Curtailed Sampling Plans

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Abstract

In this review article we will discuss about the curtailed sampling plans. The curtailed sampling plan is often used in quality control and in reliability. The main idea about the curtailed sampling plan, as the name suggests, is to stop the sampling process and take the final decision namely either to accept or to reject the lot, before inspecting the entire sample. It is a sequential process, and it can be seen as a 'common sense modification' of a sequential process. The curtailed sampling plans can be broadly classified into two parts, one is known as the attribute curtailed sampling plans and the other is known as the variable curtailed sampling plans. In the literature it has been observed that the attribute curtailed sampling plan has been further subdivided namely single sample attribute curtailed sampling plan and (sequential) double sample curtailed sampling plans. About the variable curtailed sampling plan there are mainly two approaches namely the classical approach and Bayesian approach. The main aim of this article is to provide a gentle introduction of the above topic, provide the history and recent development of this area. We would like to mention that in this article we do not provide any new results, but we will provide several open problems for future research.

Key Words and Phrases: sequential process; attribute sampling plan; variable sampling plan; classical inference; Bayesian inference; maximum likelihood estimator; Bayesian estimator; loss function.

AMS Subject Classifications: 62L05, 62N05, 62L10.

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1 Introduction

The curtailed sampling plan (CSP) is a sequential process often used in statistical quality control, reliability analysis and in process control. It is used mainly to decide to stop the sampling process before it reaches the predetermined sample size. It basically helps to improve the efficiency by reducing the number of samples taken while still maintaining an acceptable level of confidence in the process. It is very common if the sampling procedure is very expensive or it is destructive in nature. According to Johnson [17] it can be called as a "common sense method of modification" of a sequential process. This method of modification simply amounted to stop inspecting the sample any further as soon as the final decision, either acceptance or rejection of the lot, was clear, what might be results of the remaining observations. Suppose a routine one sample acceptance test rejects a lot if five or more defective items are found in a sample of hundred items. In this case the sampling is performed sequentially and it would stop if one of the following events takes place; (a) as soon a five defective items have been observed, (b) if there are no defective items among the first 96 items, (c) if there are fewer than two defective items among the first 97 items, (d) if there are fewer than three defective items among the first 98 items, (e) if there are fewer than four defective items among the first 99 items. In this case it is clear one need not go for 100% inspection, and a decision can be taken before that. Hence, it can be called a curtailed sampling plan.

It should be mentioned that the curtailed sampling plan is directly related to stopping rule and sequential testing. The CSP can be considered as an integral part of these two areas. An extensive amount of literature exits which deals with these topics, see for example Wald and Wolfowitz [45], but they do not mention explicitly about the curtailment. In this article we do not want to discuss anything which does not directly mention about the curtailment. We restrict our discussions only on those work where the curtailment has been mentioned explicitly mainly due to the limitation of page. The first article on this topic can be traced back to Barnard [2], see also Vajda [43] in this respect. Since then an extensive amount of papers have appeared till date. In recent time Bayesian curtailed sampling plan (BCSP) has received a significant attention. The main aim is to develop optimal BCSP under different censoring schemes. Flexible lifetime distributions also have been considered by different authors.

The CSP can be broadly classified into two parts; (a) attribute CSP and (b) variable CSP. Further, it is observed that the attribute CSP can be subdivided as (i) single sample attribute CSP and (ii) (sequential) double sample attribute CSP. About the variable CSP there are mainly two approaches (i) classical approach and (ii) Bayesian approach. The work on CSP has initiated with the attribute sampling plan both for one sample and for (sequential) two or multiple sample plans. Let us denote by 'p' the proportion in the population that the attribute is present. A significant amount of attention has been paid in the efficient estimation of p, as well as some of the testing problem associated with it both for the single sample as well as for (sequential) two or multiple sample also. Interestingly it has been mentioned in Montgomery [28] that why a single sample CSP will always produce a biased estimate of p. It is always possible to rectify that problem in case of (sequential) double or multiple sample CSP. Another important issue in attribute CSP is to compute the average sample number for both one sample and (sequential) two sample CSP. Several authors have addressed this problem, see for example Phatak and Bhatt [31], Cohen [7], Shah and Phatak [42], for one sample CSP. Craig [9] and Bar-Lev and Boukal [1] considered (sequential) two sample CSP.

The variable CSP has received much more attention than the attribute CSP as expected. The first article in this area can be traced back to Garner [12], see also [29] in this respect. Garner [12] has discussed about the variable CSP in the general set up. His approach was mainly classical in nature. Since then an extensive amount of work has appeared in this area and most of the attention has been paid in case of lifetime measurement. Interestingly, majority of these work are of the Bayesian nature. The main aim in all these work is to develop the Bayes optimal sampling plan under suitable loss function. Different censoring schemes have been considered; namely Type-I censoring, Type-II censoring, random censoring, Type-I hybrid censoring, Type-II hybrid censoring, progressive censoring, interval censoring etc. and majority of these work has been developed for one parameter exponential distribution mainly due to its analytical tractability. Lam [20], see also Lam [19], has first developed the optimal Bayesian sampling plan for Type-I censored data, when the lifetime of the experimental units follows one-parameter exponential distribution. Since then several papers have come out dealing with different aspects of this problem, see for example [3], [4], [5], [6], [15], [16], [21], [22], [23]. The CSP for step-stress model has been discussed by Prajapati and Kundu [33]. Prajapati et al. [39, 40] considered the CSP for two sample problem and Prajapati et al. [38] developed CSP in case of competing risks.

The main aim of this article is to provide a gentle introduction of this area. We would like to provide the history, its development and the recent advances in this area. We are not going to present any new results, but we would like to mention several open problems for future research.

The rest of the paper is organized as follows. In Section 2 we provide the attribute CSP both for one sample and for (sequential) two sample. In Section 3 we provide variable CSP for different censoring schemes and for one sample problem. In Section 4 we discuss variables CSP for two sample problem. In Section 5, we discuss variable sampling plans in case of competing risks and CSP for step-stress model has been considered in Section 6. Finally in Section 7, we conclude the paper.

2 Attribute CSP

2.1 Single Sample Attribute CSP

The usual single sample attribute CSP is defined by three numbers; (i) the lot size N, (ii) the sample size n, (iii) the acceptance number c and one decision rule either accept the lot or reject the lot. It is a sequential process and accept the lot if the number of defective items in the sample is less than or equal to the acceptance number, otherwise reject it. In this case although the sample size is fixed the curtailment can be done before inspecting all the items in the sample and it can be done either in the acceptance or at the rejection stage. The CSP can be described as follows:

Plan: Inspect randomly selected units of the lot sequentially one at a time, until either k defective items have been observed or $n - k + 1$ non-defective items have been observed. Accept the lot if there are $n - k + 1$ non-defectives. Reject the lot if there are k defectives.

Both k and n are predetermined and in general k is much smaller than n. Here, k is known as the rejection number, and it is related to the acceptance number c by $c = k - 1$. There are three problems which are of interest. The first problem is to choose optimally n and k given the producer's and consumer's risks. The second problem of interest is to find an efficient estimator of p, the proportion of defective items in the lot. Since it is a CSP, the third problem is to obtain the average sample number (ASN).

Patil [30] obtained the optimal values of n and k given producer's and consumer's risks. Let us define a discrete random variable D, the number of defective items in a lot, which takes values $0, 1, \ldots, k-1$, when a lot is accepted and the values $k, k+1, \ldots, n$, when a lot is rejected. Now based on the assumption that the lot size N is very large compared to the sample size n, i.e. the probability of any inspected item of the lot to be defective is p, which remains constant in the sequential process. Therefore,

$$P(D=d)=\begin{cases}\binom{n-k+d}{n-k}p^d(1-p)^{n-k+1} & \text{if} \quad d=0,1,\ldots,k-1\\[2ex] \binom{d-1}{k-1}p^k(1-p)^{d-k} & \text{if} \quad d=k,k+1,\ldots,n.\end{cases} \tag{1}$$

Hence, $\sum_{d=0}^{k-1} P(D=d)$ gives the probability of acceptance of a lot, and $\sum_{d=k}^{n} P(D=d)$ gives the probability of rejection of a lot. Since, a lot will be either accepted or rejected, therefore,

$$\sum_{d=0}^{k-1} P(D=d)+\sum_{d=k}^{n} P(D=d)=1.$$

Now to compute the maximum likelihood estimator of p, it is assumed that total T lots have undergone inspection under this CSP. Each, either accepted or rejected, lot will give rise to two observations namely; the total number of items which has been inspected, and the number of defective items out of these inspected items. Let a_i denote the number of accepted lots out of T lots in which the number of defectives in the sample was i and let r_j denote the number of rejected lots out of T lots in which the number of inspected items was j. Hence, the likelihood function can be written as

$$L(p)=\prod_{i=0}^{k-1}\left[\binom{n-k+i}{n-k}(1-p)^{n-k+1}p^i\right]^{a_i}\prod_{j=k}^{n}\left[\binom{j-1}{k-1}(1-p)^{j-k}p^k\right]^{r_j}. \tag{2}$$

The maximum likelihood estimator of p can be obtained by maximizing (2) with respect to the unknown parameter p. It can be obtained in explicit form. If we define

$$A=k\sum_{j=k}^{n} r_j+\sum_{i=0}^{k-1} i a_i \quad \text{and} \quad B=(n-k+1)\sum_{i=0}^{k-1} a_i+\sum_{j=k}^{n} r_j(j-k),$$

then $\widehat{p}$, the maximum likelihood estimator of p, becomes

$$\widehat{p}=\frac{A}{A+B}.$$

The asymptotic variance of $\widehat{p}$ is given by

$$V(\widehat{p})=-\left[E\left(\frac{d^2}{dp^2}\ln L(p)\right)\right]^{-1}.$$

It can be shown that, see for example Pathak and Bhat [31] that

$$V(\widehat{p}) = \frac{p^2(1-p)}{TJ}.$$

Here, if we define

$$B(p,n,k) = \sum_{m=0}^{k} \binom{n}{m} p^m (1-p)^{n-m},$$

then

$$J = \frac{p}{1-p}(n-k+1)B(p,n+1,k-1) + k[1-B(p,n+1,k)].$$

Further, if M denotes the number of inspected items, then

$$M = \begin{cases} n-k+D & \text{if the lot is accepted} \\ D & \text{if the lot is rejected.} \end{cases}$$

From (1) we can obtain $E(M) =$ the average sample number (ASN) as

$$\begin{aligned} \text{ASN} &= \sum_{m=n-k+1}^{n} m\binom{m-1}{n-k} p^{m-(n-k+1)}(1-p)^{n-k+1} + \sum_{m=k}^{n} m\binom{m-1}{k-1} p^k (1-p)^{m-k} \\ &= \frac{1}{1-p}(n-k+1)B(p,n+1,k-1) + \frac{1}{p}k[1-B(p,n+1,k)] = \frac{J}{p}. \end{aligned}$$

2.2 (Sequential) Double Sample CSP

The (sequential) double sample CSP was originated with a proposal of Dodge and Roming [10]. In the usual notation, a general (sequential) double sampling plan for attributes is specified by five design parameters namely; n_1, n_2, r_1, r_2 and a. Here n_1 and n_2 are two sample sizes at two different stages, r_1 and a are the rejection and acceptance number at the first stage and r_2 is the rejection number at the second stage. There are two kinds of truncation possible for double sampling. In the first, truncation can occur before the first sample is completely inspected. It can happen if r_1 defectives items before a non-defective items are found with the rejection of the lot. Similarly, it can happen if a non-defective items before r_1 defective items are found with the acceptance of the lot. In the second case, the truncation can occur at the second stage. It can happen if in the first sample after inspecting all the n_1 items, less than r_1 defective items and less than a non-defective items are found then the second sample will be inspected. In the second sample if r_2 defective items before $n_2 - r_2$ non-defective items are found with the rejection of the lot or $n_2 - r_2$ non-defective items before r_2 defective items are found with the acceptance of the lot.

Hence, a (sequential) double sampling plan is defined by five design parameters. With five such parameters, the problem of finding an 'optimal' plan that satisfies certain requirements becomes a difficult problem. However, there exist many plans which only meet the desired Type-I error level, and their numbers may be reduced by imposing some restrictions on these design parameters. For example (sequential) double sampling inspection plans of MIL-STD-105E [26] are constructed under the restriction $n_1 = n_2$ and give choices of the remaining design parameters usually $r_1 = r_2$. They had presented extended tables for the optimal design parameters. The uniformly most powerful (UMP) test on the proportion of the defective items in the lot in case of (sequential) double sampling plan has been presented by Bar-Lev and Boukai [1] and the average sample number has been provided by Craig [9]. For some related issues about the double sampling plan one is referred to Hamaker and Van Strik [14] and see the references cited there in.

3 Variable CSP for One Sample

Similar to the attribute CSP, the variable CSP is necessary when the testing is costly, when each test takes a long time to perform or when failure of the test characteristic indicates a potential hazardous condition. Garner [12] has provided a specific real life example where the variable CSP has been used formally to save both time and money. The excessive chamber pressure of a rocket might cause a blow-up which, in turn, might extensively damage a test stand. Thus, if it can be shown that a lot will be rejected with certainty after $r < n$

of the required n tests have been completed, monetary savings are assured by eliminating unnecessary tests and having a decision in the shortest time possible. But there can be several other curtailment in case of lifetime experiment. In case of lifetime experiment different censoring schemes play the role of curtailment of the experiment either in time or the number of inspected items or both. Suppose the specification for lot acceptance places a limitation on the characteristic being tested, so that when the results of the sample have been obtained, the relationship,

$$\overline{X} + ks \leq U,$$

must hold. Here, $\overline{X}$ is the arithmetic mean of the n observed values of the characteristic being measured, s is the standard deviation, k is a pre-fixed constant and U is a pre-fixed upper limit assigned by the specification. Now suppose after $r < n$ sample has been tested, if it can be shown that the lot will be rejected regardless of the outcome of the remaining $n - r$ items, the lot can be rejected immediately. It can be shown that, see for example Garner [12], that a lot will be rejected if and only if $\sqrt{V_r B(k^2C - A^2)/C}$ is real and

$$\overline{X}_r + \sqrt{V_r B(k^2C - A^2)/C} > U. \tag{3}$$

Here, $\overline{X}_r$ and V_r are the mean and variance of the first r observations, $A = (n - r)/n$, $B = (r - 1)/(n - 1)$ and $C = r(n - r)/(n(n - 1))$. It may be mentioned that the above condition (3) is obtained by finding the most conservative values of the remaining $n - r$ observations based on first r observations. It can be shown that the most conservative values of the remaining $(n - r)$ observations can be obtained when they are all equal and it is

$$X = \overline{X}_r - \left(\frac{(n-1)(r-1)nV_r}{nk^2r^2 - r(n-r)(n-1)} \right)^{1/2}.$$

An extensive amount of work has been done on CSP under specific assumption on the lifetime distribution. The problem remains the same, i.e. given a batch of N products, it is required to inspect for acceptance or rejection of the batch. The quality of an item is measured by its survival time. The standard value a_0 of the survival time is specified by the producer as the minimum acceptable value for a satisfactory item. If the survival time is greater than a_0, the item can be accepted without any additional loss, i.e. it can be sold at the normal price and if it is less than a_0, it could be sold at a reduced price, which effectively means a loss to the producer. Lam [19] first consider the optimal CSP for Type-II censored data when the lifetime of the experimental units follows an exponential distribution with mean θ. If $X_{(1)} < \ldots < X_{(r)}$ denote the first r observations before the experiment stops, then the decision function based on the observed data $\boldsymbol{X} = (X_{(1)}, \ldots, X_{(r)})$ takes the following form

$$\delta(\boldsymbol{X}) = \begin{cases} d_0 & \text{if} \quad \widehat{\theta} \geq T \\ d_1 & \quad \text{otherwise} \end{cases} \tag{4}$$

Here T is the minimum acceptable survival time based on which the batch will be accepted (d_0) or rejected (d_1), and

$$\widehat{\theta} = \frac{\sum_{i=1}^{r} X_{(i)} + (n-r)X_{(r)}}{r}, \tag{5}$$

the maximum likelihood estimator (MLE) of θ. It is assumed that $\lambda = \theta^{-1}$ has a conjugate gamma prior. Based on the assumption that the loss function is a quadratic loss function, the Bayes risk of the above decision function can be obtained in explicit form. Finally, the optimal sampling plan can be obtained by minimizing the Bayes risks. The optimal sampling plan has to be obtained numerically, and numerical algorithm has been provided to detect the optimal Bayesian sampling plan. In the same paper the result has been extended for the two-parameter exponential distribution when both scale (λ) and location (μ) are present. Here the MLE of θ becomes

$$\widehat{\theta} = \frac{\sum_{i=1}^{r} X_{(i)} + (n-r)X_{(r)} - nX_{(1)}}{r}, \tag{6}$$

where as the decision function takes the same form as in (4). In this case based on the joint conjugate prior of (λ, μ) as proposed by Varde [44] the Bayes risk can be obtained in explicit form when the loss function is quadratic. The optimal sampling plan can be obtained by minimizing the Bayes risks.

Lam [20], see also Lin et al. [25], further considered CSP for Type-I censoring when the lifetime of the experimental units follows exponential distribution. In this case also the decision function takes the same form as in (4) where as the MLE of θ for $M > 0$, becomes

$$\widehat{\theta} = \frac{\sum_{i=1}^{M} X_{(i)} + (n-M)\tau}{M},$$

here τ denotes the time where the experiment stops and $M = \max\{i : X_{(i)} \le \tau\}$. Similarly as before, based on the conjugate gamma prior on $\lambda = \theta^{-1}$, the Bayes risk of the decision function can be obtained in explicit forms. The optimal CSP also can be obtained along the same way. One major issue about Lam's procedure is that the MLE does not exist when $M = 0$. To overcome that Prajapati et al. [35] considered a shrinkage estimator of θ as follows;

$$\widehat{\theta}_{M+c} = \frac{\sum_{i=1}^{M} X_{(i)} + (n-M)\tau}{M+c}, \tag{7}$$

which always exists for $c > 0$. Under the same prior assumption and based on the quadratic loss function, the Bayes risk can be obtained in closed form. The optimal CSP has been obtained. The authors have also considered higher degree polynomial and non-polynomial loss functions also. Prajapati et al. [34] also provided an optimal CSP based on the MLE of λ directly, which always exists.

An extensive amount of work has been done for other censoring schemes also, although most of the work are for one-parameter exponential distribution. For example Liang et al. [22] considered efficient CSP for random censoring, Liang and Yang [23], Lin et al. [24] provided the optimal CSP for hybrid censoring, generalized hybrid censoring cases have been addressed by Prajapati et al. [36, 37]. Prajapat et al. [32] provided the optimal CSP for Type-I hybrid censoring case when the lifetime of the experimental units follow two-parameter exponential distribution and Chen and Lam [4] considered the case when the lifetime becomes two-parameter Weibull distribution. Although, an extensive amount of work has been done in case of exponential distribution, not much work has been done for other lifetime distributions. It will be important to develop optimal CSP for other lifetime distributions like gamma, log-normal, Birnbaum-Saunders distributions etc.

4 Variable CSP for Two Sample

The joint censoring scheme of two samples is quite useful to conduct comparative life testing experiment of different products. Suppose there are two consignments of similar products, and one needs to take a decision whether to accept or reject both or one of them. Mondal and Kundu [27] introduced a joint progressive censoring scheme applied on two different samples called the balanced joint progressive censoring (BJPC) scheme. Suppose there are two kinds of products from product line A and product line B and it is required to study the relative merits of these two kinds of products. A sample of size m is drawn from the product line A and similarly a sample of size n is drawn from the product line B. They will be called sample A and sample B, respectively. Let k be the total number of failures to be observed from the life testing experiment and $R_1, \ldots, R_{k-1}$, be pre-specified non-negative integers such that $\sum_{i=1}^{k-1}(R_i + 1) < \min\{m, n\}$. Suppose the first failure occurs from sample A at the time point W_1, then at W_1, R_1 units are removed randomly from the remaining $(m-1)$ surviving units of sample A, and $R_1 + 1$ units are chosen randomly from the n surviving units of sample B and they are withdrawn from the experiment. Next, if the second failure comes from sample B at time point W_2, $R_2 + 1$ units are withdrawn from the remaining $m - R_1 - 1$ units of sample A and R_2 units are withdrawn from the remaining $n - R_1 - 2$ units of sample B randomly at W_2 time point. The experiment is continued until the k-th failure occurs. At the k-th failure time point W_k, the experiment is terminated with the removal of all the remaining surviving units from both the samples.

Prajapati et al. [39] considered the CSP for two samples when the lifetime of the experimental units follow exponential distributions with mean θ_1 and θ_2, respectively. The major advantage of such implementation is to take decision on the acceptance or rejection of one batch or both the batches, in a single life testing experiment. It can save time significantly. Under the BJPC scheme, failure of all the items from both the samples are not observed, which reduces experimental cost. Prajapati et al. [39] used shrinkage estimators of θ_1 and θ_2 as follows:

$$\widehat{\theta}_{1s} = \frac{U}{k_1 + c} \quad \text{and} \quad \widehat{\theta}_{2s} = \frac{U}{k_2 + c},$$

where $U = \sum_{i=1}^{k-1}(R_i + 1)w_i + \left(n - \sum_{i=1}^{k-1}\right) w_k$, k_1 and k_2 are the number of failures observed from sample A and sample B, respectively. Based on the shrinkage estimators, the decision function takes the following form

$$\delta = \begin{cases} (d_{10}, d_{20}) & \text{if} \quad \widehat{\theta}_{1s} > T_1, \widehat{\theta}_{2s} > T_2 \\ (d_{10}, d_{21}) & \text{if} \quad \widehat{\theta}_{1s} > T_1, \widehat{\theta}_{2s} < T_2 \\ (d_{11}, d_{20}) & \text{if} \quad \widehat{\theta}_{1s} < T_1, \widehat{\theta}_{2s} > T_2 \\ (d_{11}, d_{21}) & \text{if} \quad \widehat{\theta}_{1s} < T_1, \widehat{\theta}_{2s} < T_2 \end{cases}$$

Here $d_{10}(d_{11})$ denotes the action of accepting (rejecting) a batch from product line A and $d_{20}(d_{21})$ denotes the action of accepting (rejecting) a batch from product line B. Further, T_1 and T_2 denote the minimum acceptable mean survival time from product A and B, respectively. Based on the Dirichlet-Gamma prior on $(\theta_1^{-1}, \theta_2^{-1})$, the Bayes risk of the above decision function can be obtained in explicit form when the loss function is squared error. Hence, optimal CSP can be obtained numerically. The results have been further extended in case of Weibull distribution by Prajapati et al. [40].

5 CSP for variables: Step-Stress Model

Accelerated life test (ALT) has been used regularly by reliability engineers to reduce the lifetime of the experimental units or to accelerate the degradation of their performance. Such testing aims to quickly obtain data that, adequately modelled, analyzed and yield desired information on the lifetime of the experimental units or their performances under normal operating condition. The step-stress test is a special case of the ALT, where the stress changes during the experiment in a regular interval. The stress changing time can be fixed or random. Depending on the type of the experimental items, several stress factors like temperature, voltage etc. can be used during the experiment. If in a step-stress experiment, the stress changes only once, it is known as the simple step-stress experiment. An extensive amount of work has been done in analyzing data obtained from a simple step-stress experiment, see for example the monograph by Kundu and Ganguly [18] in this respect.

Prajapati and Kundu [33] provided the optimal CSP for simple step-stress model when the stress changing time is random. The problem can be formulated as follows. Suppose n items are put in a life testing experiment at the initial stress level s_0. After n_1 failures, the stress level is changed to s_1. Finally the experiment stops after $n_1 + n_2$ failures, where $n_1 + n_2 < n$. Based on the failure data, the problem is to accept or reject the lot. It is assumed that the lifetime distributions of the experimental units at the two different stress levels follow exponential distributions with mean $1/\lambda_1$ and $1/\lambda_2$, respectively and they satisfy the cumulative exposure model assumptions, see for example Kundu and Ganguly [18]. Based on the assumptions that (λ_1, λ_2) has Dirichlet-Gamma prior and the loss function is quadratic, the Bayes risk can be obtained in explicit forms. The optimal CSP can be obtained numerically. In the same paper the authors have provided the order restricted optimal CSP also.

There are several important open problems in this area. For example it will be interesting to develop optimal CSP when the stress changing time is fixed. Another important problem is to develop optimal CSP for other lifetime distributions. More difficult problem will be to develop optimal CSP for general step-stress models.

6 CSP for Variables: Competing Risks

In many life testing experiment the failure might occur due to different causes. For example, failure of an automobile may be due to a surface defect or an interior defect. Thus it seems, several risk factors, compete with each other, for causing instant failure of an unit. In such a situation an investigator is often interested in the assessment of a specific cause, in presence of the other causes. In the statistical literature it is well known as the competing risks problem. In this case the data consists of the failure time an indicator denoting the cause of failure. There are two most popular models which deal with the competing risks data namely; the latent failure times model of Cox [8] and cause specific hazard function model of Prentice et al. [41].

Prajapati et al. [38] provided the optimal CSP for competing risks data when the data are Type-II censored. The problem can be formulated as follows. Suppose n items are put in a life testing experiment, and each item can fail due to two causes. Stop the experiment when m, where $m < n$, failure are observed. Since it is a competing risks data, one observes the cause of failure also along with the time of failure. Now based on these observations one has to take the decision either to accept the lot or reject the lot.

It is assumed that the latent lifetimes of the experimental units follow exponential distributions and they satisfy the latent failure times model assumptions of Cox [8]. Further, the model parameters have the Dirichlet-Gamma prior. If the loss function is linear, then Bayes decision function can be obtained in explicit form, and the optimal CSP can be obtained numerically. The authors have extended the results when the lifetime distributions are Weibull and also for Type-I hybrid censored data. There are several open problems in this area. First of all how the results can be extended if there are more than two causes of failures. Also how the results can be generalized for other lifetime distributions.

7 Conclusions

In this paper we have provided a gentle introduction of CSP. We have discussed briefly about the existing literature on attribute CSP and variable CSP. Recently an extensive amount of work is going on in developing optimal BCSP under different censoring schemes. We have briefly addressed these problems. Although we have not provided any new results, we have presented several open problems and recent references. Hopefully they will be useful for the readers for further research in this area.

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Related Topics: Censoring, exponential random variables, gamma random variables, competing risks, stopping rule, sequential testing; order statistics.