Sankhya A

Discussion of D. Kundu's article in Sankhya A: Bayesian Inference of a Stationary Proportional Hazard Class Process

--Manuscript Draft--

Manuscript Number:	SANK-D-25-00114
Full Title:	Discussion of D. Kundu's article in Sankhya A: Bayesian Inference of a Stationary Proportional Hazard Class Process
Article Type:	Original Research Articles
Keywords:	D. Kundu, C.R. Rao
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Funding Information:	
Abstract:	No abstract. This is an invited discussion.

Discussion of D. Kundu's article in Sankhya A: Bayesian Inference of a Stationary Proportional Hazard Class Process

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Keywords and phrases: D. Kundu, C. R. Rao

The paper is devoted to consideration of a discrete time and continuous state space stationary stochastic process $\{X_n;\ n \geqslant 1\}$ whose elements are non-negative r.v.'s with distributions from the so-called proportional hazard class (PHC) of distributions and the consecutive observations may be equal with positive probability. This work addresses a significant gap in the literature, as existing models like the exponential process [1], Weibull process [2, 3], Pareto process [4], and others assume $\mathsf{P}(X_n = X_{n+1}) = 0$.

The proportional hazard class process (PHCP) is constructed as a minification process:

$$X_n = \min\{Y_n, Y_{n+1}, Z_n\}, \quad n \in \mathbb{N}, \tag{1}$$

where Y_n and Z_n are independent sequences of r.v.'s following distributions from the proportional hazard class with the survival functions ${S_0}^{\lambda_1}$ and ${S_0}^{\lambda_2}$, respectively, $\lambda_1, \lambda_2 > 0$, S_0 being the survival function of an arbitrary absolutely continuous distribution with positive support. This structure ensures stationarity, Markov property, and a positive probability $\mathsf{P}(X_n = X_{n+1}) = \lambda_1/(3\lambda_1 + \lambda_2) \in (0, 1/3)$.

Three cases for the baseline survival function S_0 are considered in the paper corresponding to the exponential, Rayleigh, Weibull distributions and the distribution with piecewise constant hazard function

$$h_0(t) := -S_0'(t)/S_0(t) = \sum_{j=1}^{M} \theta_j \mathbf{1}(\tau_{j-1} \leqslant t < t_j),$$

where $M \in \mathbb{N}$ and $0 = \tau_0 < \tau_1 < \ldots, \tau_M = \infty$ are suggested to be known constants and $\theta_j > 0$, $\theta_1 + \ldots + \theta_M = 1$ are unknown.

A major innovation of the paper is the data-driven method for selecting the number M and locations $\{\tau_j\}$ of cut-points in the PCHP. By modeling cut-points as arrival times of a non-homogeneous Poisson process with intensity $u(t) = \beta u^{\beta-1}$, $\beta > 0$, the author provides an efficient way to estimate the cut-points by the expected values of the corresponding arrival times and uses Akaike and Bayesian information criteria to select the number of cut-points M. This addresses a critical limitation in existing PCHP literature, where M and $\{\tau_j\}$ are typically assumed to be known.

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The joint survival copula's structure of the considered minification process (Marshall-Olkin type) facilitates dependency analysis, including Kendall's and Spearman's rank correlation coefficients.

The paper develops a comprehensive Bayesian inference framework using flexible priors: beta–gamma prior for (λ_1, λ_2), accommodating positive and negative correlation between parameters, gamma prior for the Weibull shape parameter α, Dirichlet prior for the hazard coefficients $\theta_1, \ldots, \theta_M$ in the PCH model, the three above parameter sets are assumed to be independent. Furthermore, the author proposes efficient importance sampling algorithms for estimating parameters and constructing highest posterior density (HPD) credible intervals. This approach is computationally feasible and avoids reliance on asymptotic approximations.

The bootstrap-based goodness-of-fit test using the statistic $W_n = \max\limits_{1 \leqslant i \leqslant n} |X_{i:n} - a_i|$, where $a_i = \mathsf{E}_{H_0} X_{i:n}$, is practical for validating model assumptions. Applications to gold price and USD/INR exchange rate data demonstrate the PHCP's effectiveness.

Kundu's paper rises interest to many new problems such as investigation of the accuracy of the suggested bootstrap-based goodness-of-fit test, investigation of the maxification instead of minification process in (1), extensions to lag-2 and higher dependencies $X_n = \min\{Y_n, \ldots, Y_{n+k}, Z_n\}$.

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