Performance of Error Estimates on Noisy Photoacoustic Data for Universal Backprojection Based Photoacoustic Tomography

Prabodh Kumar Pandey¹, Naren Naik ^{2,3}, Prabhat Munshi ^{4,5}, Asima Pradhan^{1,3}

¹ Department of Physics, Indian Institute of Technology, Kanpur, Kanpur, India, 208016,
²Dept. of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India-208016,
³Center for Laser and Photonics, Indian Institute of Technology Kanpur, Kanpur, India-208016,
⁴Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur, India-208016,
⁵Nuclear Engineering and Technology Programme, Indian Institute of Technology Kanpur, Kanpur, India-208016.

Corresponding author: nnaik@iitk.ac.in

Abstract: We provide universal backprojection based photoacoustic error estimates for any detection geometry. We further study its performance on noisy data and provide an algorithm for choice of optimum frequency window for reconstruction.

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1. Introduction

Irradiating a sample with laser pulse leads to thermoelastic expansion and hence a pressure rise (photoacoustic source), that is proportional to absorbed optical energy [1]. This initial pressure rise propagates in the medium as pressure waves (photoacoustic waves), which can be detected by ultrasonic transducers. Photoacoustic tomography (PAT) targets to reconstruct the PA source using the PA signals acquired at a detector grid, which partially or completely covers the irradiated sample. Recovery of the PA source is carried out using analytical as well as numerical approaches such as Radon transforms, time reversal, finite elements and Fourier domain based algorithms etc. Our work is based on universal back-projection (UBP) based photoacoustic reconstruction algorithm [2]. The UBP algorithm has similarities with the filtered backprojection(FBP) algorithm, used for reconstructions in X-ray CT [3].

Error estimates for FBP based computerized tomography (CT) have been developed and are being used to discriminate between "good" and "bad" data [4–6]. However, the error estimates have rather been unexplored in the case of PAT, apart from an error estimate with respect to the cut-off time, in time reversal based PAT for non trapping speeds [7]. In [8], we derived and numerically validated UBP based PA error estimates for planar detection geometry. Our present work comprises of error estimates for any detection geometry w.r.t. bandlimited Ram-Lak reconstruction for UBP based PAT. We further study the developed error estimates for noisy data and propose a method to choose the optimum cut-off frequency window, where the contribution from noise is dominated by the actual data.

2. Error estimates

In UBP based PA reconstruction, the accuracy of reconstruction is governed by experimental as well as computational parameters (detection geometry, detector size and spacing and sampling frequency, window function and cut-off frequency). We have concentrated on the inherent error that transpires due to the variety of filters used to filter the data. The filter functions needed for the filtering of PA signals are of the form:

$$H(k) = \begin{cases} W(k/k_c) & : |\vec{k}| < k_c \\ 0 & : otherwise. \end{cases}$$
(1)

where $W(k/k_c)$ is a window function ($W(k/k_c) = 1$ for band-limited Ram-Lak window) and k_c is the cut off frequency (chosen sufficiently large). If the forward data, generated over the detection surface S_0 at detector co-ordinates \vec{r}_0 is filtered using a filter H(k), the filtered reconstruction of the PA source $p_0(\vec{r})$ can be given by [2]:

$$p_{0_W}^r(\vec{r}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \int_{S_0} dS_0 \tilde{p}(\vec{r}_0, k) H(k) [\hat{n}_0^{S_0}.\vec{\nabla}_0 \tilde{G}_k^{(D)}(\vec{r}, \vec{r}_0)]$$
(2)



Fig. 1: Cross section of (a) original phantom **P**; Reconstruction of **P** using PA data with 10dB SNR with 2 MHz cut-off frequency using (b) Hanning window, (c) Ram-Lak Window; Reconstruction of **P** using PA data with 10dB SNR with Ram-Lak and (d) 2.67 MHz cut-off frequency, (e) 4 MHz cut-off frequency; Reconstruction of **P** using PA data with 20dB SNR with Ram-Lak and (f) 4 MHz cut-off frequency, (g) 8 MHz cut-off frequency

where $\hat{n}_0^{S_0}$ denotes the unit normal to S_0 and $\tilde{G}_k^{(D)}(\vec{r},\vec{r}_0)$ is the Green function corresponding to the Helmholtz equation. After substitution in equation (2) and on further solving we obtain the averaged error(ε)

$$\varepsilon = \frac{1}{N_x N_y N_z} \sum_{i,j,k=0}^{N_x, N_y, N_z} |p_{0_W}^r(i,j,k) - p_{0_{BL}}^r(i,j,k)| = -\frac{1}{N_x N_y N_z} \left(\frac{W''(0)}{k_c^2}\right) \sum_{i,j,k=0}^{N_x, N_y, N_z} \bigtriangledown^2 p_{0_{BL}}^r(x_i, y_j, z_k)$$
(3)

where $p_{0_{RI}}^{r}$ represents bandlimited Ram-Lak reconstruction.

3. Numerical studies and the proposed strategy for choosing optimum cut-off frequency

The k-wave toolbox [9] is used to generate the PA data for a 200 x 200 x 200 grid with 1/5 mm grid resolution. The pressure signals have been recorded on spherical detector grid (detectors distributed uniformly on a 1.5cm sphere, separated by 1.9mm) at 8 MHz sampling frequency. The phantom used in this study is **P**: a big cylinder with eleven small cylinders and two cuboids inside(Figure 1a).

The process of filtering of data always has a trade off. Choosing a small cut off window leads to loss of information while a large cut off window brings in contribution from noise in the data. Information about sharp boundaries of sources are encrypted in the higher frequency amplitudes of the data. Therefore using a smooth window function (high W''(0)) in a small cut-off frequency window may at times generate false reconstruction(Figure 1b,1c). The graphs are plotted for errors in reconstruction ε w.r.t. W''(0), which come out to be linear for different cut-off frequencies. As in the previous work [8], for spherical detection geometry too, we see that the slopes of the ε vs W''(0) fits are found to follow an order, in which the slopes increase with decreasing cut off frequencies. With noisy data sets, the ε vs W''(0) linearity is maintained but the ordering of the slopes no longer follow the aforementioned trend. With the increase in SNR, the perturbation in the slope order keeps diminishing and the order finally becomes the same as noiseless case for high SNRs. Thus, we can infer that the cut-off frequency, below which the ordering of slopes for the ε vs W''(0) plots obtained from reconstructions of phantom **P** (Figure 2), we can conclude the optimum cut-off frequencies to be 1.6 MHz, 2.67 MHz, 4 MHz and 8 MHz for the data of 5 dB, 10 dB, 20 dB and 30 dB SNRs and this choice of cut-off frequency is justified by the reconstructions provided for comparison in figures 1d-1g.



Fig. 2: Averaged error for phantom **P** in spherical geometry (noisy forward data)

4. Conclusions

As an extension of our previous work [8], we have developed UBP based PAT error estimates for any detection geometry(Eq.(3)). We have validated our result for spherical detection geometry and further, we provide a strategy to choose an optimum cut-off frequency window for PA reconstruction.

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