Noise

in Electronic Devices and Circuits

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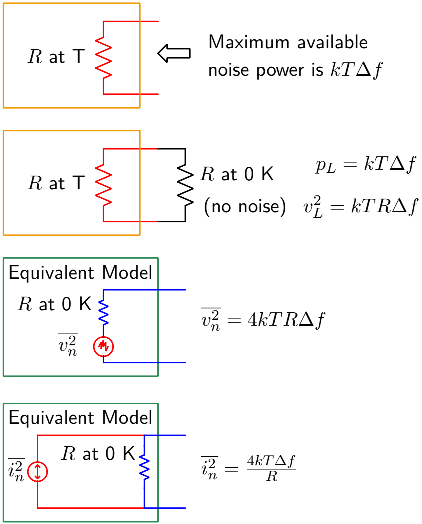
Noise

- Noise in electronic components - Random fluctuation of voltage or current with time $\Rightarrow$ a random process with zero mean value
- Since noise is a random process, we use statistical quantities to quantify the noise - power/voltage/current spectral density, mean-square (integrated) voltage/current, etc.
- Multiple origins/causes $\Rightarrow$ Multiple noise behaviours/spectral densities $S_n(f)$ $\Rightarrow$ Different types of noise
- Two important types of noise:
  1. Thermal noise (a.k.a. Johnson-Nyquist noise): Brownian motion of carriers due to temperature gives random fluctuations in current. Thermal noise power is proportional to the absolute temperature (in Kelvin). $S_n(f) \propto T$ but is independent of (or constant with) frequency. $\leftarrow$ White noise
  2. Flicker noise (a.k.a. $1/f$ noise, pink noise): —— $\leftarrow$ Still not fully understood but always present in dc-current carrying electronic components! Known spectral behaviour $\Rightarrow$ Spectral density is inversely proportional to frequency, $S_n(f) \propto \frac{1}{f}$

Thermal noise in resistors

- If a resistor R is at a temperature T, then the maximum available noise power over a (noise) bandwidth Δf is $kT\Delta f$, where k is the Boltzmann constant.
- We can extract maximum power from a resistor R by connecting a load resistor of equal value. $\leftarrow$ maximum power transfer theorem.
- We can model a noisy resistor by an equivalent noisy voltage (current) source $\overline{v_n^2}$ ($\overline{i_n^2}$) in series (parallel) with a noiseless resistor (i.e., resistor is at 0 K) using the idea of maximum available noise power.



Flicker noise in resistors

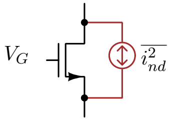
- Resistors exhibit Flicker noise only when dc current flows through them. It also depends on the material composition of resistors.
- Usually very small in thin-film resistors found in integrated circuits, but more in thick-film or discrete resistors.
- Flicker noise in a resistor is modeled by

$$\bar{v_n^2} = \frac{K_R}{f}\frac{R_\square}{A}V^2\Delta f,$$

  where K_R is a material-specific constant, $R_\square$ is the sheet resistance, A is the area of the resistor, and V is the dc voltage across the resistor.
- Sheet resistance $R_\square = \frac{\rho}{t}$, where ρ is the resistivity and t is the thickness of the thin-film. Typically, CMOS foundries mention $R_\square$ of metals and other layers instead of ρ. Resistance is $R_\square \times \frac{L}{W}$.

Thermal noise in MOSFET

- Recall that MOSFET is a tunable resistor and hence exhibit thermal and flicker noises similar to a resistor. Noise in a MOSFET is due to the channel resistance.
- We need to consider the dc resistance (slope of $I_D - V_{DS}$ in linear region) for noise instead of incremental resistance $r_{ds} = (1/\lambda I_d)$ we use in saturation.
- In saturation, the expression for g_m is same as the expression for channel resistance g_{ds0} in linear region and can be used in noise current expression.
- The factor γ is a model parameter and is equal to 2/3 for long channel device but is close to unity in short-channel devices.

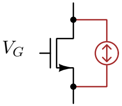


$$\overline{i_{nd}^2} = 4kT\gamma g_{ds0}\Delta f = 4kT\gamma g_m \Delta f$$

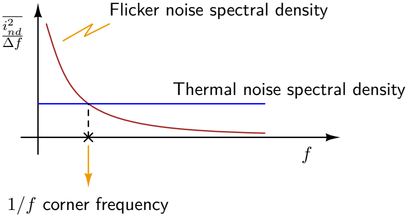
g_{ds0} is the channel conductance when $V_{DS} = 0$

γ is a constant; $\gamma = 2/3$ for long-channel MOSFET

Flicker noise in MOSFET

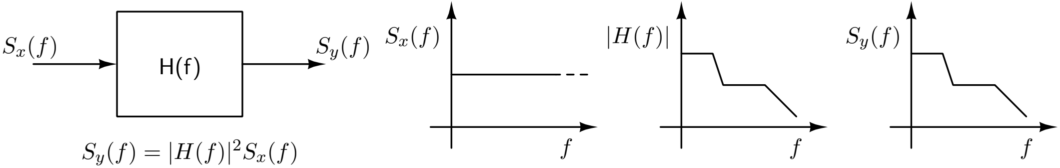


$$\overline{i^2_{nd,f}} = \frac{K_f}{f}\frac{g_m^2}{WLC_{ox}}\Delta f$$



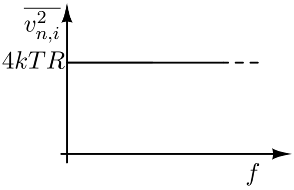
- Flicker noise in MOSFET is present only when dc current flows through it.
- Flicker noise current in a MOSFET is inversely proportional to the gate area: $W \times L$.
- K_f is a device dependant constant. K_f for a pMOS is roughly $(1/50)$ of nMOS.
- The frequency at which the $1/f$ noise density value equals to the thermal noise density is called "$1/f$ noise corner frequency," $f_c = \frac{K_f}{WLC_{ox}}\frac{g_m}{4kT\gamma}$.

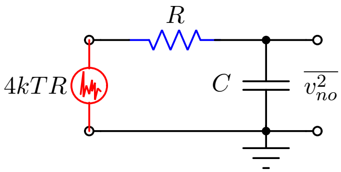
Noise through an LTI systems

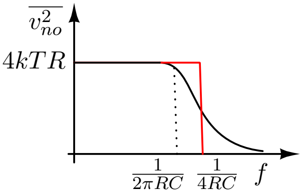


- If an LTI system with transfer function $H(s)$ is excited with noise having power spectral density (PSD) of $S_x(f)$, then the output PSD is $S_y(f) = |H(f)|^2 S_x(f)$. We assumed that the LTI system is noise-less here.
- If the input noise is white (constant for all frequencies), then the output noise will have the shape of the transfer function $|H(f)|^2$.
- If the LTI system is noisy and the output PSD due to internal noise is $S_o(f)$, then we can model the system to be a noise-less but with an equivalent input referred noise of $S_i(f) = \frac{S_o(f)}{|H(f)|^2}$.

Integrated (output) noise and noise bandwidth





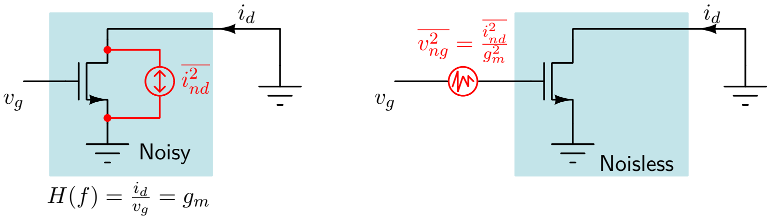


- The integrated output noise voltage in the above network is

$$\boxed{\overline{v_{no}^2} = \int_0^\infty \frac{4kTR}{1+(2\pi f RC)^2} df = \frac{kT}{C},}$$ which is independent of the resistance R.

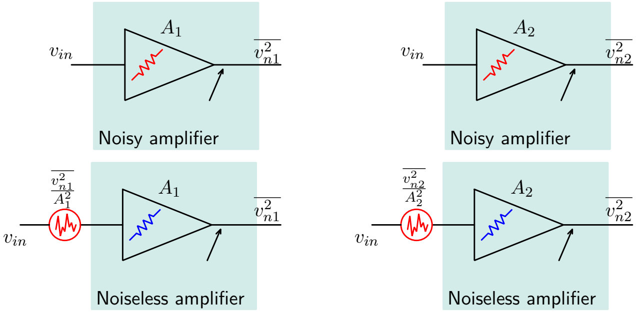
- Noise bandwidth:The bandwidth of a MODEL brick-wall-filter with amplitude equal to the peak SD. For the above example, if we assume the noise bandwidth to be B_n then the integrated noise in the brick-wall filter, $4kTRB_n$ should be equal to kT/C. $\Rightarrow B_n = \frac{1}{4RC}$

Input referred noise in a MOSFET



- We can refer the drain current noise to gate using the transfer function from gate to drain, $\overline{v_{ng}^2} = \frac{\overline{i_{nd}^2}}{g_m^2}$. Here $\overline{i_{nd}^2}$ can be thermal/ Flicker/total noise.
- Similarly for an amplifier or any other LTI circuit, we can represent the noise by referring it to the input.
- Caution: If the source is not grounded, we need to compute the transfer function first before referring $\overline{i_{nd}^2}$ to the input.

Input referred Vs. Output referred



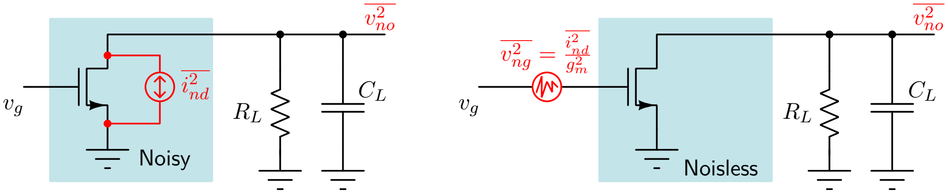
EX:

$$A_1 = 100, \overline{v_{n1}^2} = 1\mu V^2$$

$$A_2 = 200, \overline{v_{n1}^2} = 2\mu V^2$$

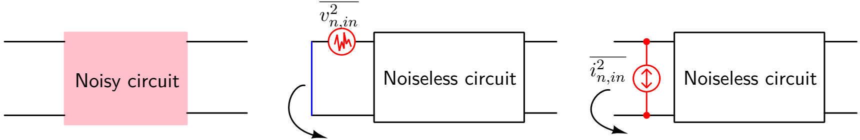
- Signal-to-Noise ratio (SNR) degrades as the signal propagates through noisy stages.
- If we want to compare two amplifiers, with different gains, for their noisy performance, we should check SNR rather than absolute noise at the output. The one with higher SNR at the output is less noisy.
- If we refer the total noise to the input, then the gain of the amplifier will not affect the signal/input power. We can simply use the absolute noise voltage/current at the input for comparison.

Input referred Vs. Output referred



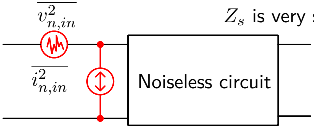
- Compute the integrated noise voltage at the input. Neglect the noise due to R_L.
- Remember that the input referred noise is just a model whereas the output referred noise is a measurable quantity. If we transfer the MOSFET noise alone to the gate, the integrated noise at the input will be ∞.
- Always compute the integrated noise at the output and then refer it to the input using peak gain.

Input referred noise models



Input needs to short circuited
or
Z_s is very small or Z_{in} is very high

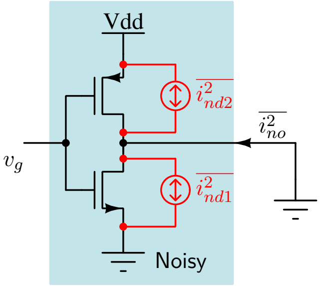
Input needs to open circuited
or
Z_s is very high or Z_{in} is very small



$\Longleftarrow$ $\overline{v_{n,in}^2}$ and $\overline{i_{n,in}^2}$ may be correlated.

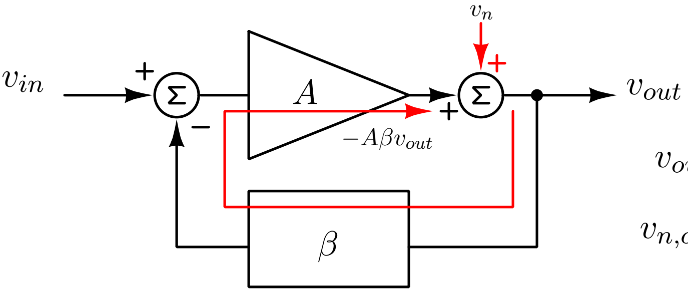
- One has to use both $\overline{v_{n,in}^2}$ and $\overline{i_{n,in}^2}$, particularly when the input impedance (Z_{in}) and source impedance (Z_s) are comparable, to model the input referred noise.
- We should observe the output noise when the input is short circuited and when the input is open circuited for modeling $\overline{v_{n,in}^2}$ and $\overline{i_{n,in}^2}$, respectively. If the input impedance is very high (looking into the gate of a CS stage at low frequencies) compared to the source impedance, we can neglect the effect of $\overline{i_{n,in}^2}$ at the input.

Circuits with multiple noise sources



- Uncorrelated noises add in mean-square (or noise powers are added): $\overline{i^2_{n,total}} = \overline{i^2_{n,1}} + \overline{i^2_{n,2}} + \overline{i^2_{n,3}} + \ldots$
- Noise due to different devices/components, even if they are identical copies of each other, are uncorrelated.
- If two noises are correlated, we can use instantaneous voltages or currents (with instantaneous polarity) while finding the total noise response.

Effect of feedback on noise

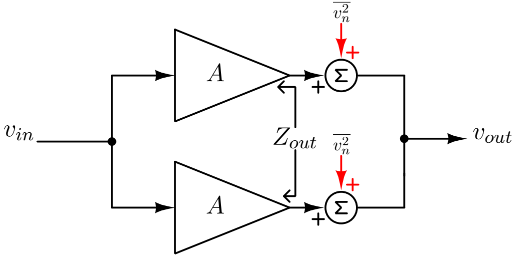


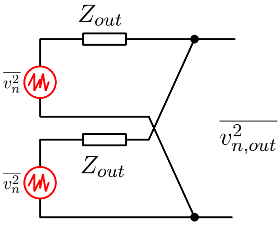
$$v_{out} = \frac{A}{1+A\beta} v_{in}$$

$$v_{n,out} = \frac{1}{1+A\beta} v_n$$

- In this example, we could deal with instantaneous noise voltage, instead of mean-square voltage, as we are dealing with just one noise source.
- Looking at the output noise alone and comparing it with open-loop amplifier noise (v_n) gives a wrong impression that feedback reduced noise.
- If we want to compare noise performance, we should compare SNRs at the output. In this case, no change in SNR due to feedback.

Reducing Noise - A trade-off with power





$$\overline{v_{n,out}^2} = \frac{\overline{v_n^2}}{4} + \frac{v_n^2}{4}$$

- One way to reduce the noise or improve the SNR at the output is to connect two identical amplifiers in parallel as shown above.
- Using the z-parameter model of the voltage amplifier, we can prove that the noise power at the output is halved whereas the signal power remained the same. The tradeoff is mainly with the power consumption. The bandwidth and area are also affected here.

References

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Thank you!

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