

$$E(n_t) = \sum_{n_t=0}^n n_t \cdot \binom{n}{n_t} p_t^{n_t} (1-p_t)^{n-n_t}$$

$$= n p_t$$

$$EC(T) = \frac{[c_1 \sum_{t=1}^T E(n_t)] + [c_2 (n - E(n_T))]}{T}$$

$$= \frac{[c_1 n \sum_{t=1}^T p_t + c_2 n (1 - p_T)]}{T}$$

DEFINING  $T^*$  AS THAT VALUE FOR WHICH  
 $EC(T^*) \leq EC(T^*-1)$  AND  $EC(T^*) \leq EC(T^*+1)$   
 ONE CAN OBTAIN  $T^*$  BY ENUMERATING  $EC(T)$   
 VALUES FOR ALL VALUES OF  $T$ .

SAY IF  $c_1 = 100$ ,  $c_2 = 10$  AND  $n = 50$  THEN

$t$	$p_t$	$T$	$\sum_{t=1}^T p_t$	$1 - p_T$	$EC(T) = (100A + 10B) \cdot \frac{50}{T}$
1	0.05	1	0.05	0.95	725
2	0.07	2	0.12	0.93	532.5
3	0.10	3	0.22	0.90	516.7 ← $T^* = 3$
4	0.13	4	0.35	0.87	546.3
5	0.18	5	0.53	0.82	612.0

### AN ASIDE

Let  $x$  be a random variable with expected value  $E(x)$   
 and variance  $\sigma^2$ .

Over  $n$  observations, let the values of  $x$  be  $x_1, x_2,$   
 $x_3, \dots, x_n$ , then the average of the observations  
 $\bar{x} = \frac{\sum x_i}{n}$ ; variance of  $\bar{x} = \frac{\sigma^2}{n}$ , and its expected  
 value is  $E(x)$ . Hence only if  $n \rightarrow \infty$  does  $\bar{x}$ 's  
 variance  $\rightarrow 0$  and  $\bar{x}$  becomes  $E(x)$ .