On Progressive Censoring

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Part of this work is going to appear in Technometrics.

Outline of the Talk

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- What is a Progressive Censoring?

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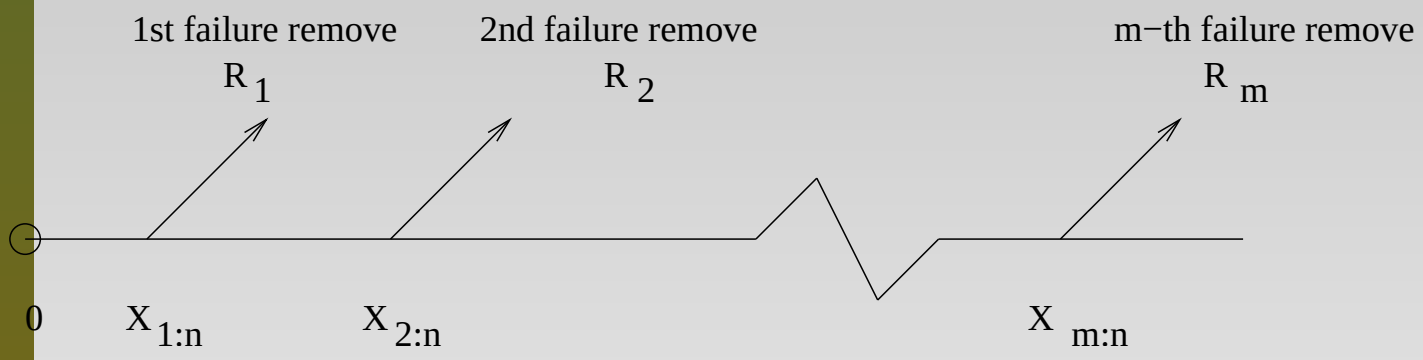
Progressive Censoring

- Put n items on test.
- Prefix m, $R_1, \ldots, R_m$, such that

$$R_1 + \ldots + R_m + m = n$$

- At the i-th failure time say $X_{i:n}$ remove R_i items from the remaining items.
- Stop the experiment at $X_{m:n}$.


1st failure remove
R 1
2nd failure remove
R 2
m−th failure remove
R m
0
X 1:n
X 2:n
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Associated Problems?

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- Inference on the Lifetime Distribution.

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- Inference on the Lifetime Distribution.
- Optimal Censoring Plans

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- Find Optimal Censoring Plans Using E or D Optimality

Proposed Methods

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- Assume Weibull Lifetime Distributions

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- Obtain Bayes Estimates and Credible Intervals using MCMC
- Propose New Optimal Censoring Plans

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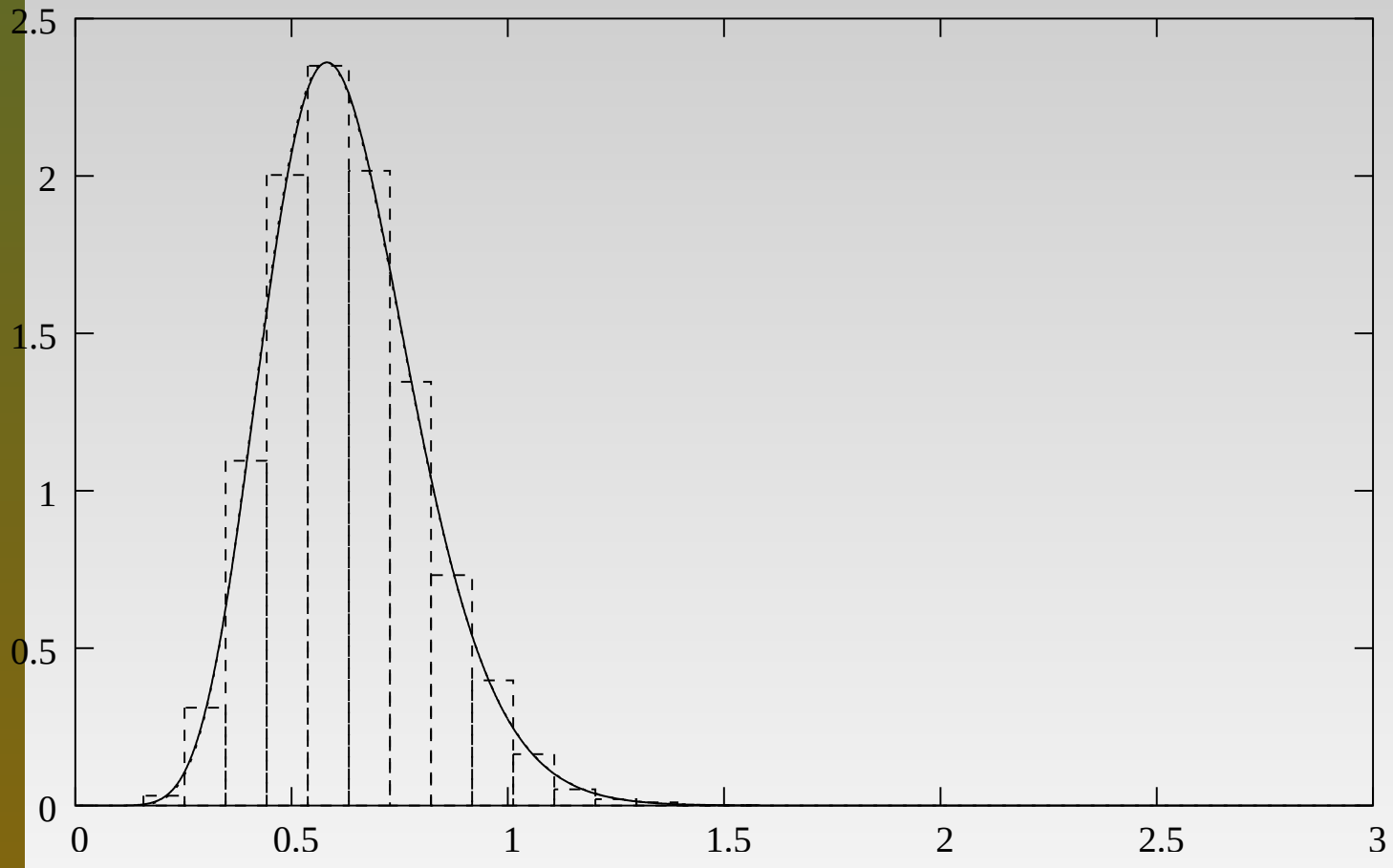

Posterior Density Function, Approximate Posterior Density Function and the Generated MCMC Samples

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- The variance of the p-th percentile estimator
- Criterion depends on p

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- Criterion 2:

$$C_2(P) = \frac{\int_0^1 V_{(P)}(\ln T_p)dW(p)}{\int_0^1 V_{(C)}(\ln T_p)dW(p)},$$

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- Choose P so that C_1 or C_2 is maximum

Some Open Problems:

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- Find a discrete optimization algorithm

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- Extend it to other censoring plans

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Thank You