Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Mid-Semester Examination 2020-21-II

Abstract Algebra (MTH 204A/B)

Date: February 25, 2021

Time: 2 hours | Maximum marks: 30

1. Let $n \geq 5$. Find all proper subgroup $H \leq S_n$ such that $[S_n : H] < n$. [4]

2. Assume that G is a finite group and X is a finite transitive G-space having at least two elements. For any $g \in G$, we let $f(g)$ denote the number of elements of X fixed by g. Show that, G is doubly transitive $\iff \sum_{g \in G} f(g)^2 = 2|G|$. [4]

3. (a) Let G be such a group of order 56 that its Sylow 7-subgroup is not normal. Show that any Sylow 2-subgroup of G is isomorphic to $\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$.

   (b) Find the number of groups, up to isomorphisms, of order 56 with a nonnormal Sylow 7-subgroup.

   (c) Find the number of groups, up to isomorphisms, of order 56 with Sylow 7-subgroup normal and a nonnormal Sylow 2-subgroup isomorphic to D_8?

   [4+3+5=12]

4. (a) Show that there is a transitive subgroup H of S_6 with the following three properties:

   (i) H is simple,

   (ii) $|H| = 60$, and

   (iii) H is contained in A_6.

   (b) Prove or disprove the following:

   For every $\varphi \in Aut(A_6)$ there exists $\sigma \in S_6$ such that $\varphi(x) = \sigma x \sigma^{-1}$, for all $x \in A_6$.

   [4+5=9]

5. Does there exist a group G, finite or infinite, such that $G' = S_4$? Justify your answer. [3]

The End