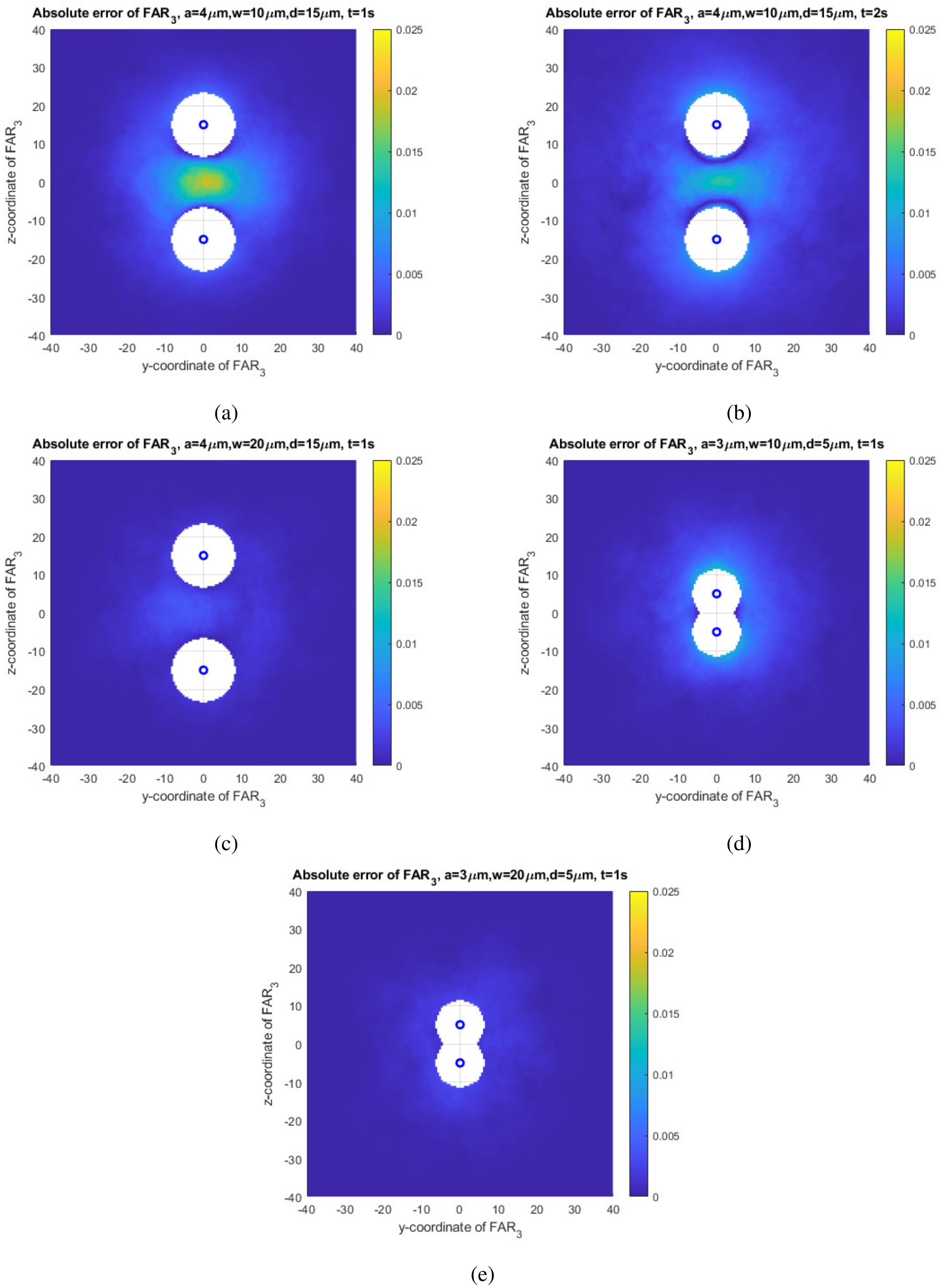
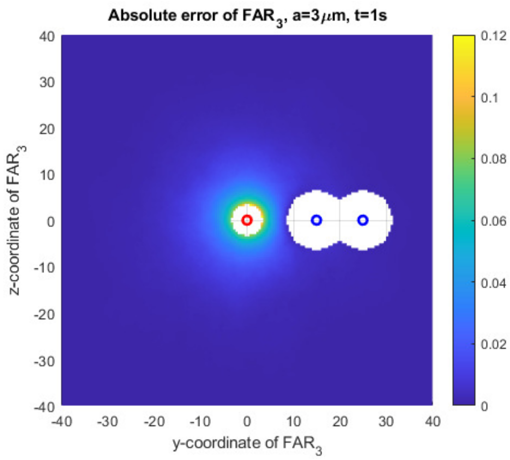


Fig. S- 4: Variation of the absolute error of the hitting probability of FAR_3 in a 3- FAR system. The small blue circles represents the center of fixed FARs.

We can see that the error is minimal in most regions in all configurations confirming the validity of derived expressions.

B. Configuration 2: Transmitter and the center of FARs are in the same plane.

In the figures shown below, we consider different orientations of the FARs when the transmitter is also in the same plane as the center of FARs (*i.e.,* $y - z$ plane).



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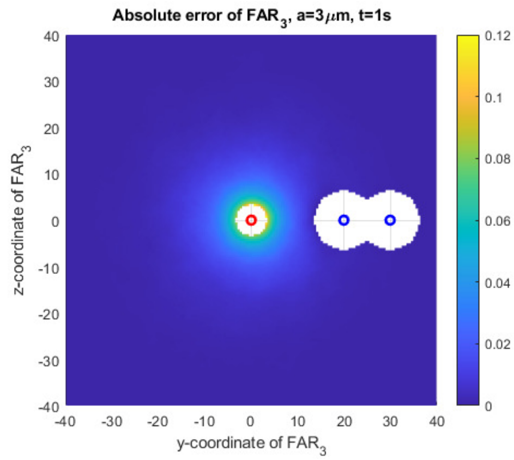

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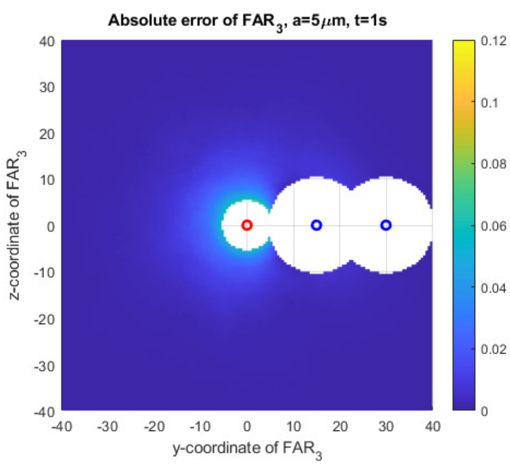

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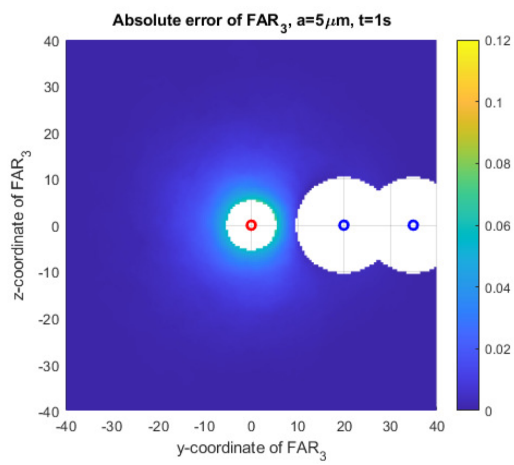
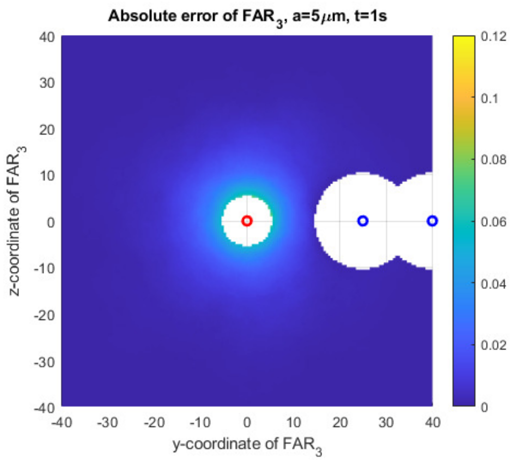


(d)

Fig. S- 5: Variation of the absolute error of the hitting probability of FAR_3 in a 3- FAR system. The small blue circles represents the center of fixed FARs, and the small red circle represents the transmitter location.



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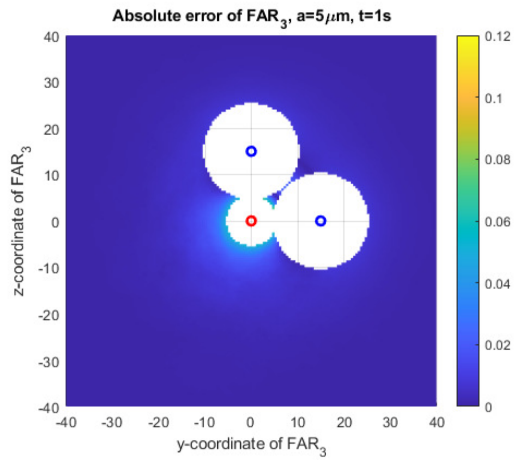

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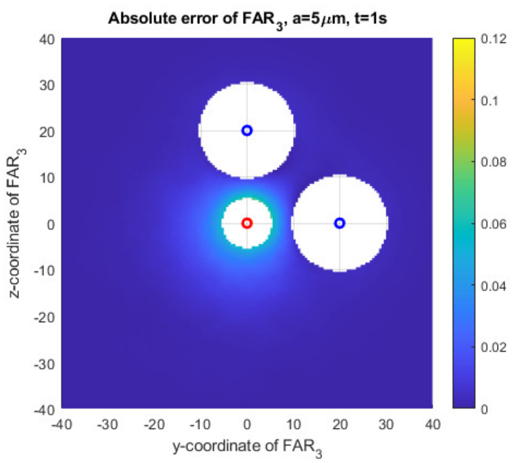

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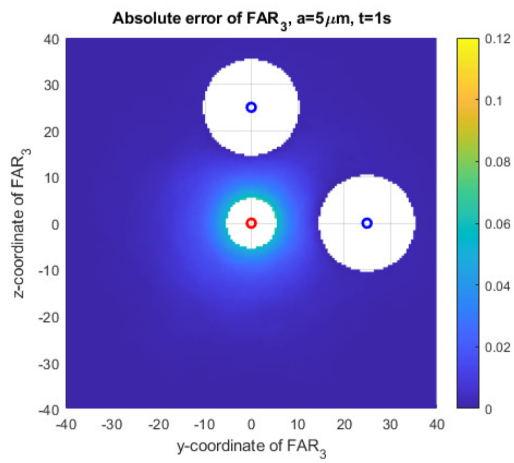

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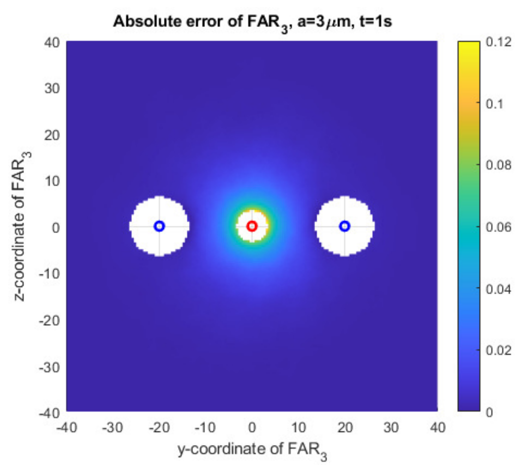


(e)

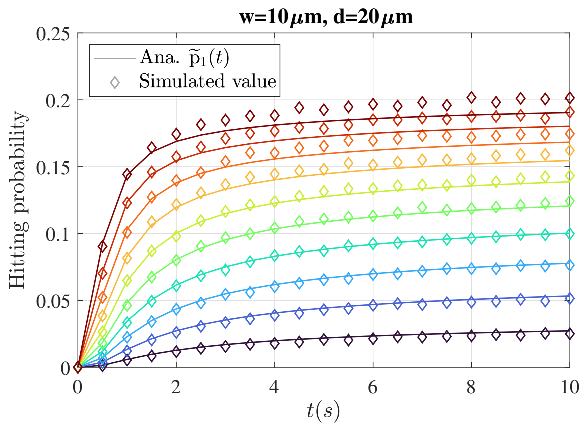
Fig. S- 6: Variation of the absolute error of the hitting probability of FAR$_3$ in a 3- FAR system. The small blue circles represents the center of fixed FARs, and the small red circle represents the transmitter location.

We fixed the absolute error threshold for an MC system as 0.002. We say that the absolute error is minimal if it falls below this threshold. For UCA case, we observed that any placement of FARs along the circle with $r_i > 5a$ resulted in minimal absolute error. For non-UCA cases, all locations with $r_i \geq 5a$ and $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| \geq 4a$ have minimal absolute error. This confirms the validity of derived expressions for these scenarios.

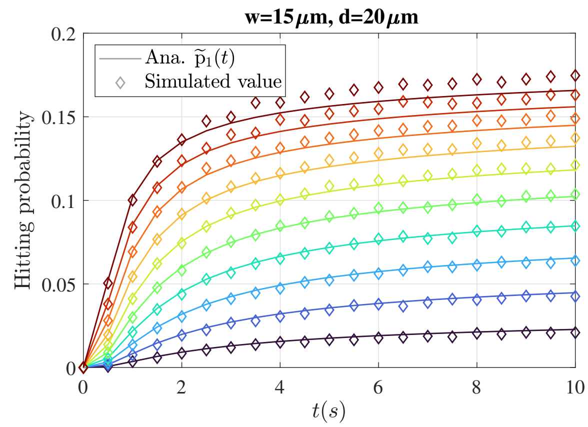
II. Hitting Probability Evaluation for Different Topology and Orientation of FARs

A. *Configuration 1: UCA of* 4*- FARs*

Here, a UCA of 4- FARs are arranged as shown in Fig. S-1.



(a)



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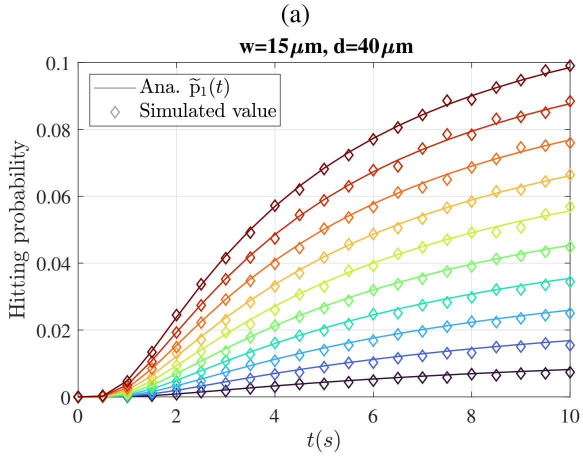

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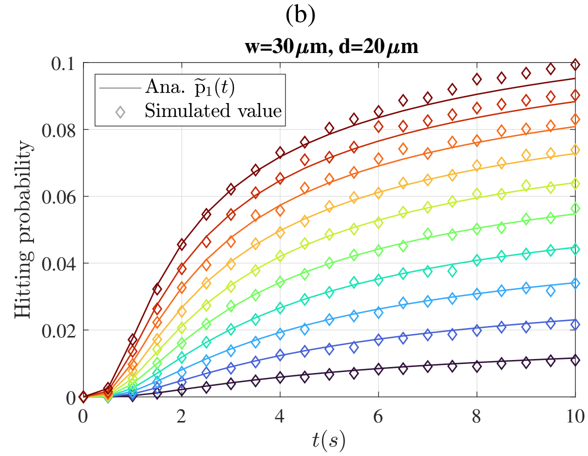
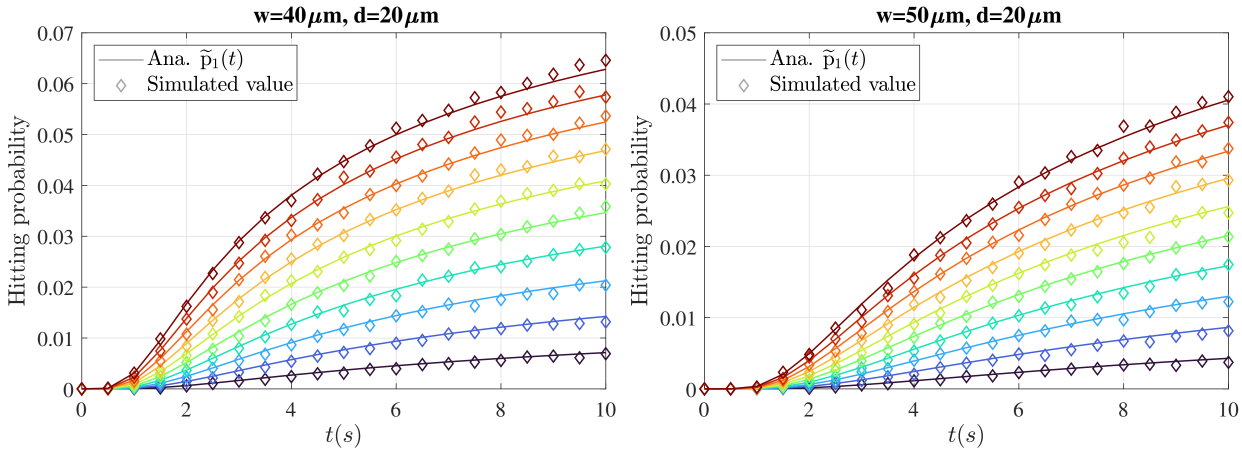


(d)

Fig. S- 7: Variation of of the hitting probability of FAR_1 with time in a UCA of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.,* black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively .



(a) (b)

Fig. S- 8: Variation of of the hitting probability of FAR_1 with time in a UCA of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.,* black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively .

We consider that the absolute error is minimal if it is below a threshold value of 0.002. From the figures, we can see that the error is minimal when the transmitter is sufficiently far away from the FARs (*i.e.,* $r_i > 5a$), regardless of the mutual proximity of receivers. This confirms the validity of derived expressions for UCA of FARs.

B. Configuration 2: Uniform linear array of FARs

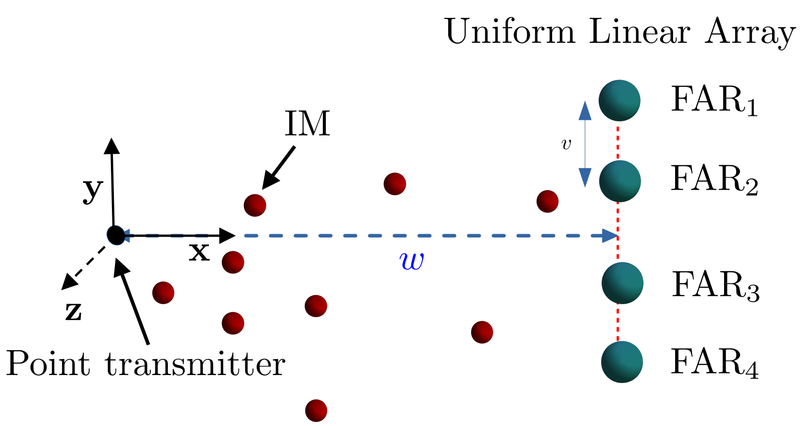
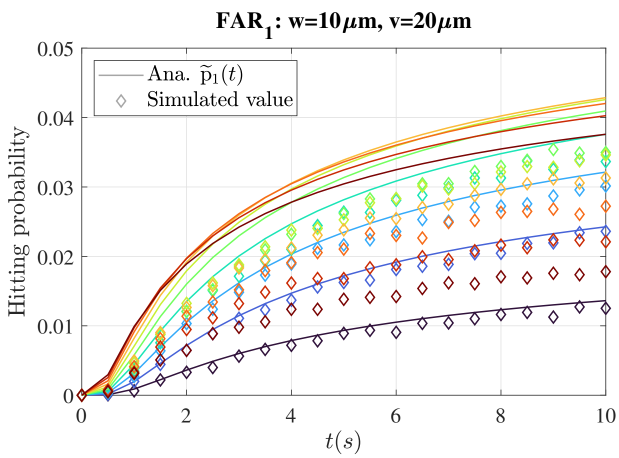
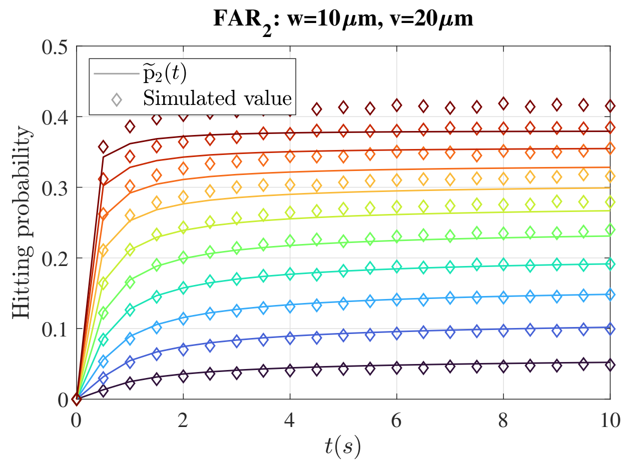


Fig. S- 9: Linear array of FARs

Now, consider a uniform linear array of FARs as shown in Fig S- 9, in which the center of the FARs lies in a line with nearby FAR distance v. The transmitter is located at a distance w from the center of the line. Note that, due to the symmetry, $\widetilde{\mathsf{p}}_1(t) = \widetilde{\mathsf{p}}_4(t)$ and $\widetilde{\mathsf{p}}_2(t) = \widetilde{\mathsf{p}}_3(t)$.



(a)



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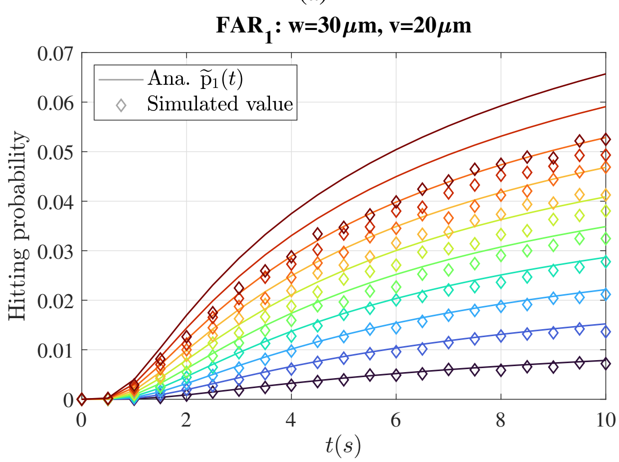

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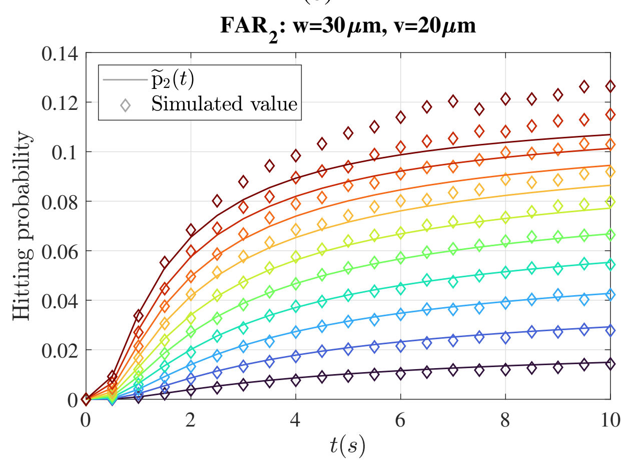
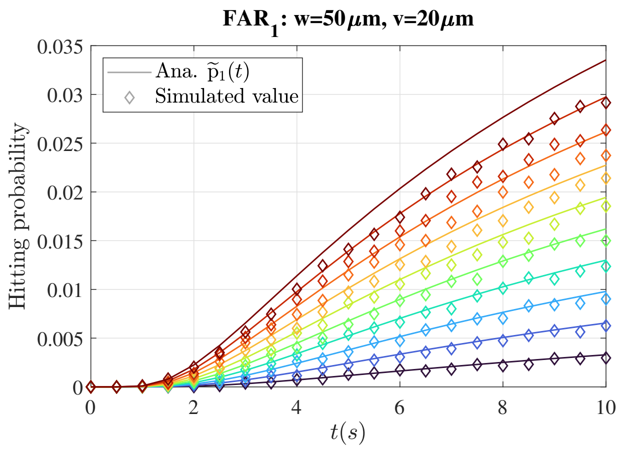


(d)

Fig. S- 10: Variation of of the hitting probability of FAR$_i$ with time in a linear array of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.,* black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively.



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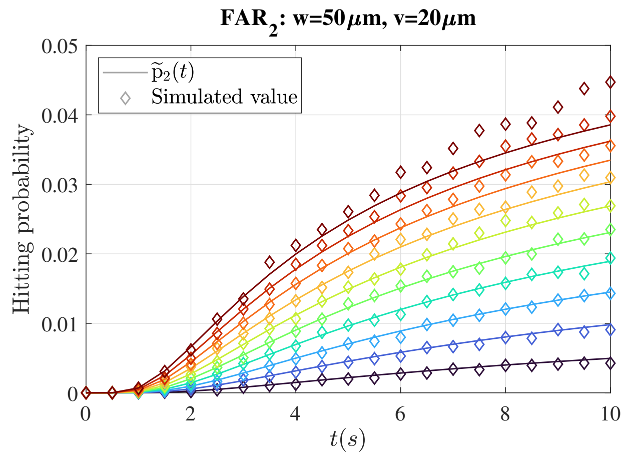

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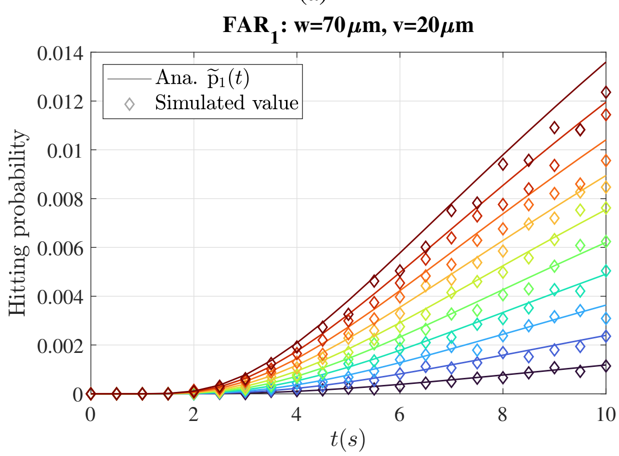

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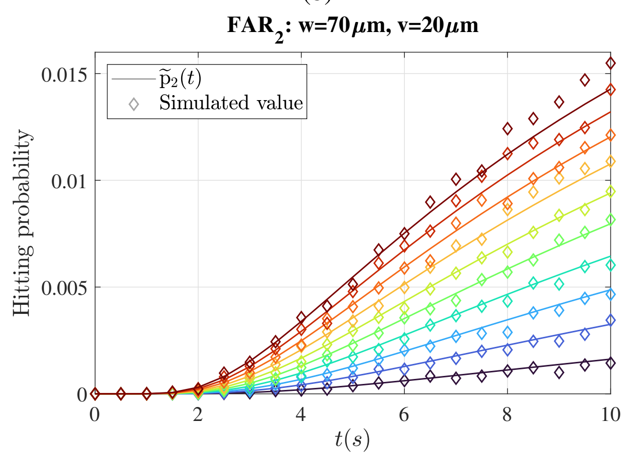

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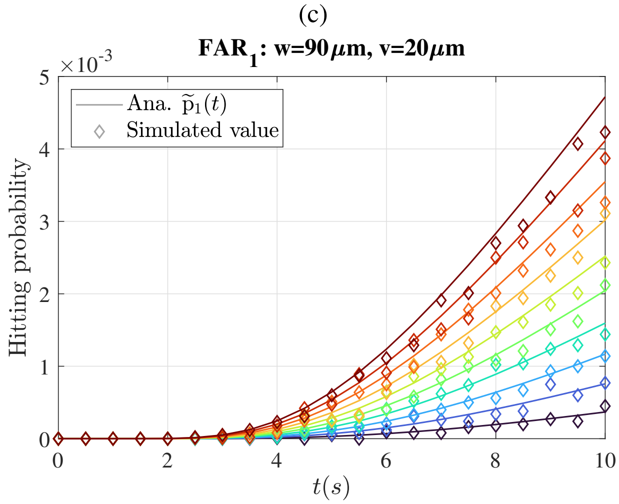

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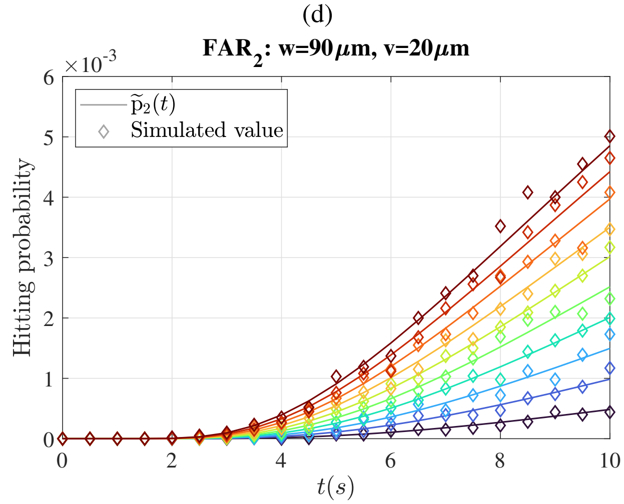

(f)

Fig. S- 11: Variation of of the hitting probability of FAR$_i$ with time in a linear array of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.*, black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively .

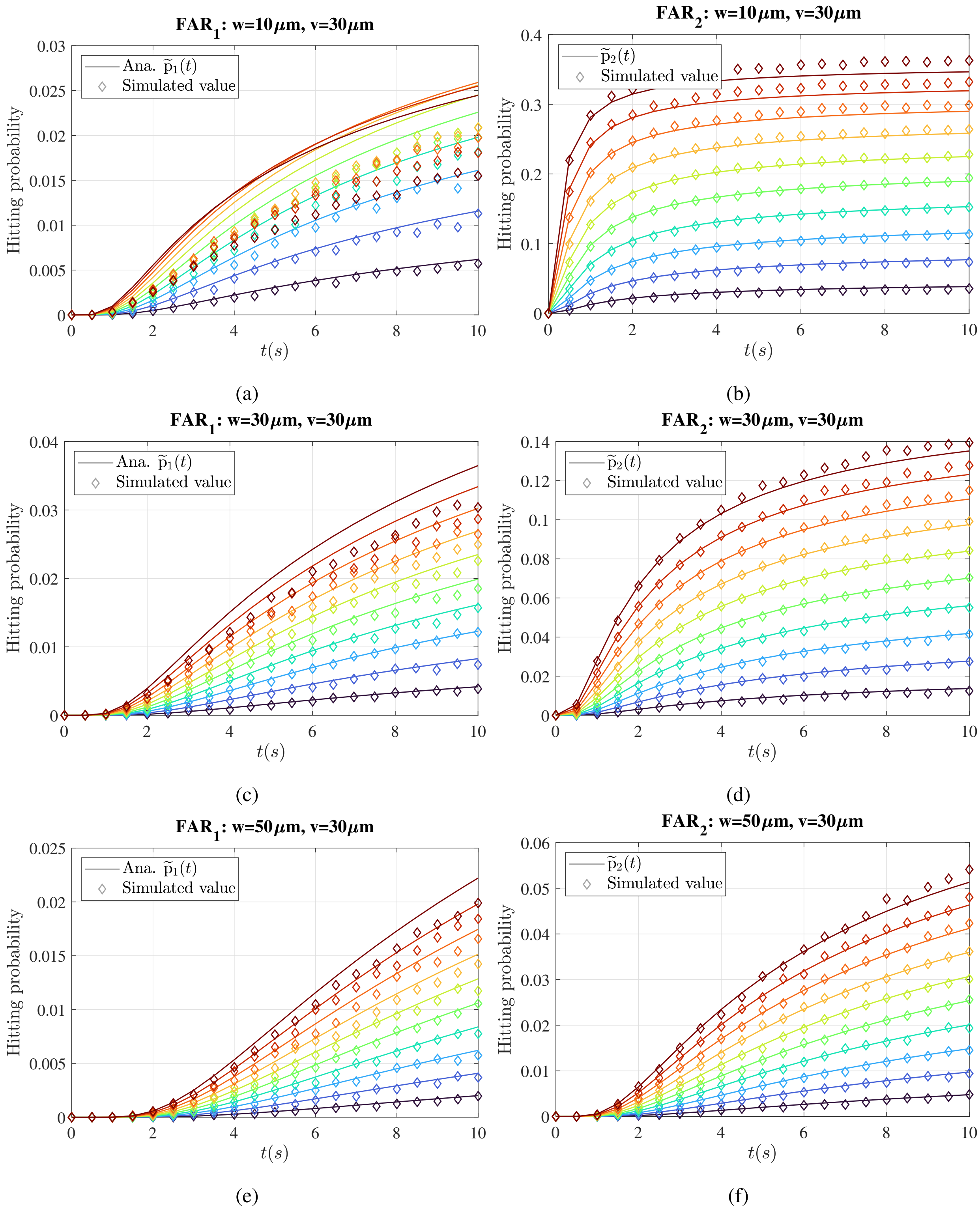

Fig. S- 12: Variation of of the hitting probability of FAR$_i$ with time in a linear array of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.*, black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively .

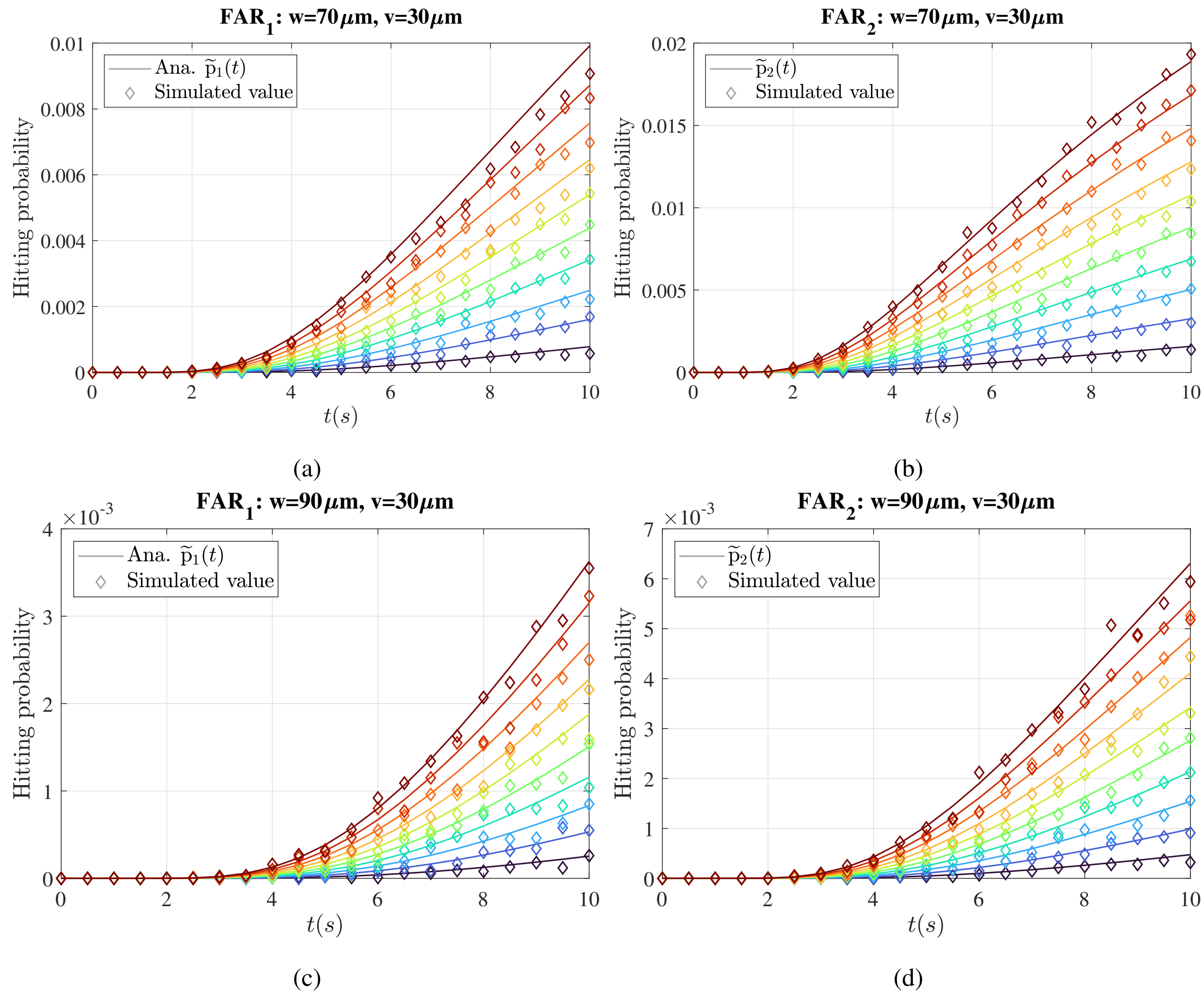

Fig. S- 13: Variation of of the hitting probability of FAR$_i$ with time in a linear array of 4- FAR system. Note that, $a \in \{1, 2, \cdots, 10\}$ with a increases from bottom curve to the top curve, *i.e.,* black curve corresponds to $a = 1\mu m$ and red curve corresponds to $a = 10\mu m$ respectively .

The figures show that the error is minimal (absolute error less than 0.002) in all configurations when the transmitter is sufficiently distant from FARs (*i.e.,* $r_i > xa$) where x is 5, regardless of the value of mutual FAR distance v. Further note that if FAR are appropriately far away (*i.e.,* $v > 4a$) from each other, the value of x decreases. This means the equation becomes accurate even at a low value of r_i.

References

[1] N. V. Sabu, A. K. Gupta, N. Varshney, and A. Jindal, "Channel characterization and performance of a 3-D molecular communication system with multiple fully-absorbing receivers," in *IEEE Trans. Commun. (Submitted)*, 2022, pp. 1–1.