A Constant Gain Kalman Filter Approach to target tracking in Wireless Sensor Networks

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Abstract—**Target tracking in wireless sensor networks is an important area of research with applications in both the military and civilian domains. One of the most fundamental and widely used approaches to target tracking is the Kalman filter. In presence of unknown noise statistics there are difficulties in the Kalman filter yielding good results. In Kalman filter operation for state variable models with near constant noise and system parameters, it is well known that after the initial transient the gain tends to a steady state value. Hence working directly with Kalman gains it is possible to obtain good tracking results dispensing with the use of the usual covariances. The present work applies an innovations based cost function minimization approach to the target tracking problem in wireless sensor networks, in order to obtain the constant Kalman gain for both the stand-alone and data-fusion modes. Our numerical studies show that the constant gain Kalman filter gives good comparative performance in both the stand-alone and data-fusion modes for the target tracking problem. This is a significant finding in that the constant gain Kalman filter circumvents or in other words trades the gains with the filter statistics which are more difficult to obtain. To the best of our knowledge, these are the only studies of a constant gain Kalman filter in wireless sensor network scenarios, that also incorporate data fusion.**

I. INTRODUCTION

A wireless sensor network (WSN) is a spatial distribution of autonomous devices capable of interacting with each other by way of information exchange while sensing environmental and habitat changes. They are capable of cooperating to aggregate information and transferring the same on a hop by hop basis [1]. Such a set of devices by versatile deployment can be used to monitor or track objects in their vicinity, measuring position and or velocity of the object. One of the most widely used target tracking algorithms is the Kalman filter (KF) [2]. However the KF solution is a formal solution in the sense that it is optimal only when the noise statistics in the form of the state and measurement noise covariances (Q and R respectively) as well as the initial state error covariance (P0) , is available a priori. Thus tuning of these parameters is important to achieve good performance of the filter algorithm.

Tuning is a still not a well researched field though some studies have been made such as the innovations adaptive estimation (IAE) based method by Mehra [3] who showed its use in correlation and covariance matching techniques. Myers and Tapley [4] formalized this method in an effective manner to provide a mechanism for online adaptive tuning for Q and
R. More recent studies provide a combination of the innovation based IAE and adaptive faded Kalman filter (AFKF) in a hybrid scheme proposed in [5]. This scheme is applied for navigation sensor fusion and may not be well suited to our requirements. Another alternative is [6] which makes use of the IAE and proposes a cost function approach. Gemson, Ananthasayanam proposed a scheme for an adaptive extended Kalman filter in [7] to obtain P0, Q and R using the minimization of the cost function based on innovation.

Constant gain Kalman filters (CGKF) have been studied in [8-11]. However these involve working with the above filter statistics P0, Q and R which may not be optimal or near optimal and then deriving the constant Kalman gains. Recently a simple cost function minimization based CGKF approach has been suggested by Anil Kumar et al [11] in a problem concerned with prediction of re-entry of risk objects wherein they have used a genetic algorithm (GA) based minimization of an innovation cost function to compute an optimal constant gain matrix. In our work in a WSN target tracking problem using a similar cost function approach [6,11]. What is further known as a fundamental observation is

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that the KF gain stabilizes to a constant value after some point of time during the filter (algorithm) operation under conditions that the covariance matrices R,Q do not change subsequently. So the conceptual change involved is that one now works with the Kalman gain rather than the error and noise covariances (P,Q,R). It is typically observed that the filter estimates obtained are more robust to variation in gain as against variation in the error and noise covariances.

Data Fusion (DF) is another critical aspect in WSN. The distributed and energy constrained architecture of these net- works require development of an efficient algorithm that fuses together the information from multiple sensors to track the target more accurately than a single sensor. The two fundamental methods studied and analyzed are the state and measurement fusion techniques (SF,MF respectively) [12]. The paper extrapolates the results of CGKF for the stand-alone (SA) case and applies them to the DF scenario as well.

The main contribution of this paper is the following 1) The application of an innovations cost function based CGKF to a target tracking problem in WSN. The results are compared to those obtained from a reference KF (where noise covariances are assumed known) and further 2) The CGKF is applied to the tracking problem in a DF mode emulating the WSN environment. To the best of our knowledge such studies have not been carried out so far.

The paper is organized as follows. Section II describes the state variable (SV) model which defines the problem and the type of target being tracked. Section III gives the theory of the CGKF. Section IV defines the same for the DF mode. Section V contains the numerical studies and results. Section VI provides a summary and discussion of the results.

II.STATE VARIABLE MODEL

A) Problem Description and Scope

The objective is to track the target successfully based on the prerequisite of sensor positions being known a priori. The target trajectory considered is assumed to be a straight line between measurement time instants with minor perturbations of random nature in the slope. These perturbations appear in the SV model described later as part of the control input. The nodes are deployed in a random manner. Node localization and subsequent localization of target based on this knowledge, is altogether a problem in itself which has been well handled by several authors. Standard techniques based on triangulation and trilateration [13,14] exist. The scope of present works is based on the fact that node positions as well as target positions are known or determined by the mentioned methods and hence we limit ourselves to employing a target tracking algorithm after having obtained this necessary and vital information. In this paper we focus on the crux of the tracking application which is the algorithm itself and its modified version incorporating DF which is a step in moving towards realization of a full tracking application within the sensor network domain.

B) State Variable Model

A two dimensional model for the target tracking problem is

State Equation:

$$X_{t+1} = AX_t + U + w_t \tag{1}$$

: where state vector is $X_t = \begin{pmatrix} x(t) \\ y(t) \\ m(t) \end{pmatrix}$, state transition matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \triangle t \\ 0 & 0 & 1 \end{pmatrix}$, control input $U = \begin{pmatrix} \triangle t \\ 0 \\ 0 \end{pmatrix}$ and w_t represents system noise. Here we have considered the state vector to include X and Y coordinates of the target as well as the slope and in effect direction of the moving target.

Measurement Equation:

$$Y_t = CX_t + n_t \tag{2}$$

: where Y_t is the measurement vector,$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $n_t$is measurement noise

It is important to mention that sensor nodes depending on their cost and configuration may not be capable of providing coordinate measurements which are independent of one another as is described above (refer to the measurement matrix C). The prerequisite for the present work is that this information or an algorithm to this effect is either inbuilt into the node as in [15] or is calculated as in [16] prior to employing the tracking algorithm.

III. CONSTANT GAIN KALMAN FILTER

A) Motivation

Under conditions when the system, measurement and state error covariances Q, R and P0 makes the filter to stabilize after sometime and stabilizes the gain matrix.

This motivates us to determine this value of the gain matrix and use it to track the target effectively right from the beginning. Typical CGKF settings have been analyzed and described in works such as [8-11]. The fact that the Gain stabilizes is observed from plot of the Filter Gain K versus the predicted error covariance$P$. A typical example of this plot is shown in the Figure. 1.

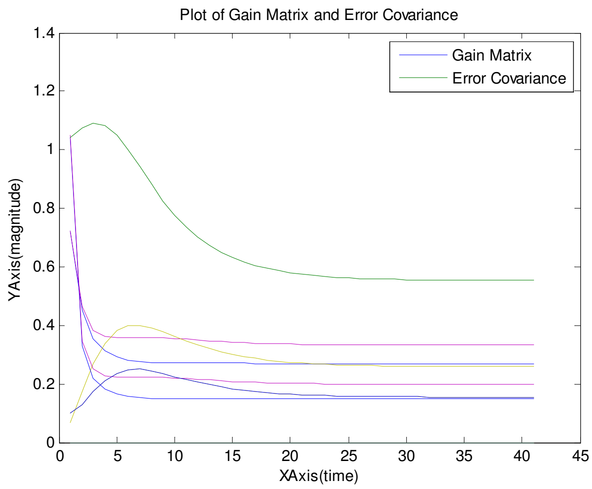

Fig. 1. Gain *K* vs error covariance matrix *P*

It is seen from the figure. 1 that 1) The gain K stabilizes to a steady state value at a certain given point of time. 2) As the gain reaches a steady state value the predicted error covariance$P$ also reduces to a steady state value indicative of the fact that the filter is tracking the target quite well at this point of time. Thus the above reasons show the advantage in the use of the CGKF approach in tracking the target.

B) The Estimation Scheme

The generic KF updates are

$$\hat{X}_t = \bar{X}_t + K_t v_t \tag{3}$$

where the the innovations sequence is $v_t = Y_t - C\bar{X}_t$. The standard KF computes the gain matrix K_t using the noise and state error covariances while we proceed to estimate this optimal value of K_t (denoted henceforth by K^* ,representative of the constant gain) for the CGKF, by solving an optimization problem. It is well known that the innovations sequence of a KF converges to a zero mean white Gaussian process. Maximizing this probability leads us to the following solution

$$(K^*, \mathcal{R}^*) = argmin_{K,\mathcal{R}} J(K, \mathcal{R}) \tag{4}$$

where the cost function is

$$J(K, \mathcal{R}) = \frac{1}{N} \sum_{t=1}^{N} (v_t^T \mathcal{R} v_t + log(|\mathcal{R}|)) \tag{5}$$

and $\mathcal{R}$ represents the covariance of the innovations v_t . In order to solve the optimization problem we can either use local gradient based methods (such as Newton type schemes) or global schemes such as a GA [17]. As the filter tracks the target the gain K is seen to stabilize to a value given by the solution of the above problem. Alternatively a residual instead of an innovations based cost function approach may be adopted wherein v_t in equation 5 is replaced by $v_t = Y_t - C\hat{X}_t$. But in our studies and analysis of results it was found that both approaches give comparative performance. Hence we proceed in our current work to show results based on an innovations cost function approach. Once we have computed the optimal K^* the KF recursions without the usual covariances become

Predict :

$$\overline{X}_{t+1} = A\hat{X}_{t+1} + U \tag{6}$$

:

Update:

$$\hat{X}_{t+1} = \overline{X}_{t+1} + K^*(Y_{t+1} - C\bar{X}_{t+1}) \tag{7}$$

: We observe that the typically expensive covariance time update step is not needed in the constant gain approach. The KF is considered to be running at one of the nodes ideally as part of the cluster head node (CHN) within the vicinity of the target location .

IV. DATA FUSION ASPECTS

The necessity of DF in WSN is integral to its distributed architecture.The WSN consists of distributed group of nodes which collaborate to combine individual information states of a target in a manner so that the consolidated information state so obtained is better (less uncertainty regarding the target) than the information states of the ISN. In order to obtain the consolidated information state or in other words to combine the estimates or measurements of the individual nodes , DF is employed.

Typically fusion is either state based [18], measurement based [19] or a combination thereof [20]. State fusion techniques combine state estimates from the ISN. Measurement fusion techniques combine the raw measurements of the target obtained from the ISN at the CHN Level. In hybrid (state & measurement) fusion techniques [19] CHN uses measurements from neighbouring sensors as well as their state estimates in forming its own estimates , over time the sensors reach a consensus on the state. A distributed KF is implemented in the form of a combination of sensor nodes which constitute the WSN. Two types of nodes exist based on computational resources 1) Individual sensor nodes (ISN) - These are capable of making measurement of the target position and depending on the type of fusion method may be considered (SF or MF technique) to run a light weight form of the tracking algorithm (SF). 2) CHN - These are capable of running a more complex form of the tracking algorithm based on the fusion method employed. Thus these nodes unlike the ordinary nodes need to first fuse either states or measurements of the target or a combination of both prior to employing the target tracking algorithm. The two basic methods of DF are described in [12]. In our work we have currently obtained results in a measurement fusion based framework given below for completeness [18]

$$Y_t = \frac{\sum_{i=1}^{N}(w_t^i Y_t^i)}{\sum_{i=1}^{N} w_t^i} \tag{8}$$

$$C_t = \frac{\sum_{i=1}^{N}(w_t^i C_t^i)}{\sum_{i=1}^{N} w_t^i} \tag{9}$$

$$R_t = \frac{\sum_{i=1}^{N}(w_t^i R_t^i)}{\sum_{i=1}^{N} w_t^i} \tag{10}$$

where Y_t, C_t and R_t are the composite measurement vector, measurement matrix and measurement noise covariance matrix respectively obtained by combining respective components from the N sensors sensing the target at that specific time instant. w_t^i is the weight allotted to the i^{th} sensor. This represents the probability of the correctness of the specific parameter with regard to the i^{th} sensor. The possible choices for the weights are based on [19,21] and given by

$$w_t^i = \frac{1}{R_t^i} \tag{11}$$

$$w_t^i = \frac{1}{(d_t^i)^r} \tag{12}$$

where R_t^i represents the measurement noise covariance matrix of the i^{th}senor , d_t^i the distance of the i^{th}sensor from the target and r represents the path loss exponent. In our simulations we have used (11) for the simple reason that distance d_t^i varies for every sensor while R_t^i is a fixed modeling parameter representative of the measurement error covariance. In practical applications this value would be calibrated and considered uniform for all sensors, hence here we choose this as a choice for the weight as opposed to the path loss exponent option. All the other steps in the standard KF or CGKF remain the same. We observe that in the SV model for DF only measurement vector, measurement matrix and the measurement noise covariance matrix are enhanced accordingly while all other steps remain same. The algorithm runs in the CHN based on measurements obtained from ISN.

In Raol, [pg 72-73,12] a comparison between state and measurement fusion techniques has been provided which shows that measurement fusion techniques are found to be more accurate than their state fusion counterparts. The same findings have been corroborated by us for the state and consensus fusion methods (hybrid method combining both state and measurement fusion ,[20]), wherein during the course of simulations it was observed that tracking was not successful for fusion cases involving more than two senors. Thus in the present work we focus on a presenting results for one of the measurement fusion techniques namely weighted fusion which is less computationally expensive while not compromising on performance.

V) NUMERICAL STUDIES

Stand alone mode

The two dimensional numerical studies have been carried out an a set of forty data points with the following system and measurement covariances matrices being used to generate the simulated track

$Q = .01I,\ R = .1I$

Error metric used is Percentage Fit Error (PFE) defined as $PFE = \frac{|X_t - \bar{X}_t|}{|X_t|} \times 100$ which represents the difference between the estimated and actual track. The error metric shown in tables is the average value computed over 1000 runs while the plots correspond to one specific run wherein results are presented in the form of plots of the simulated target trajectory, simulated measurements and the estimated track against time. Figure. 4 illustrates the plot of the actual track and error against time. This proves that the algorithm is able to track the target reasonably well since the magnitude of error in estimation of the target trajectory is much less in comparison to that of the actual track. Figures. 2-4 and Table. I show that the performance of the CGKF is comparable to the reference KF which uses complete knowledge of system parameters unlike the CGKF which works with only the constant gain. This provides the necessary justification in using the CGKF for the target tracking problem in SA mode as well.

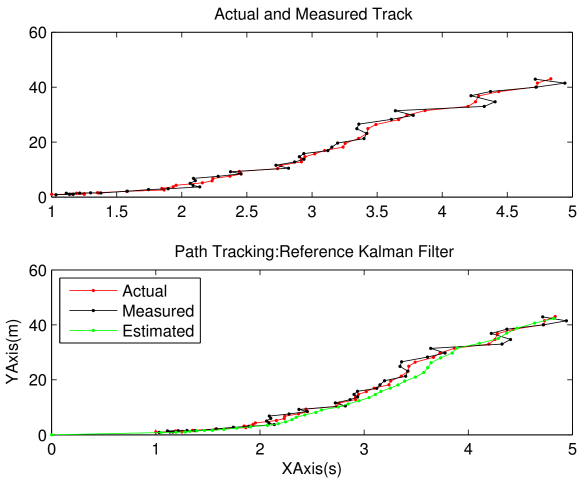

Fig. 2. Reference Kalman filter:SA mode

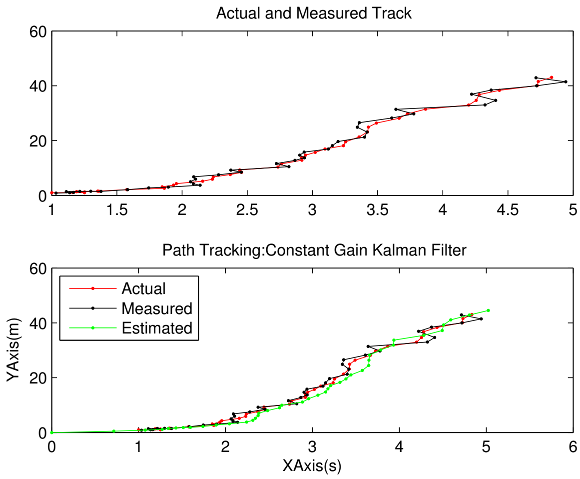

Fig. 3. Constant gain Kalman filter:SA mode

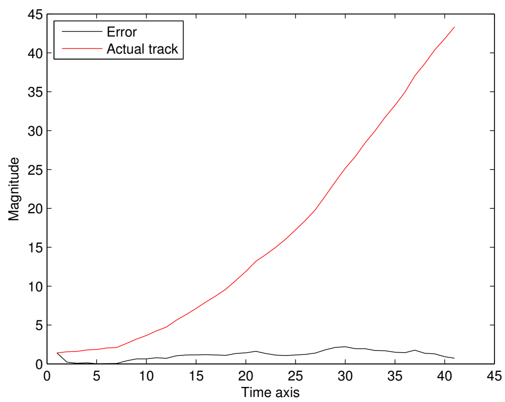

Fig. 4. Error vs actual track:SA mode

Filter	PFE
Reference KF	20.6703%
CGKF	15.6099%

TABLE I
ERROR METRIC: SA MODE

Filter	PFE:2 Sensor fusion	PFE:4 Sensor fusion
Reference KF	19.3873%	16.7898%
CGKF	17.6856%	16.2528%

TABLE II
ERROR METRIC:DATA FUSION MODE

Data fusion mode

The two dimensional numerical studies for fusion have been carried out based on two and four sensors for a set of forty data points. The following system and measurement covariances matrices have been used to generate the simulated track.

$Q = .01I$, $R = .1I$

The error metric shown in tables is the average value computed over 1000 runs while the plots correspond to one specific run wherein results are presented in the form of plots of the simulated target trajectory, simulated measurements and the estimated track against time. Results are presented for the case of two and four sensor DF case, in Figures. 5-10. From Figure. 7 and Table. II. The measurement track displayed in the figures is the composite weighted measurement, based on measurements obtained from the sensors. Thus we present results based on the weighted method here which is less computationally intensive than the We observe that the magnitude of error is much less in comparison with the actual track indicating the satisfactory performance of the tracking algorithm. Following are the deductions.

1) The CGKF performance in the case of two and four sensor fusion is comparable with that of the reference KF which justifies our use of the CGKF for the target tracking problem in WSNs.

2) The PFE for four sensor fusion case is comparatively less than the corresponding values for the two sensor fusion case thereby justifying the use of DF in WSN in order to increase our tracking efficiency.

3) It has been noticed that even when the steady state gains obtained based on the filter statistics is substantially reduced the filter is tracking the target well after the transient. This shows the robustness of working with the Kalman gain instead of the statistics.

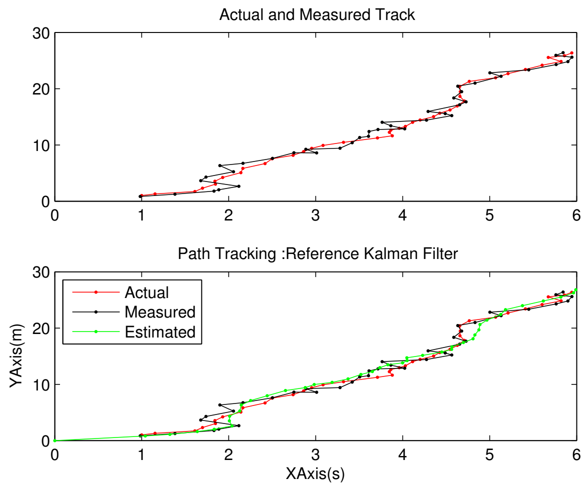

Fig. 5. Reference Kalman filter: 2 sensors DF mode

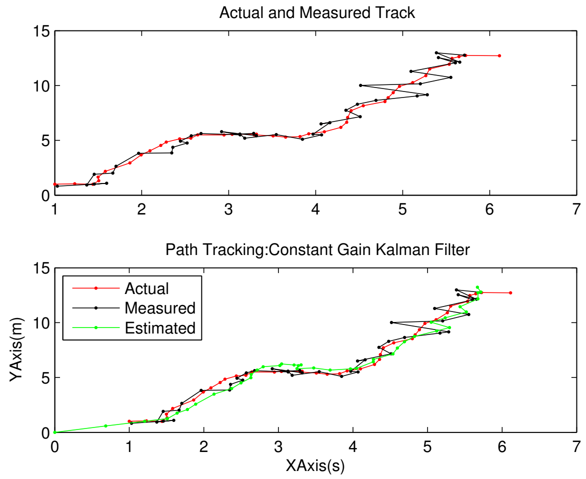

Fig. 6. Constant gain Kalman filter: 2 sensors DF mode

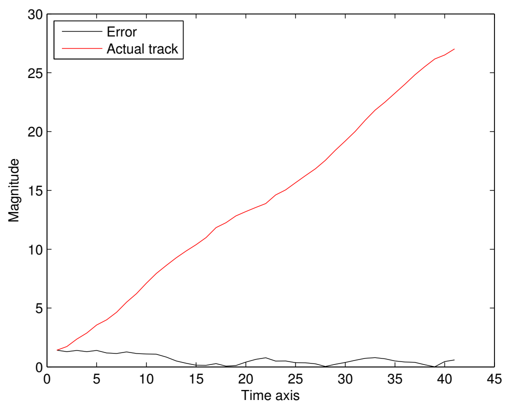

Fig. 7. Error vs actual track:2 sensors DF mode

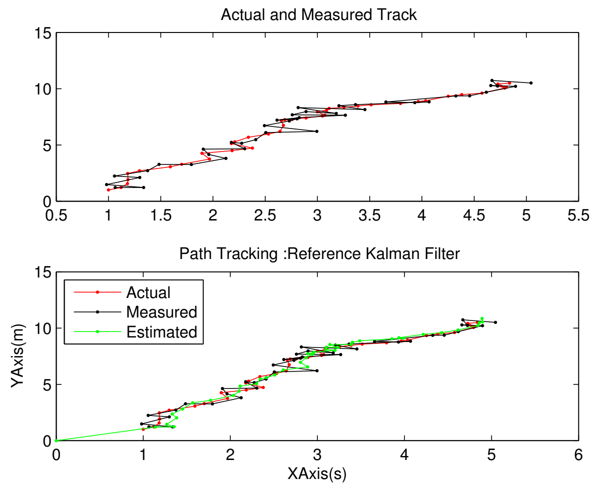

Fig. 8. Reference Kalman filter: 4 sensors DF mode

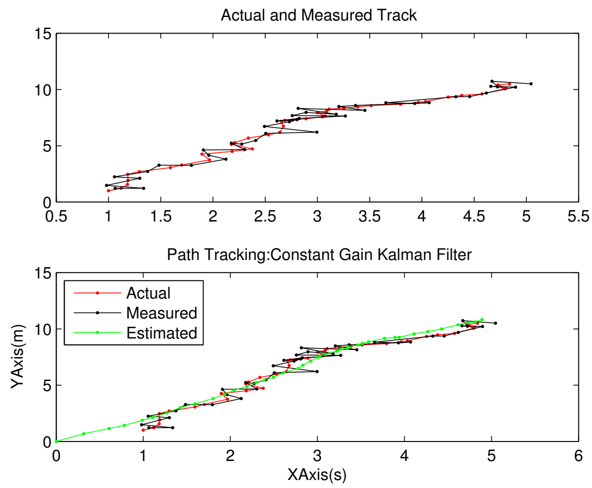

Fig. 9. Constant gain Kalman filter: 4 sensors DF mode

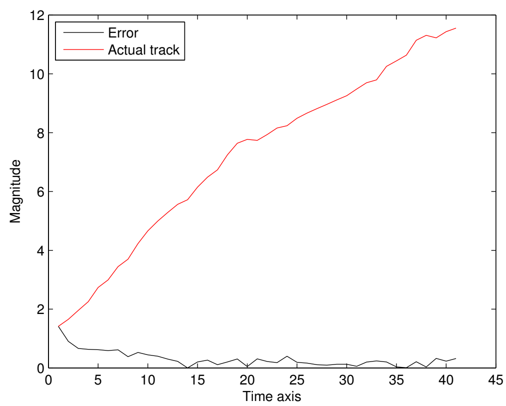

Fig. 10. Error vs actual track:4 sensors DF mode

VI. Conclusion

The results obtained show the CGKF performing comparably with a reference KF in both SA and DF modes of operation. This is a significant finding since the CGKF circumvents, or in other words trades the gains with the filter statistics which are more difficult to obtain. The present results prove that the CGKF is successful in target tracking applications where uncertainty regarding noise statistics generally exist. To the best of our knowledge, these are the only studies of a constant gain Kalman filter in wireless sensor network scenarios, that also incorporate data fusion.

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