

# Shape based reconstructions for $SP_3$ -modeled 3D fluorescence optical tomography

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**Abstract:** We present first fully-nonlinear 3D  $SP_3$  based FOT reconstructions in a shape based framework. We solve for the excitation-wavelength absorption coefficient of anomalies along with their boundaries using a compactly supported parametric level-set shape-representation. Preliminary results obtained with a trust region based iteratively Tikhonov-regularized Gauss-Newton algorithm for tumour mimicking phantoms show good localization and parameter reconstructions.

**OCIS codes:** 100.3190, 110.6955

## 1. Introduction

In early cancer detection, location and shape of physiological markers in the pre-cancerous tissue is a critical issue which motivates the use of fluorescence from intrinsic or extrinsic fluorophores in the affected regions. Fluorescence optical tomography (FOT) is an emerging imaging modality that enables us to capture functional as well as morphological features in the region of interest.

Typically, in optical tomography, light propagation through tissue is modelled by the radiative transfer equation (RTE). However, its applicability is restricted owing to its computational complexity. The simplified spherical harmonics ( $SP_N$ ) approximation to the RTE [1], is simpler than the higher order approximations such as  $P_N$  and  $S_N$ , and more accurate than the diffusion approximation. The formulation and solution of the  $SP_3$ -approximated fully-nonlinear reconstruction in FOT has been proposed by us for the first time in [2]. In [2] pointwise and shape-based (using non-compactly supported radial basis function (RBF) level-set boundary representation) reconstructions have also been demonstrated in 2D.

We have now extended our reconstruction scheme to three-dimensional FOT settings. In a three dimensional (3D) tomographic reconstruction problem, as in 2D, shape based representation of unknown coefficient(s) drastically reduces the number of unknowns. In 3D, we use compactly supported RBFs for level-set boundary representation [4]. To the best of our knowledge, these are the first fully-nonlinear 3D reconstructions in  $SP_3$  approximated FOT. The shape and contrast of the anomalies are reconstructed using an iteratively regularized Gauss-Newton method in a trust region framework [3].

## 2. Problem definition

The  $SP_3$  approximation to the coupled RTE, which describe the light propagation in tissue over a domain  $V$  is given by the following set of coupled equations [1] for the composite moments  $\phi$ , as:

$$-\underline{\nabla} \cdot C^\nabla \underline{\nabla} \phi + C^\phi \phi = 0 \quad \text{in } V; \quad C^{\nabla b}(n, \underline{\nabla} \phi) + C^b \phi = C^S \quad \text{on } \partial V \quad (1)$$

where  $C^\nabla$ ,  $C^\phi$ ,  $C^b$ ,  $C^{\nabla b}$ ,  $C^S$  and  $\phi^{x/m}$  are defined in [2] as per [1]. The measurements are the exiting partial current  $j^m$  on the boundary obtained by solving equation 1 using the finite element method. The Frechet derivative of the measurements with respect to the fluorophore absorption coefficient ( $\mu_{axf}$ ) using an adjoint method is derived in Naik *et al.* [2]. The level-set based representation of spatially varying  $\mu_{axf}$  is given by:

$$\mu_{axf}(x, \gamma) = \mu_{axf}^i H(\phi(x, \gamma)) + \mu_{axf}^o (1 - H(\phi(x, \gamma))) \quad (2)$$

where  $\gamma = \{\alpha_j, \zeta_j, \chi_j\}$  are the level set parameters [4],  $\zeta_j$  is the dilation factor,  $\chi_j$  is the RBF center coordinates and  $\alpha_j$  is the weighting factor.  $\mu_{axf}^o$  and  $\mu_{axf}^i$  are the fluorophore absorption coefficient values (assumed to be piecewise constant) in healthy and cancerous tissue respectively.  $H(\phi)$  is the Heaviside function with  $\phi$  being the level-set function. The Frechet derivative of the measurements with respect to the shape parameters is obtained via the chain rule, as:  $\frac{\partial j^m}{\partial p} = \frac{\partial j^m}{\partial \mu_{axf}} \frac{\partial \mu_{axf}(x, \gamma)}{\partial p}$ ; where  $p \in \{\mu_{axf}^i, \mu_{axf}^o, \gamma\}$  is the parameter set to be reconstructed. We solve

for  $p$  by minimizing the regularized nonlinear least squares problem using Gauss-Newton update in a trust region framework which is given by [3, 5] :  $(\mathbf{J}^T \mathbf{J} + \tau \mathbf{I} + \lambda \mathbf{I}) \delta p = -(\mathbf{J}^T \mathbf{r} + \tau(p - p_c))$ , where  $\mathbf{J}$ ,  $\tau$ ,  $p_c$  and  $\mathbf{r}$  are the Jacobian, regularization parameter, apriori information and residual respectively and trust region parameter  $\lambda$  is calculated based on the trust region radius. The measurements and derivatives are rescaled using logarithmic transformation [5], prior to reconstruction.

### 3. Numerical Results

We consider a computational domain of size of  $2 \times 2 \times 2 \text{ cm}^3$ . A total of 12 sources (2 on each face) and 120 (20 on each face) detectors are used to collect the data. Synthetic partial-current data at detector locations are generated with a mesh with 384000 tetrahedrons and 68921 nodes and corrupted with 1% noise. Reconstructions are performed on a coarser mesh with 48000 tetrahedrons and 9261 nodes. The shape reconstructions are shown in figure 1. Numerical studies are performed on two phantoms; a two object phantom (TX) and a single bean shaped phantom (BX) described in table 1. Other background optical properties are modelled as in Naik *et al.* [2].

Table 1: Description of the object(s), and initial (*init*) estimates and reconstructed (*rec*) values of  $\mu_{axf}$

	object	approx. dimensions ( $\text{cm}^3$ )	$\mu_{axf}^i$	$\mu_{axf}^o$	$\mu_{axf}^{i,init}$	$\mu_{axf}^{o,init}$	$\mu_{axf}^{i,rec}$	$\mu_{axf}^{o,rec}$
Phantom TX	Two object	$0.2 \times 0.4 \times 0.5$ (per obj)	0.3	0.006	0.006	0.0012	0.1696	0.0057
Phantom BX	Bean	$0.3 \times 0.4 \times 0.7$	0.3	0.006	0.006	0.0012	0.1305	0.0058

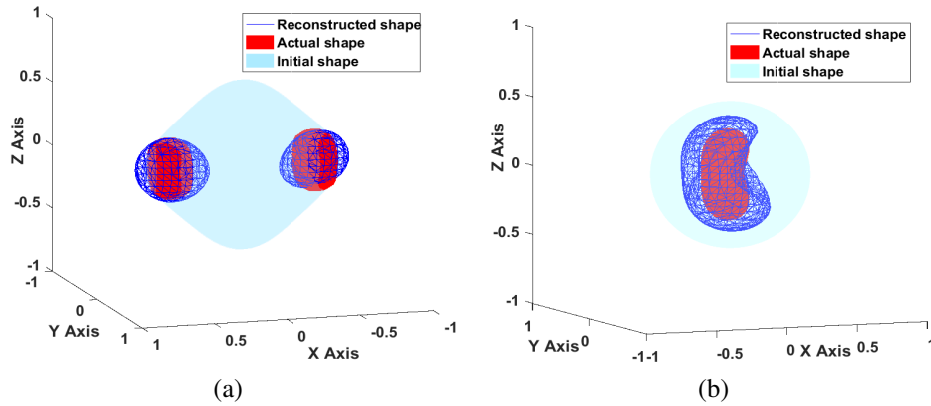


Fig. 1: Reconstruction of (a) two object phantom (b) bean phantom. Red colour denotes the true object, mesh in dark blue, the reconstructed object and the sky blue shade denotes the initial estimate

### 4. Conclusion

First fully-nonlinear 3D  $SP_3$  based FOT reconstructions in a shape based framework are presented. Numerical studies on tumor mimicking phantoms with single and double inclusions show a good localization of the objects shape reconstructions.

### References

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