Modelling of Soil behaviour

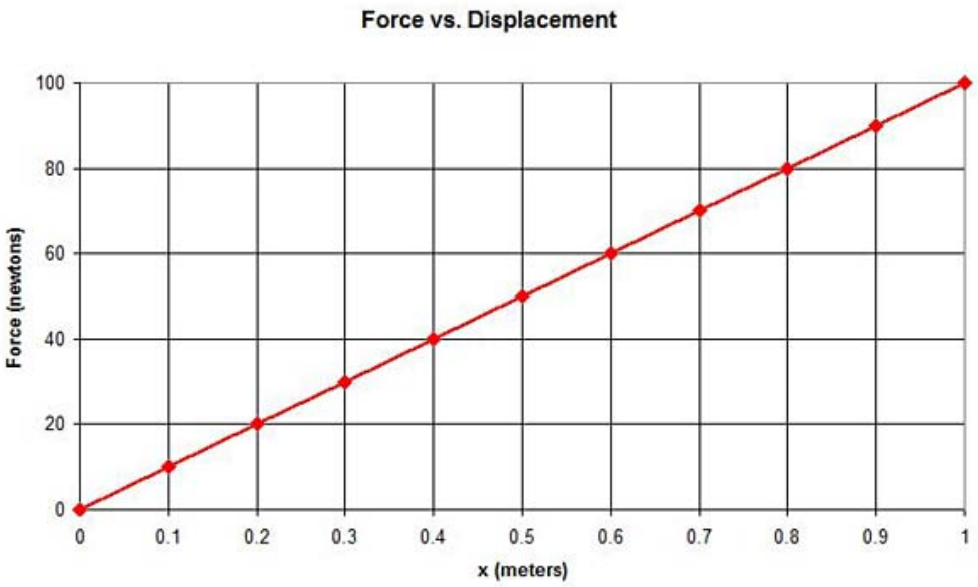
Sarvesh Chandra

TWO APPROACHES

- CONTINUUM APPROAH - Elastic, Elastoplastic, Hypoplastic, Non-homogeneous, anisotropic, layered soils --- Complex Mathematics
- MOELLING APPROACH - Simple, Determining Model Parameters is a problem --- Simple Mathematics

The Winkler Model -Winkler (1867)

- $P(x,y) = k\, w(x,y)$
- Discrete, independent, linear elastic springs
- Simple to use
- Lacks continuity amongst springs
- Soil behaviour is linear in general



Winkler Model

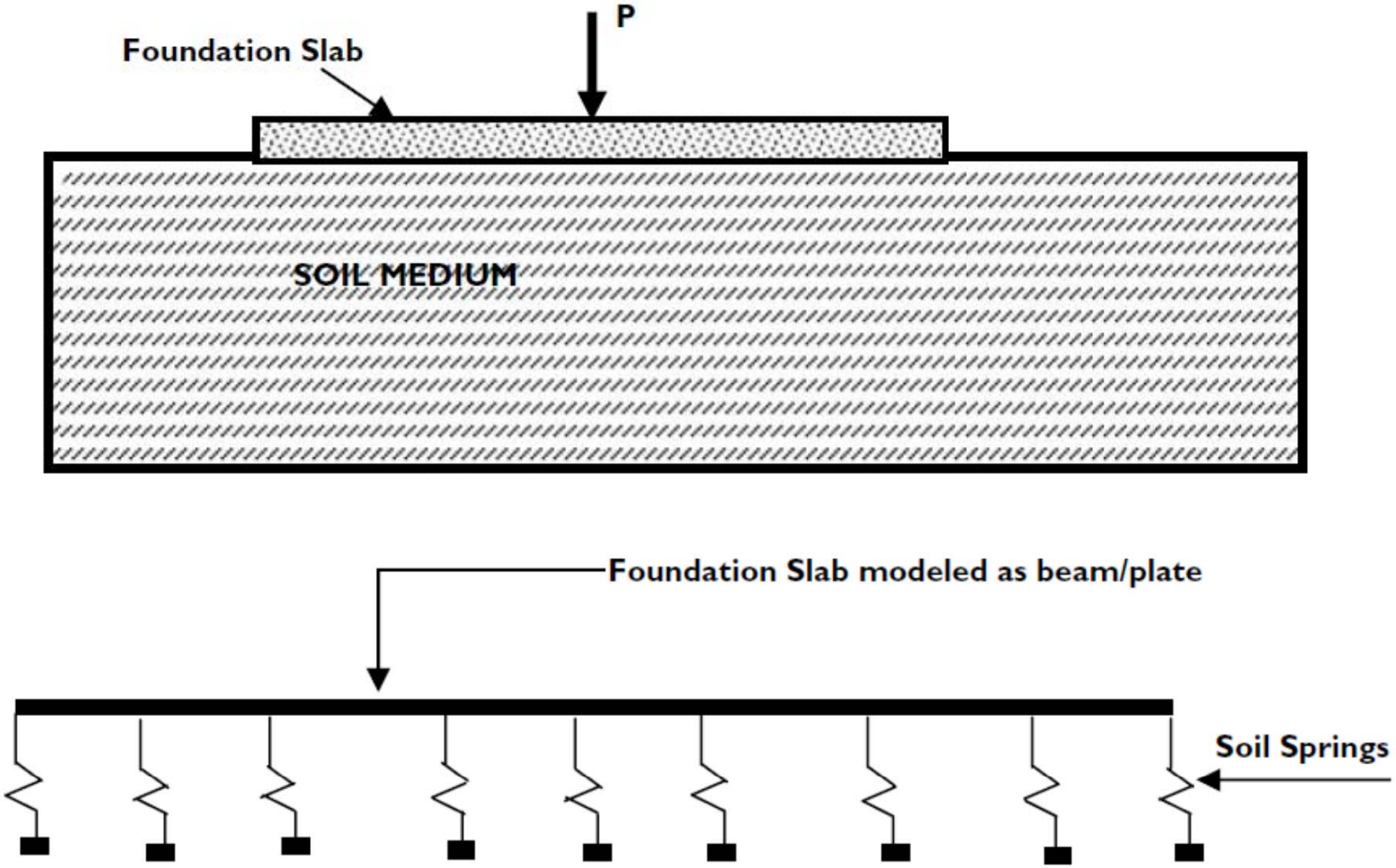


Figure 4.5.2 Equivalent foundation resting on Winkler spring bed.

Winkler Model

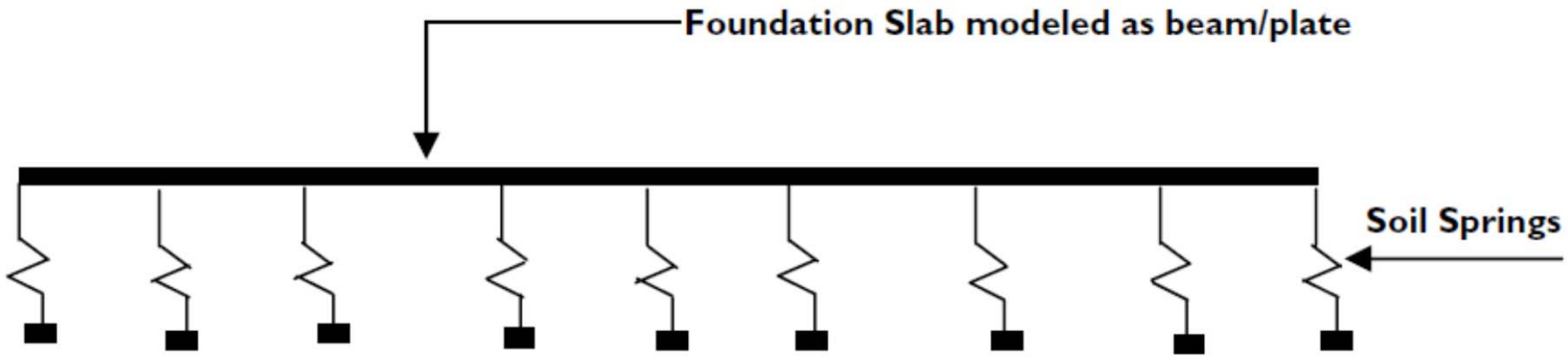
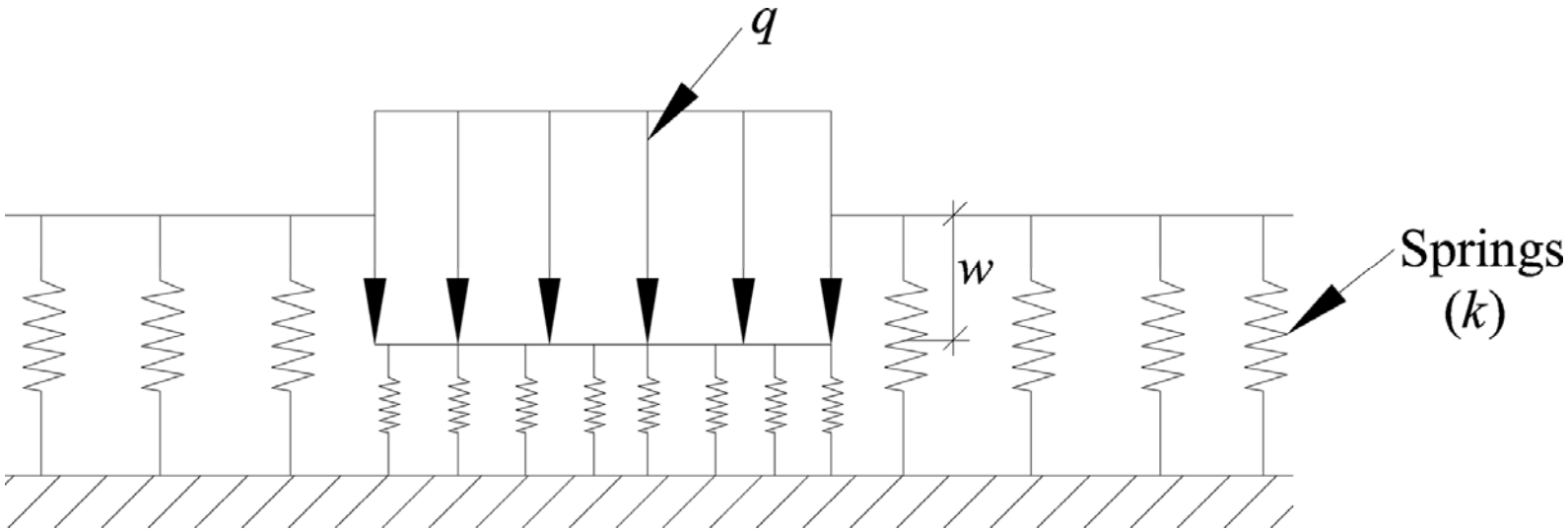


Figure 4.5.2 Equivalent foundation resting on Winkler spring bed.

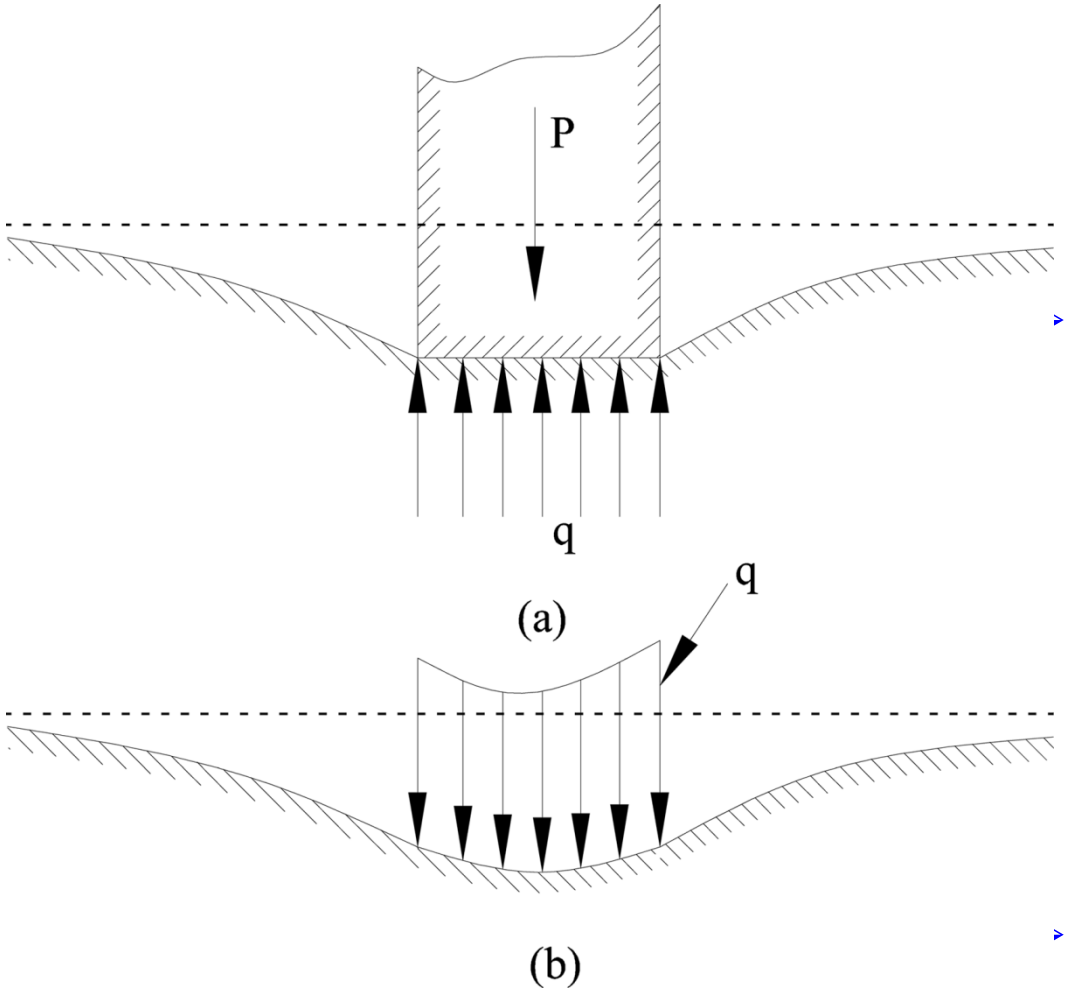
- Winkler's idealization represents the soil medium as a system of identical but mutually independent, closely spaced, discrete, linearly elastic springs.
- According to this idealization, deformation of foundation due to applied load is confined to loaded regions only.
- Figure shows the physical representation of the Winkler foundation.
- The pressure–deflection relation at any point is given by p = kw, where k = modulus of subgrade reaction.

Winkler Model



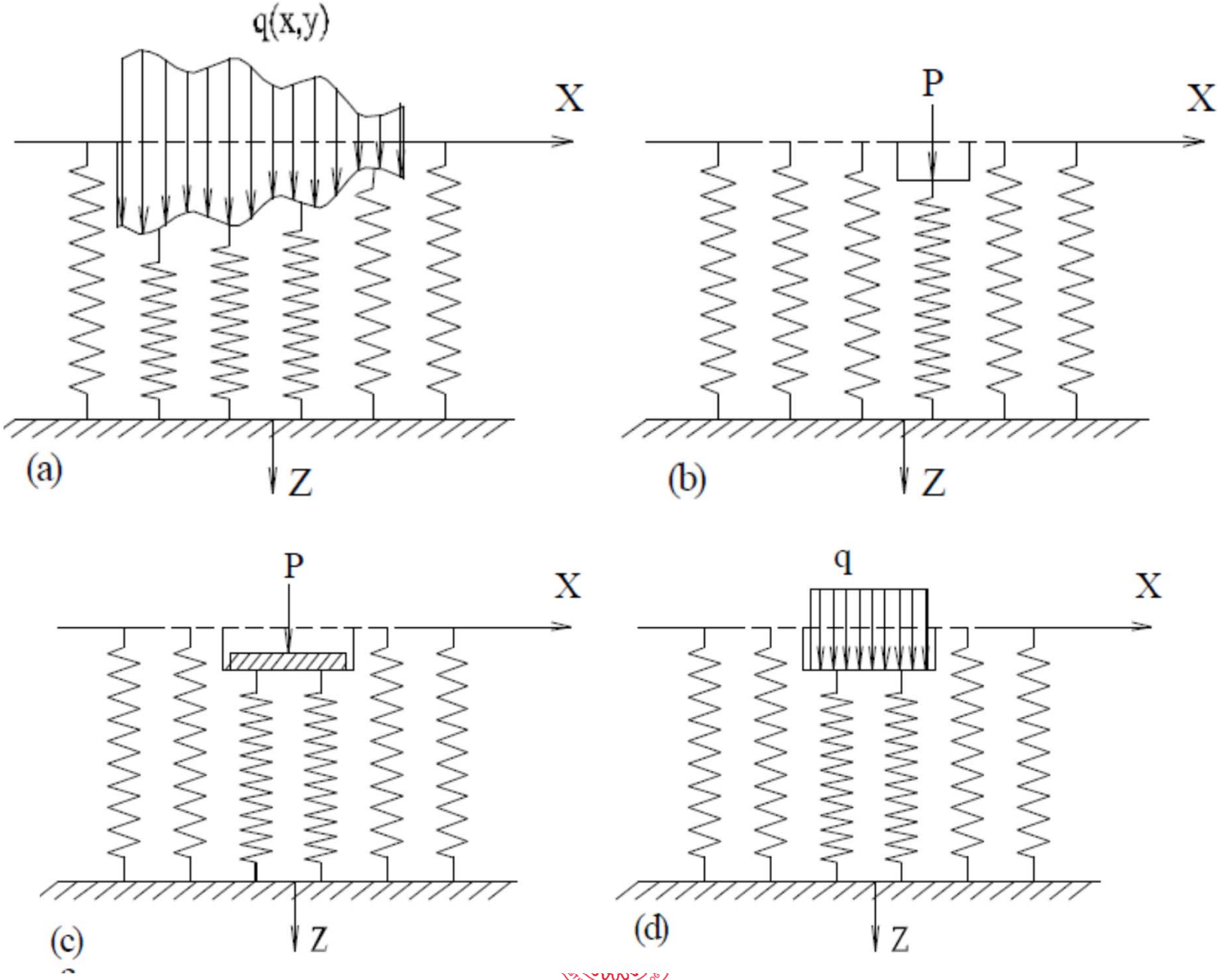
- Winkler, assumed the foundation model to consist of closely spaced independent linear springs.
- If such a foundation is subjected to a partially distributed surface loading, q, the springs will not be affected beyond the loaded region.

Winkler Model

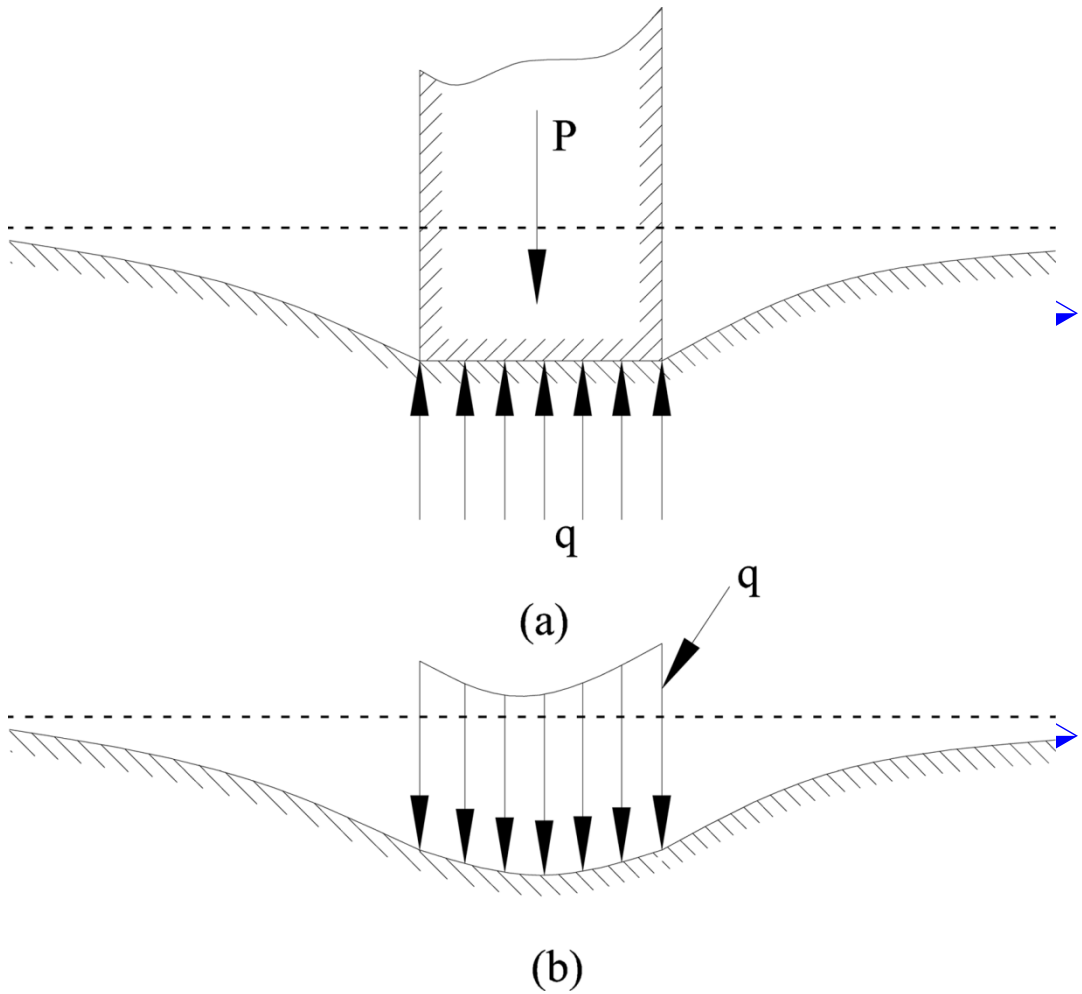


- For such a situation, an actual foundation is observed to have the surface deformation as shown in Figure.
  - Hence by comparing the behaviour of theoretical model and actual foundation, it can be seen that this model essentially suffers from a complete lack of continuity in the supporting medium.
  - The load deflection equation for this case can be written as p = kw

Winkler Models

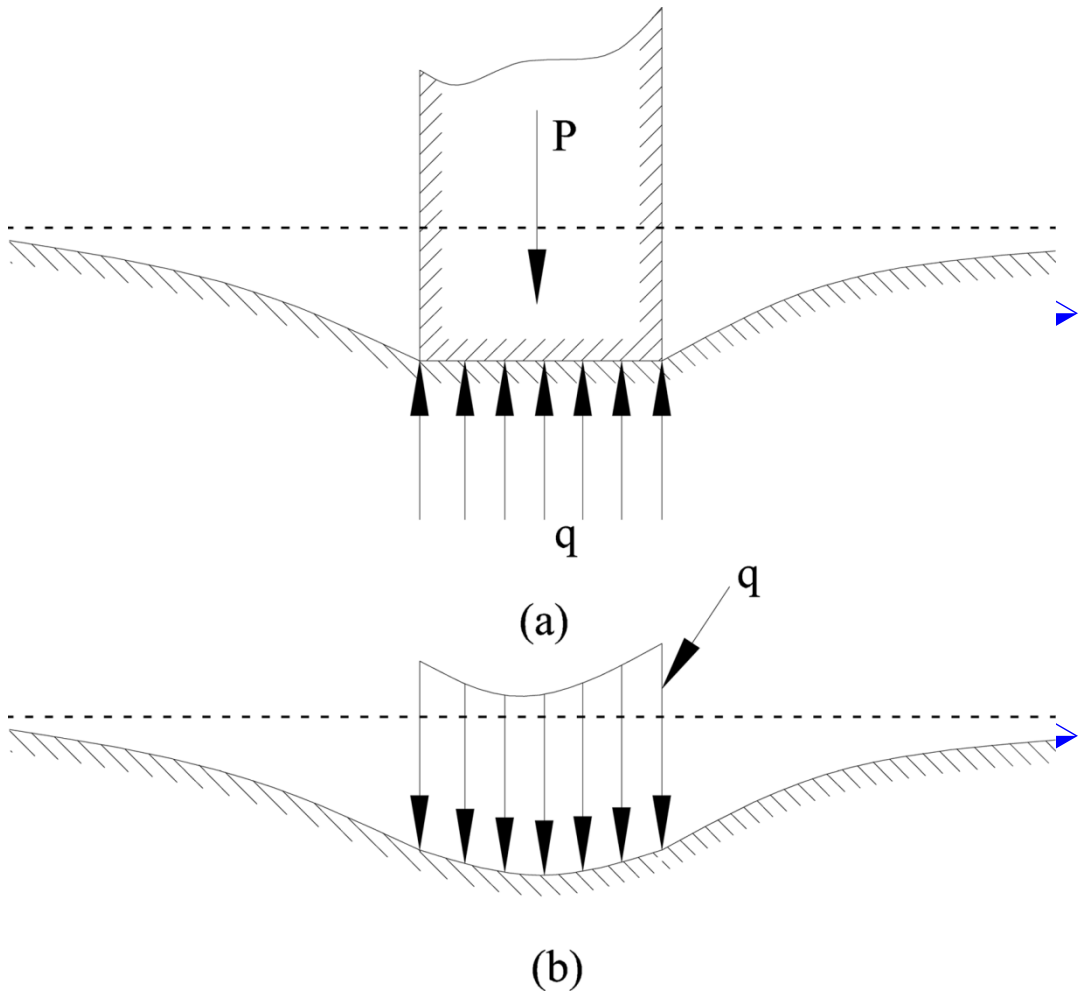


Limitations of Winkler Model



- According to this idealization, deformation of foundation due to applied load is confined to loaded regions only.
- A number of studies in the area of soil–structure interaction have been conducted on the basis of Winkler hypothesis for its simplicity.
- The fundamental problem with the use of this model is to determine the stiffness of elastic springs used to replace the soil below foundation.

Limitations of Winkler Model



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Limitations of Winkler Model

- A number of studies in the area of soil–structure interaction have been conducted on the basis of Winkler hypothesis for its simplicity. The fundamental problem with the use of this model is to determine the stiffness of elastic springs used to replace the soil below foundation.
- The problem becomes two-fold since the numerical value of the coefficient of subgrade reaction not only depends on the nature of the subgrade, but also on the dimensions of the loaded area as well.

Limitations of Winkler Model

- Since the subgrade stiffness is the only parameter in the Winkler model to idealize the physical behaviour of the subgrade, care must be taken to determine it numerically to use in a practical problem.
- Modulus of subgrade reaction or the coefficient of subgrade reaction k is the ratio between the pressure p at any given point of the surface of contact and the settlement y produced by the load at that point:

Terzaghi (1955) introduced the Coefficient or Modulus of Subgrade Reaction

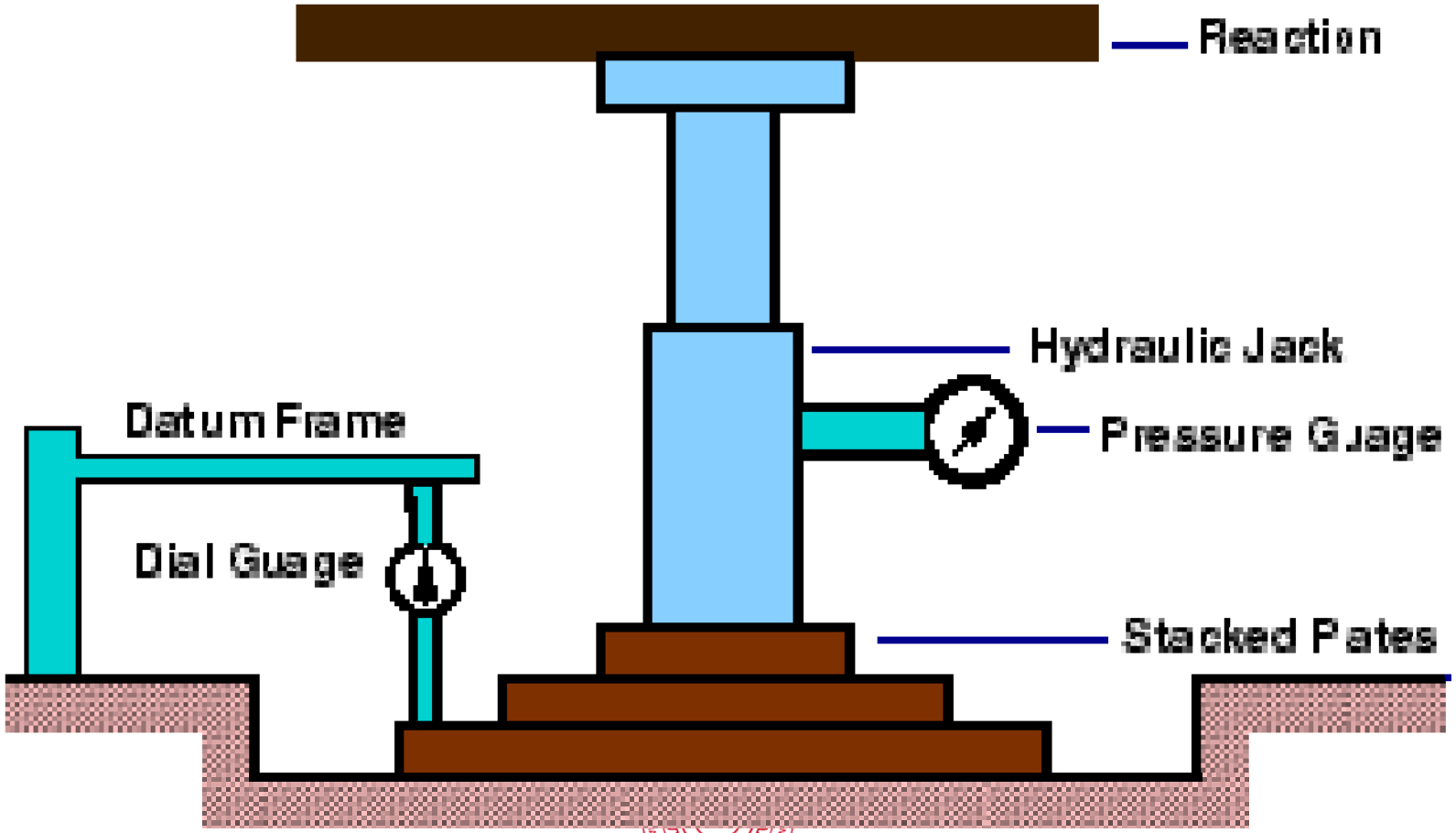
$$k_s = \frac{q}{y}$$

kg/m

- Width of Footing
- Shape of Footing
- Embedment Depth of Footing

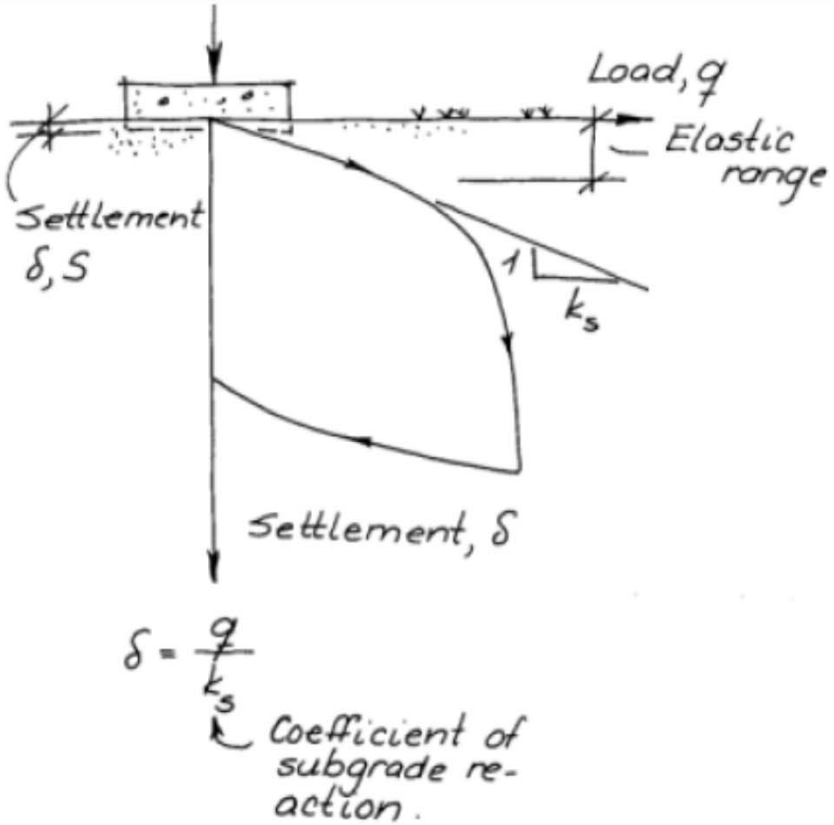
Limitations of Winkler Model

- The value of subgrade modulus may be obtained in the following alternative approaches:



Coefficient of Subgrade Reaction

- Plate load test for coefficient of subgrade reaction



Coefficient of Subgrade Reaction

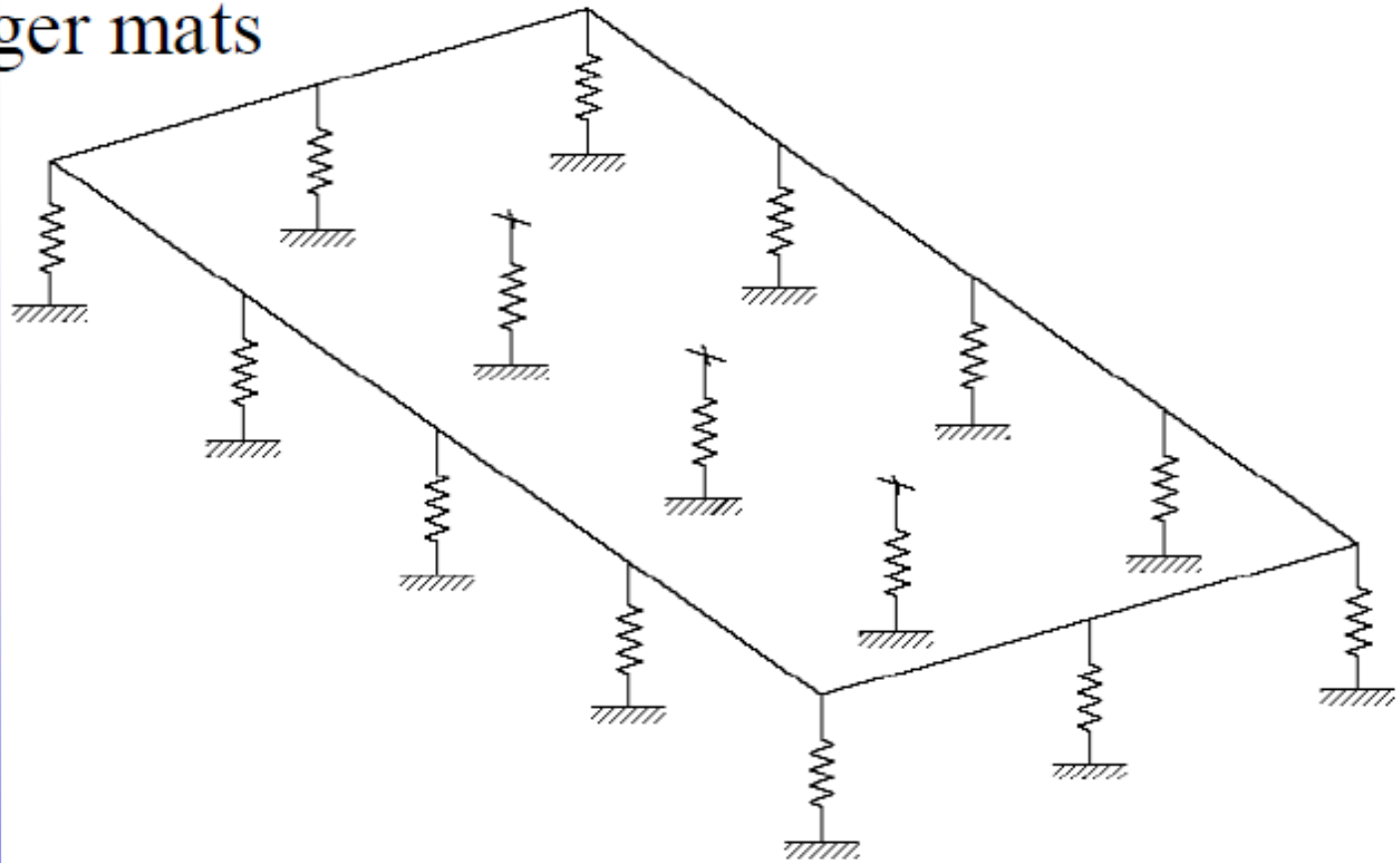
- Definition of Coefficient of Subgrade Reaction

$$k_s = \frac{q}{\delta}$$

- k_s = coefficient of subgrade reaction, units of force/length^3 (the units are the same as the unit weight, but not the significance!)
- q = bearing pressure
- δ = settlement

Coefficient of Subgrade Reaction

- Application of coefficient of subgrade reaction to larger mats



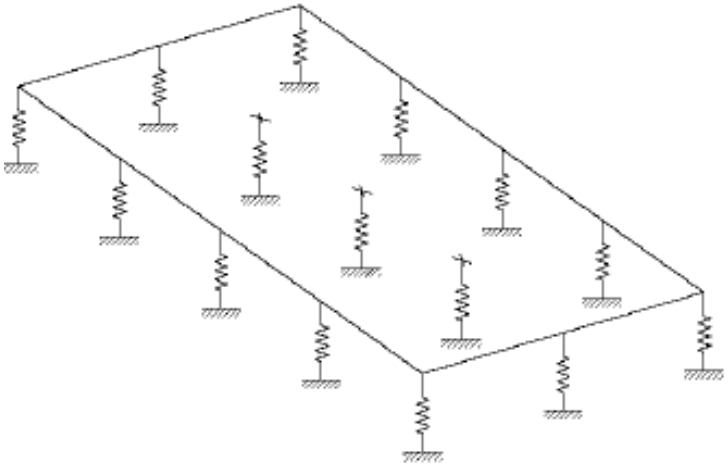
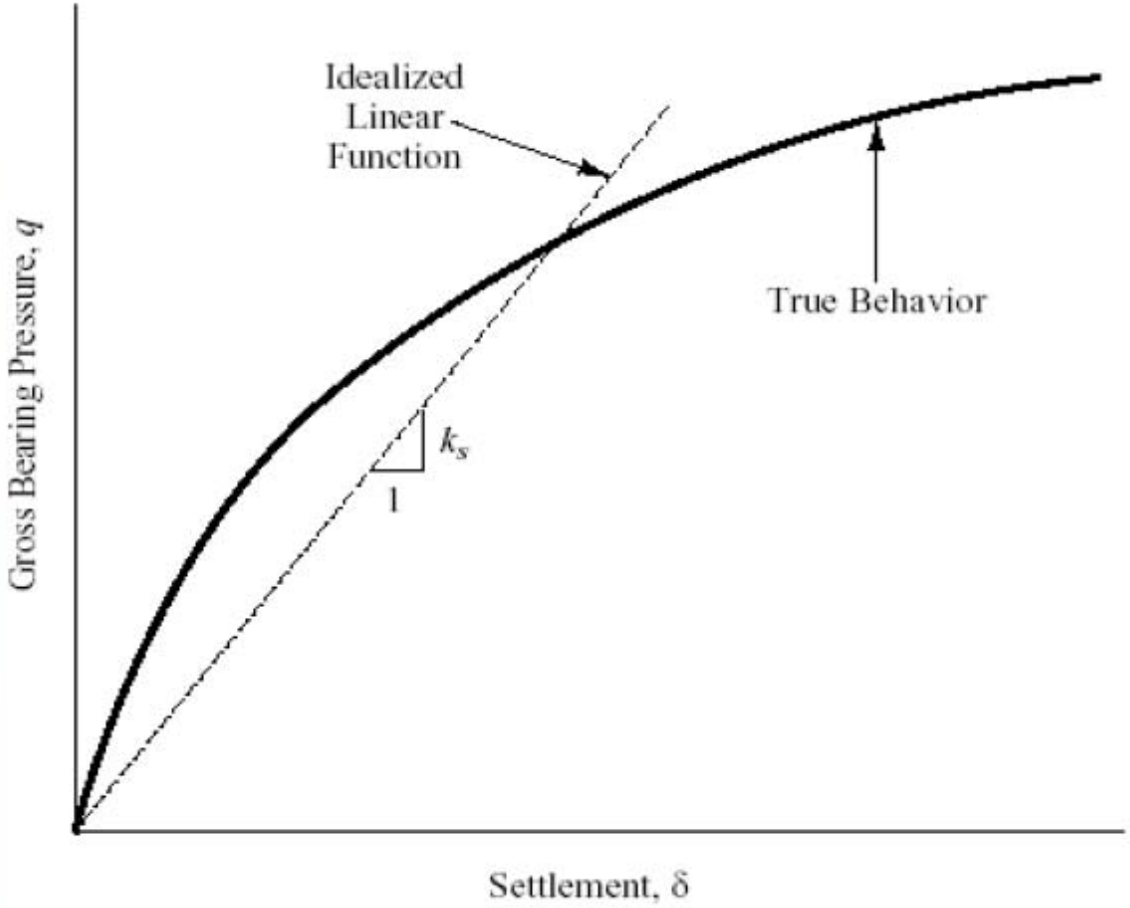


Figure 10.6 The coefficient of subgrade reaction forms the basis for a "bed of springs" analogy to model the soil-structure interaction in mat foundations.

Coefficient of Subgrade Reaction

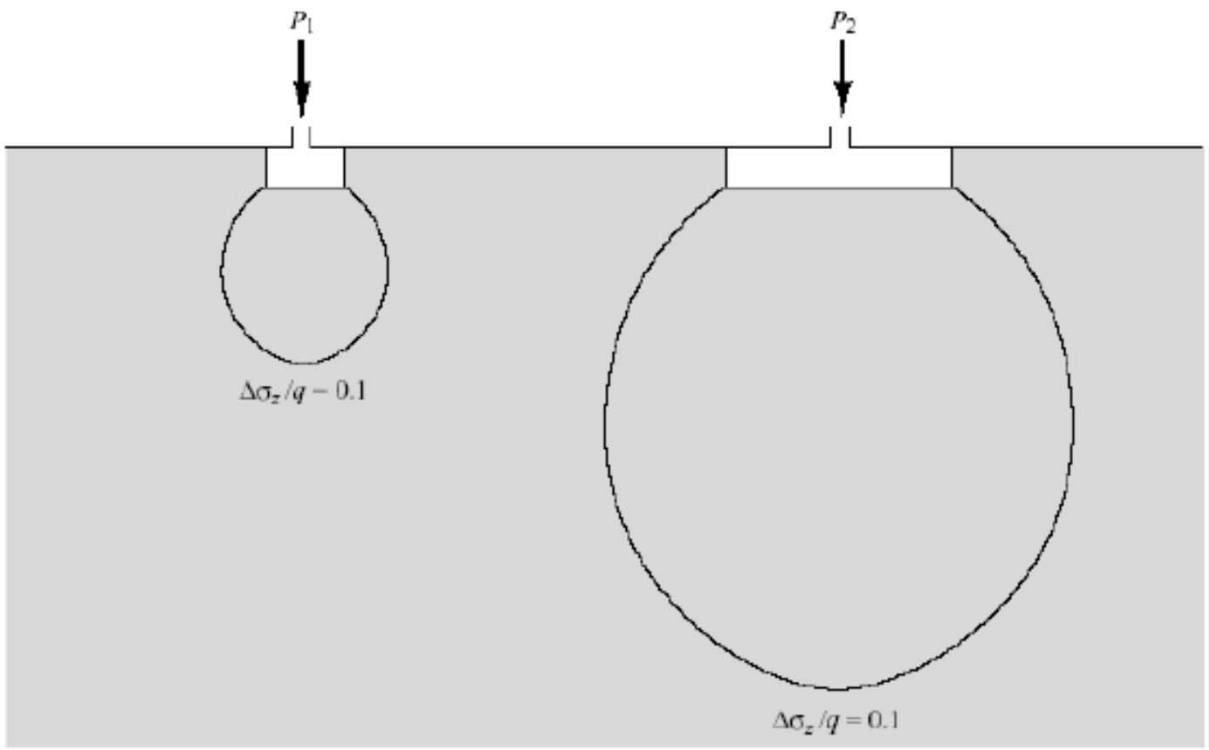
- Portions of the mat that experience more settlement produce more compression in the springs
- Sum of these springs must equal the applied structural loads plus the weight of the mat

Non-Linear Characteristics of Soil Deformation



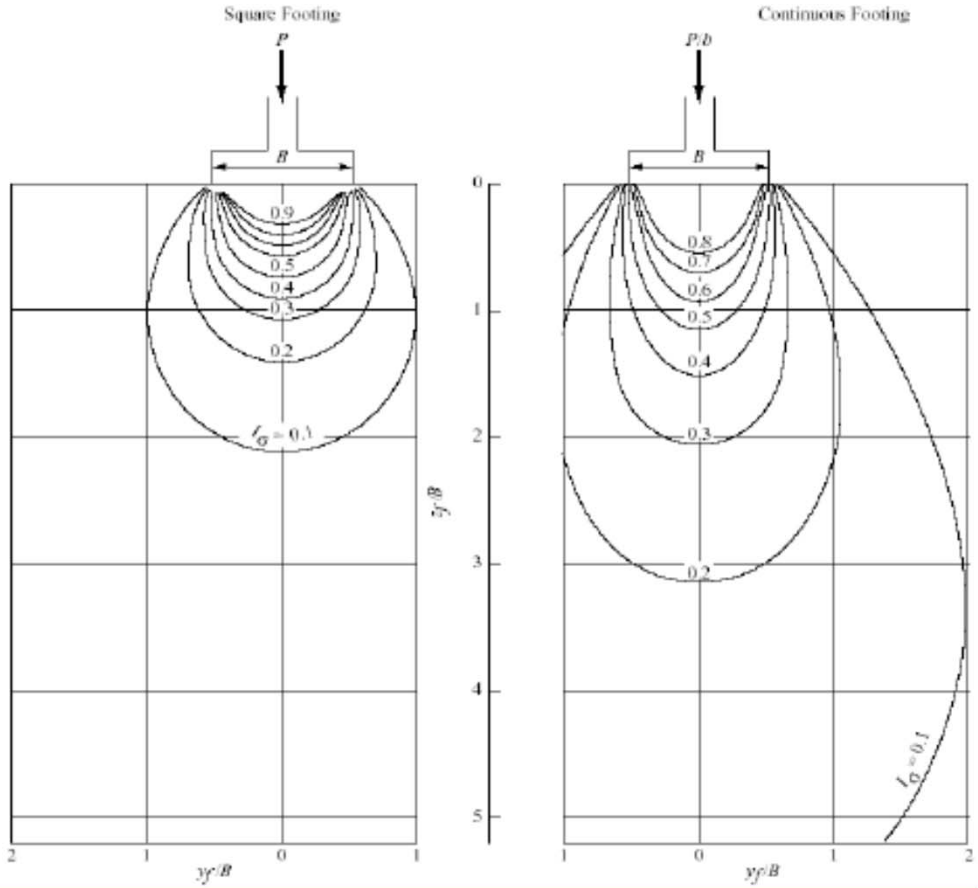
Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - Width of the loaded area; wide mat will settle more than a narrow one because more soil is mobilised by a wide mat



Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - Depth of the loaded area below the ground surface
    - Change in stress in the soil due to q is a smaller percentage of the initial stress at



Two Parameter Elastic Models

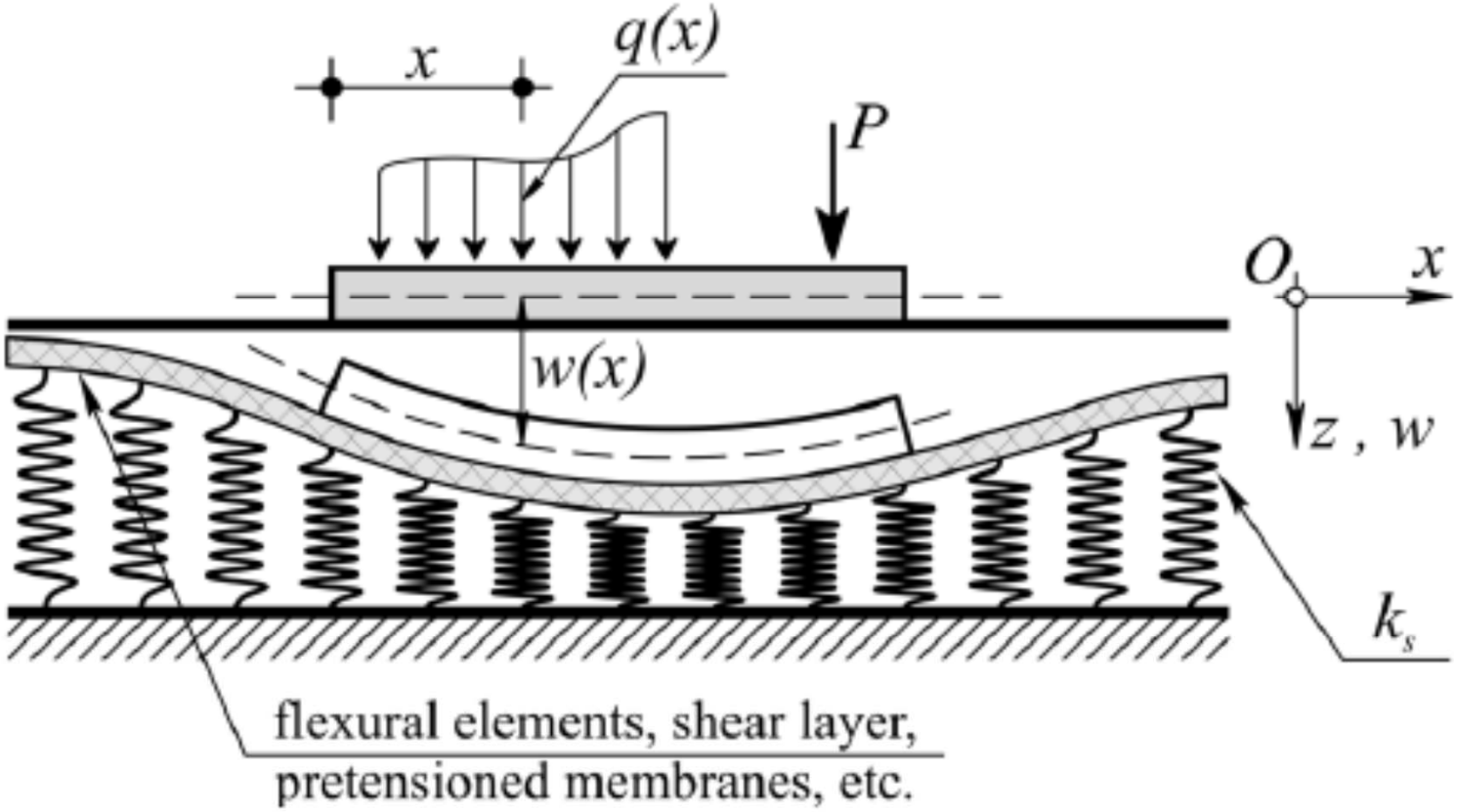
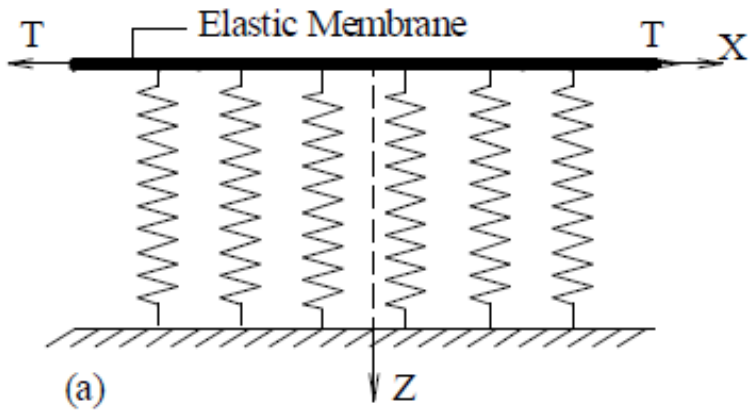


Fig. 2 – Beam resting on two-parameter elastic foundation.

Filanenko Borodich Model

This model requires continuity between the individual spring elements in the Winkler's model by connecting them to a thin elastic membranes under a constant tension T.



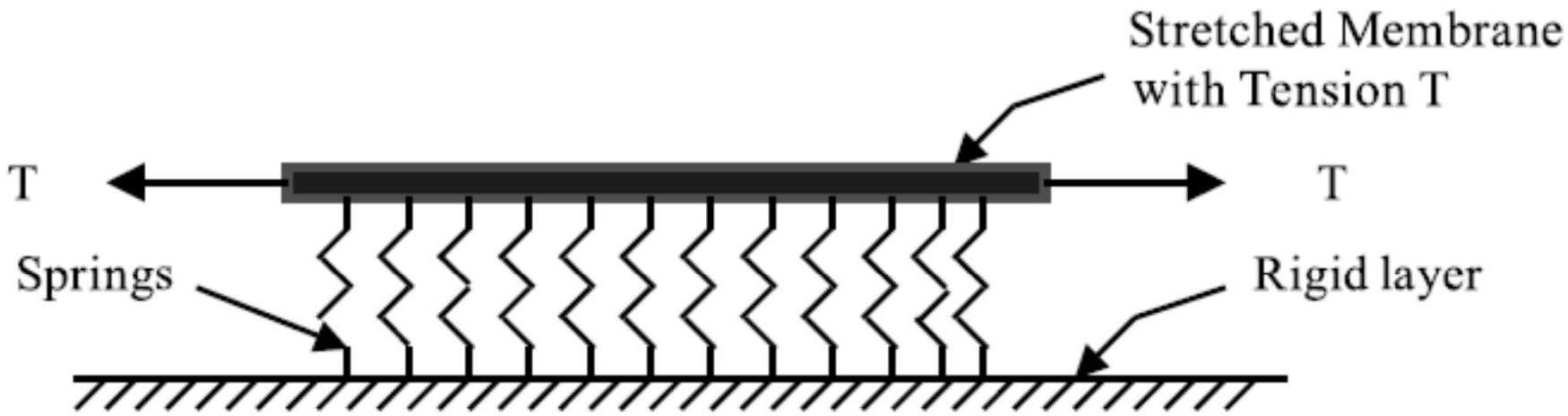


Fig. 3. Filonenko–Borodich foundation [52].

Filanenko Borodich Model

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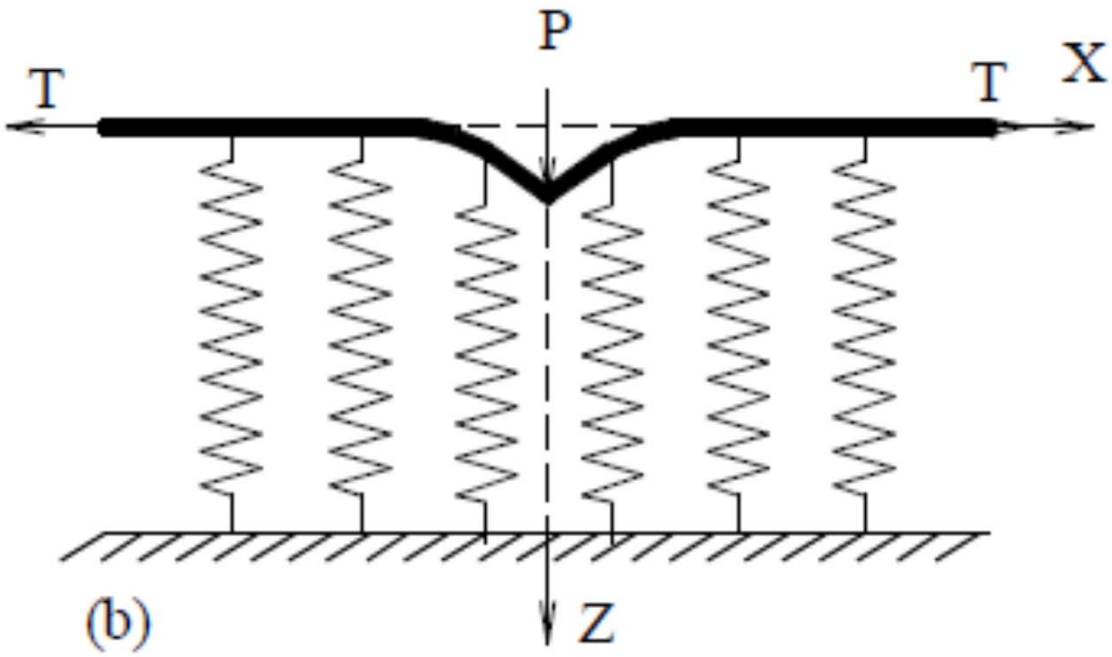
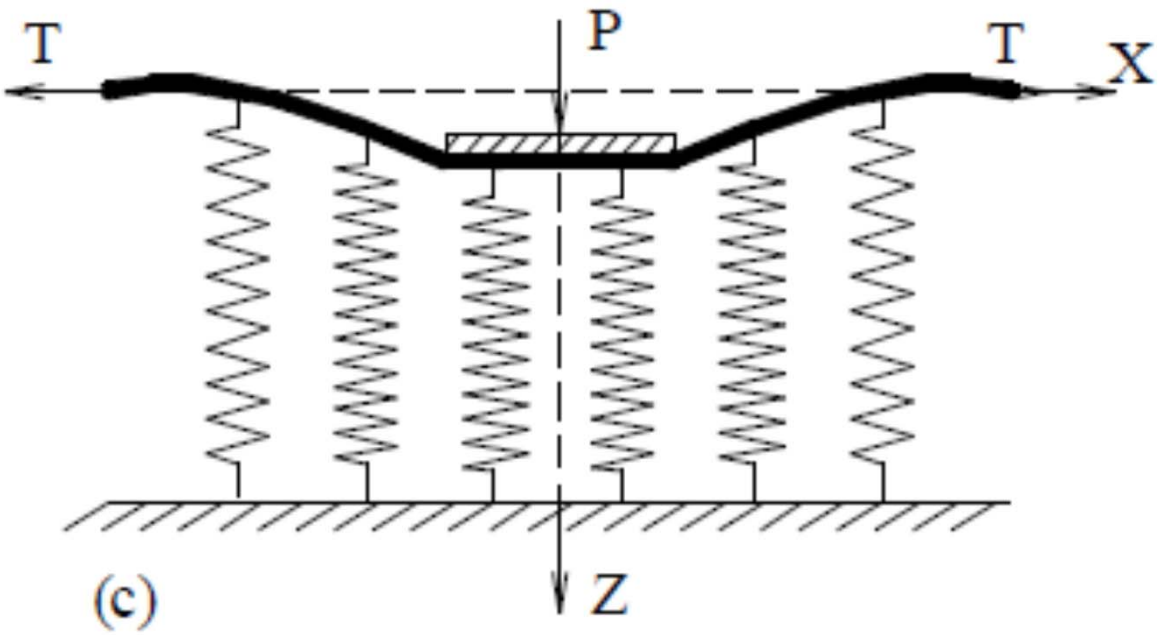


Concentrated Load

Filanenko Borodich Model

This model requires continuity between the individual spring elements in the Winkler's model by connecting them to a thin elastic membranes under a constant tension T.



Rigid Load

Filanenko Borodich Model

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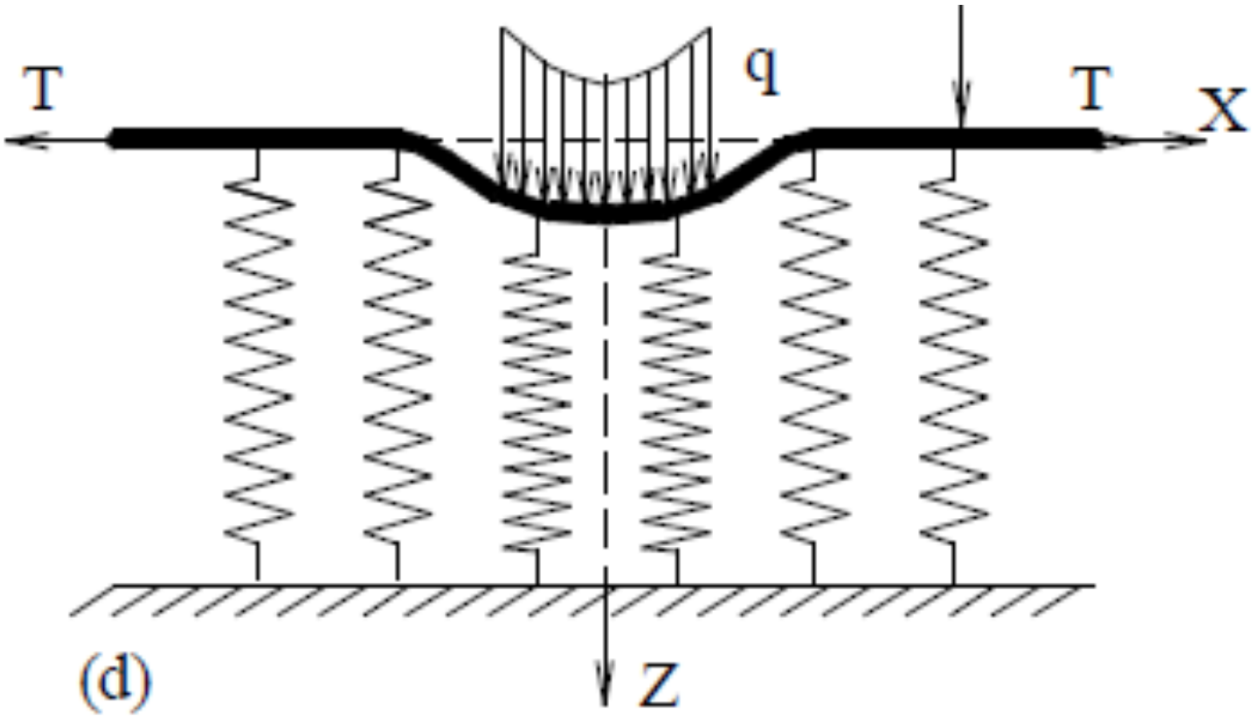


Uniform Flexible Load

Filanenko Borodich Model

The response of the model can be expressed mathematically as follows:

$$p = kw - T\nabla^2 w, \text{ for rectangular or circular foundation}$$

$$= kw - T\frac{d^2 w}{dx^2}, \text{ for strip foundation}$$

where, $\nabla^2 \equiv$ Laplace operator $\equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $T =$ tensile force.

Hence, the interaction of the spring elements is characterized by the intensity of the tension T in the membrane.

Hetenyi's Model

This model suggested in the literature can be regarded as a fair compromise between two extreme approaches (viz., Winkler foundation and isotropic continuum). In this model, the interaction among the discrete springs is accomplished by incorporating an elastic beam or an elastic plate, which undergoes flexural deformation only

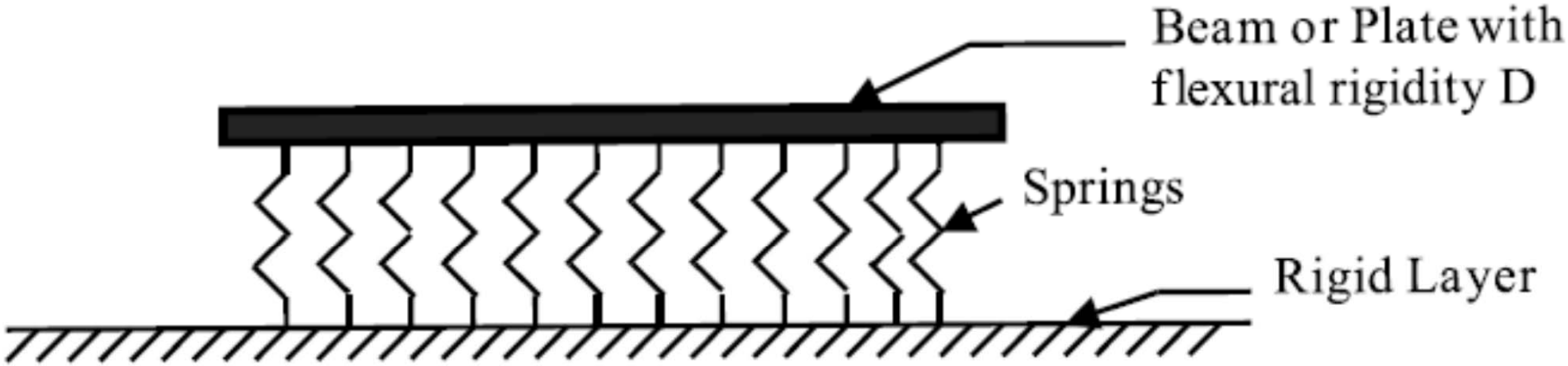


Fig. 4. Hetenyi foundation [30].

Hetenyi's Model

Thus the pressure–deflection relationship becomes

$$p = kw + D\nabla^4 w \qquad \nabla^4 \equiv \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} \tag{4}$$

where,

D = flexural rigidity of the elastic plate

$= (E_p h_p^3)/(12(1-\mu_p)^2)$;

p is the pressure at the interface of the plate and the springs; E_p and μ_p are Young's modulus and Poisson's ratio of plate material; h_p is the thickness of the plate and

Pasternak Model

- In this model, existence of shear interaction among the spring elements is assumed which is accomplished by connecting the ends of the springs to a beam or plate that only undergoes transverse shear deformation.
- The load–deflection relationship is obtained by considering the vertical equilibrium of a shear layer.

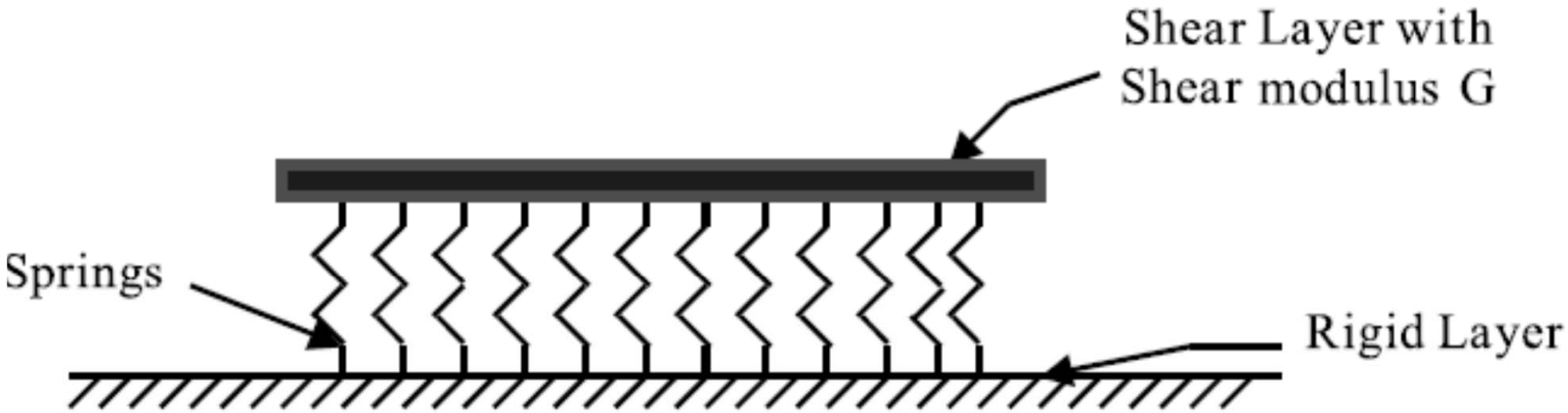


Fig. 5. Pasternak foundation [55].

Pasternak Model

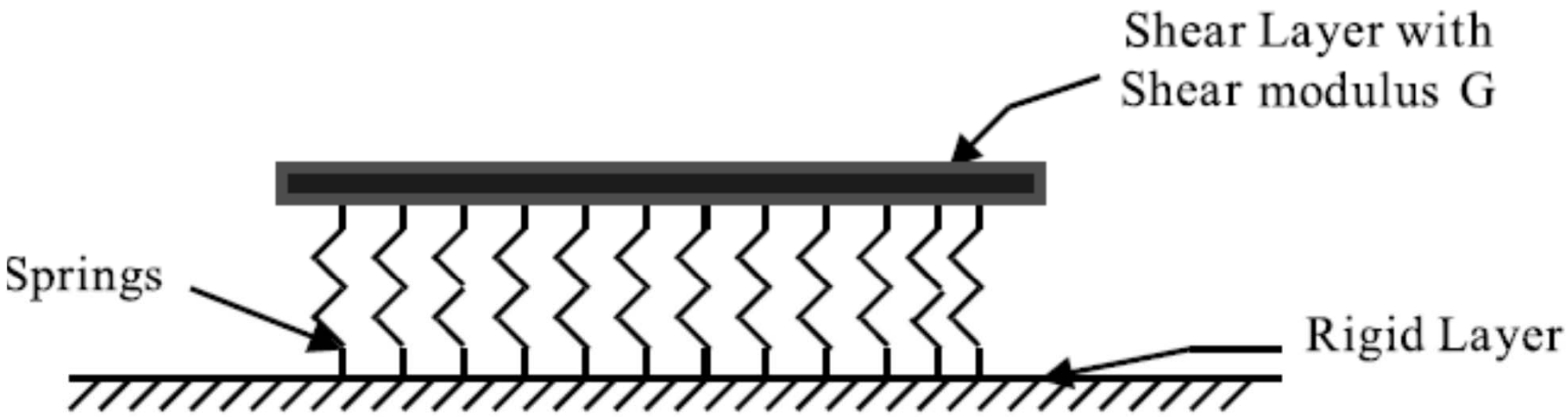


Fig. 5. Pasternak foundation [55].

The pressure–deflection relationship is given by

$$p = kw - G\nabla^2 w$$

where, G is the shear modulus of the shear layer.

Pasternak Model

The continuity in this model is characterized by the consideration of the shear layer.

A comparison of this model with that of Filonenko–Borodich implies their physical equivalency (“T” has been replaced by “G”).

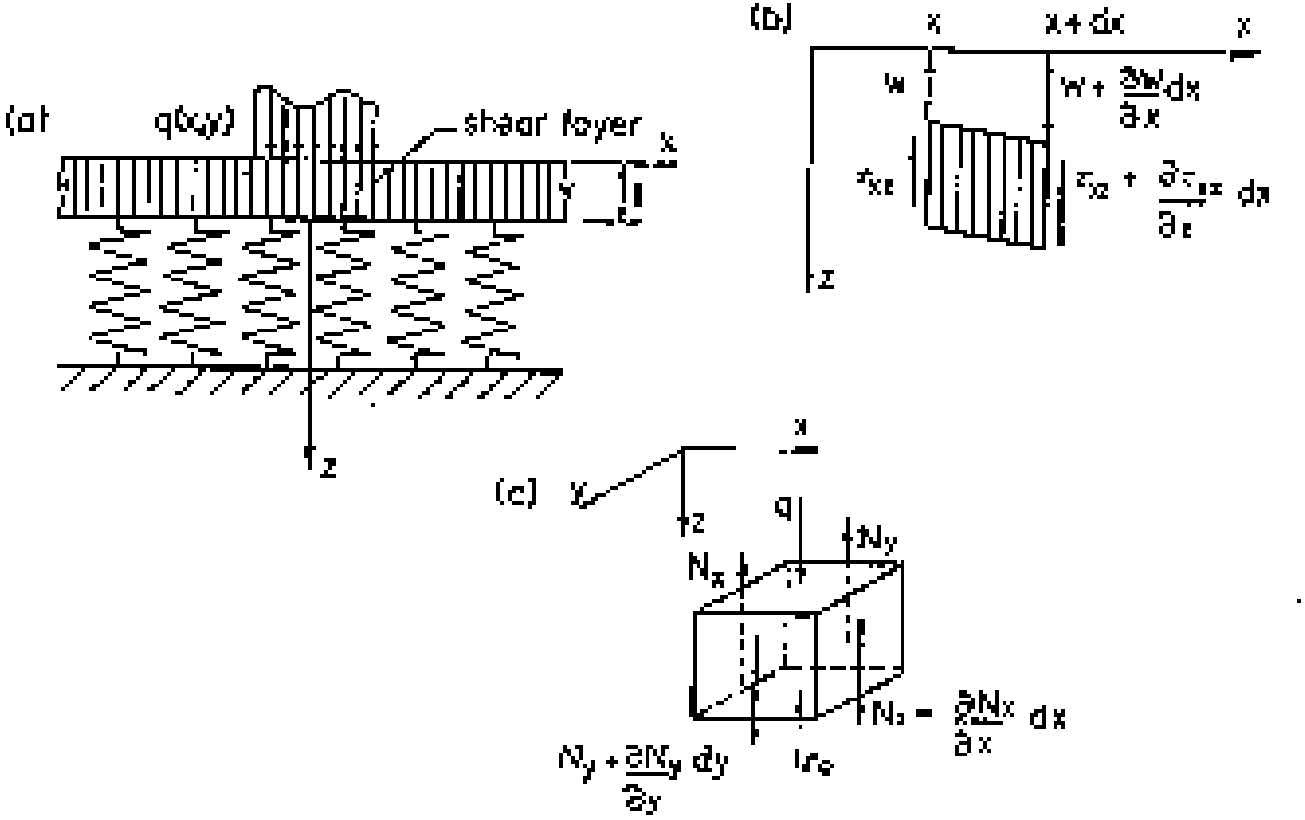


Fig. 2.6. The Pasternak model. (a) The basic model, (b) stresses in the shear layer, (c) forces acting on the shear layer.

Pasternak Model:

Pasternak (1954) proposed model which assumes the existence of shear interaction between the spring elements. The springs are connected to an isotropic layer of incompressible vertical elements which deform in transverse shear only. The total shear forces per unit length of the shear layer are given by:

$$N_x = \int_0^1 \tau_{xz} dz = G_p \frac{\partial w}{\partial x} \; ; \; N_y = \int_0^1 \tau_{yz} dz = G_p \frac{\partial w}{\partial y}$$

For the force Equilibrium in the z-direction:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + q - r_0 = 0$$

Using the condition $r_0 = k\,w$, we have

$$q(x,y) = k\,w(x,y) - G_p \nabla^2 w(x,y)$$

The surface deflection profiles are very similar to those obtained for the Filonenko-Borodich model.

Kerr Model

A shear layer is introduced in the Winkler foundation and the spring constants above and below this layer is assumed to be different as per this formulation.

The following figure shows the physical representation of this mechanical model. The governing differential Fig. 4. Hetenyi foundation [30]. equation for this model may be expressed as follows.

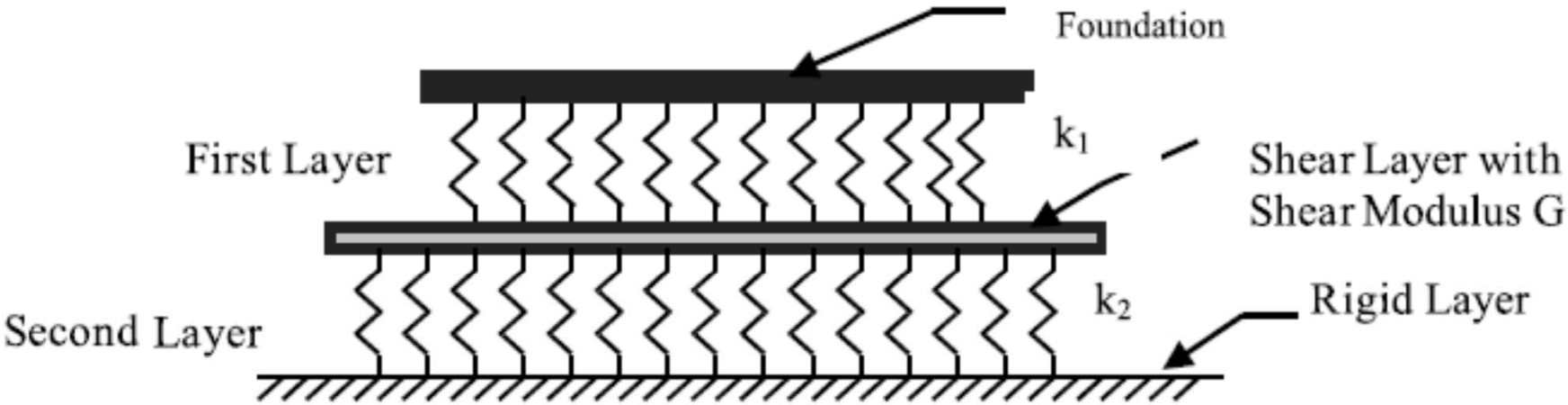


Fig. 6. Kerr foundation [50].

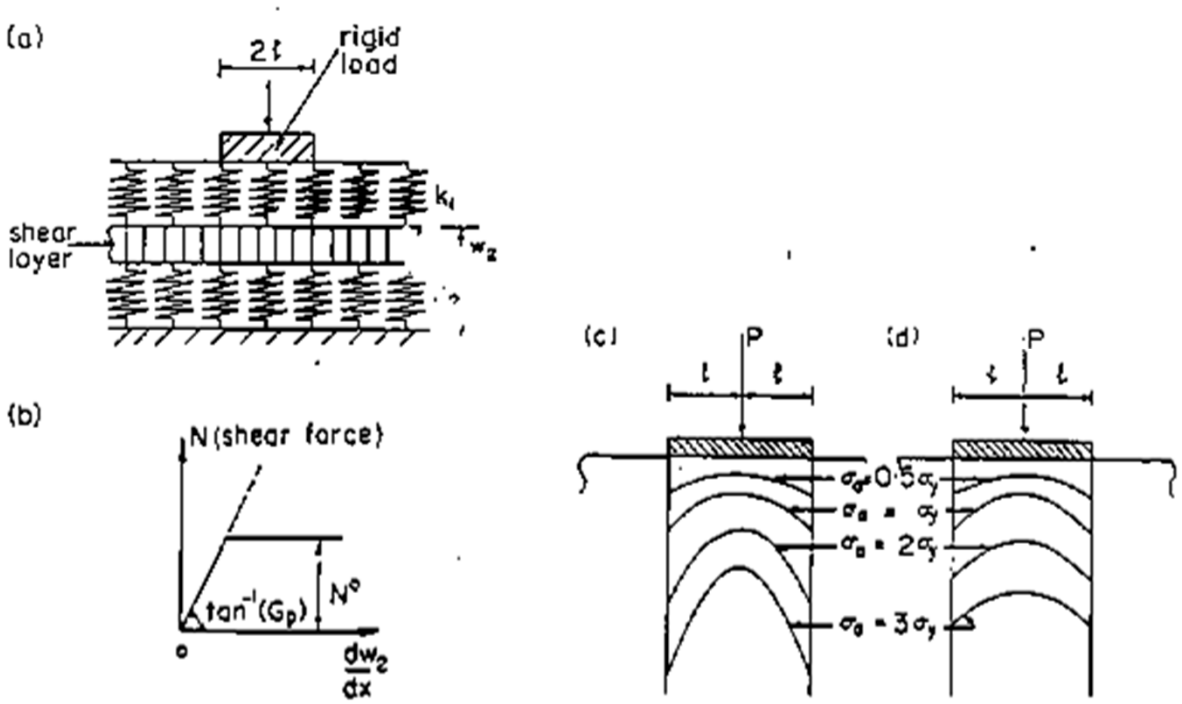
Kerr Model

The governing differential equation for this model may be expressed as follows.

$$\left(1+\frac{k_2}{k_1}\right)p=\frac{G}{k_1}\nabla^2 p+k_2 w-G\nabla^2 w \tag{7}$$

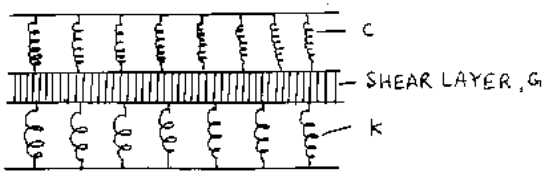
where, k_1 is the spring constant of the first layer; k_2 is the spring constant of the second layer; w is the deflection of the first layer.

Elasto-Plastic Model (Rhines, 1969)

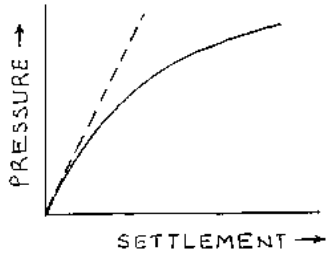


Kerr Model:

Kerr (1964) reviewed the various models reported in the literature and presented a model in 1965. This model is an extension of Pasternak Model and in this model the shear layer is sandwiched between two spring layers. The model parameters are c, k and G. To reduce the number of coefficients to two, it is recommended to take c=3k.

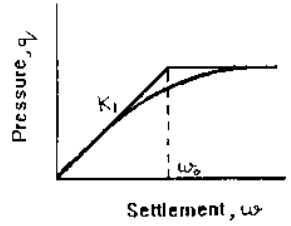


Non-linear behaviour of soil:



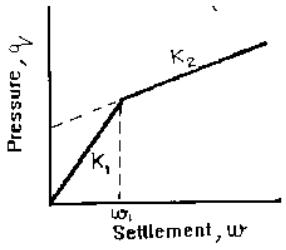
STARZEWSKI, K. (1959)

Elasto - Plastic Representation of a Nonlinear response



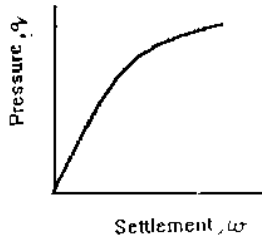
$q = k_1 w$ for $w < w_0$
$q = k_1 w_0$ for $w > w_0$

Bilinear Representation:



$q = k_1 w$ for $w < w_1$
$q = k_2 w + q_0$ for $w > w_1$

Nonlinear Model (Chandra, 197



A Polynomial relationship of the type:
$q = k_1 w - k_2 w^3$ was suggested. To determine k_1 and k_2, plot q/w versus w^2.

Modelling of Reinforced Granular Beds

Different type of reinforcements

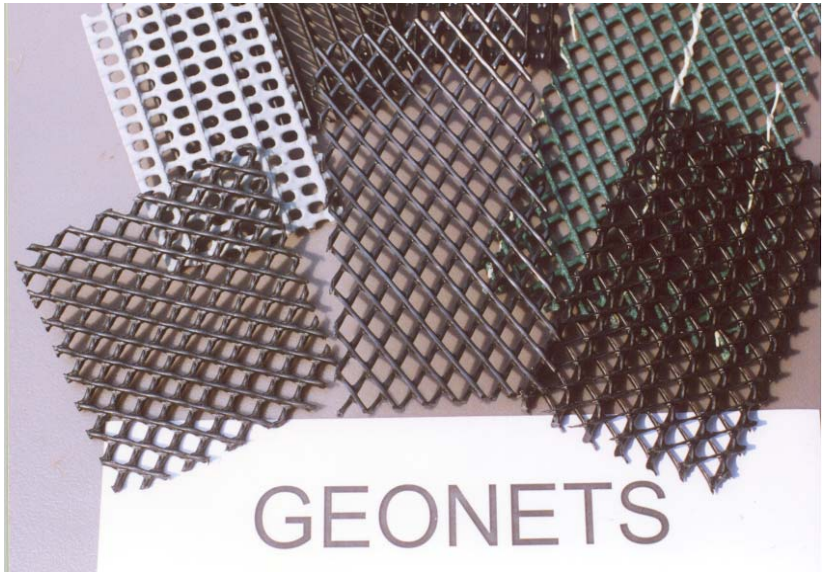
- *Geotextiles (GT)*

- Very versatile in their primary function

- *Geogrids (GG)*

- Focuses entirely on reinforcement applications, e.g., walls, steep slopes, base and foundation reinforcement

- *Geonets (GN)*



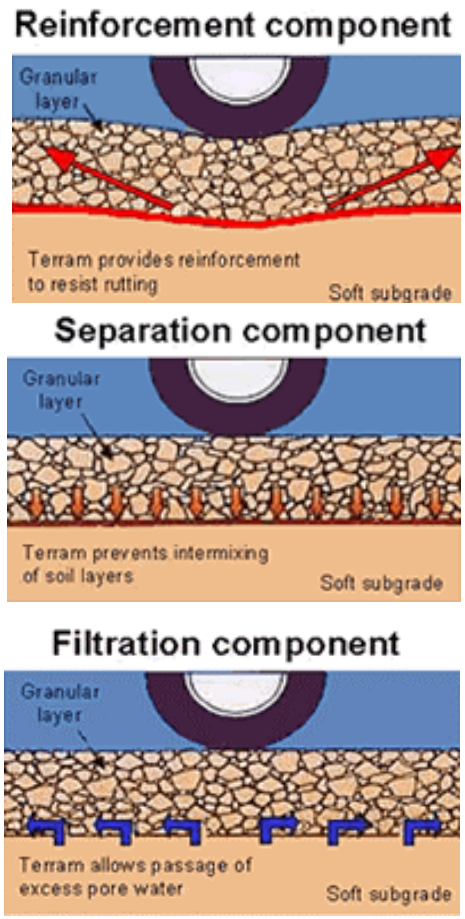
- Function is always in drainage

- *Geomembranes (GM)*

- Function is always containment
- Represents a barrier to liquids and gases

Major Functions of Geosynthetics

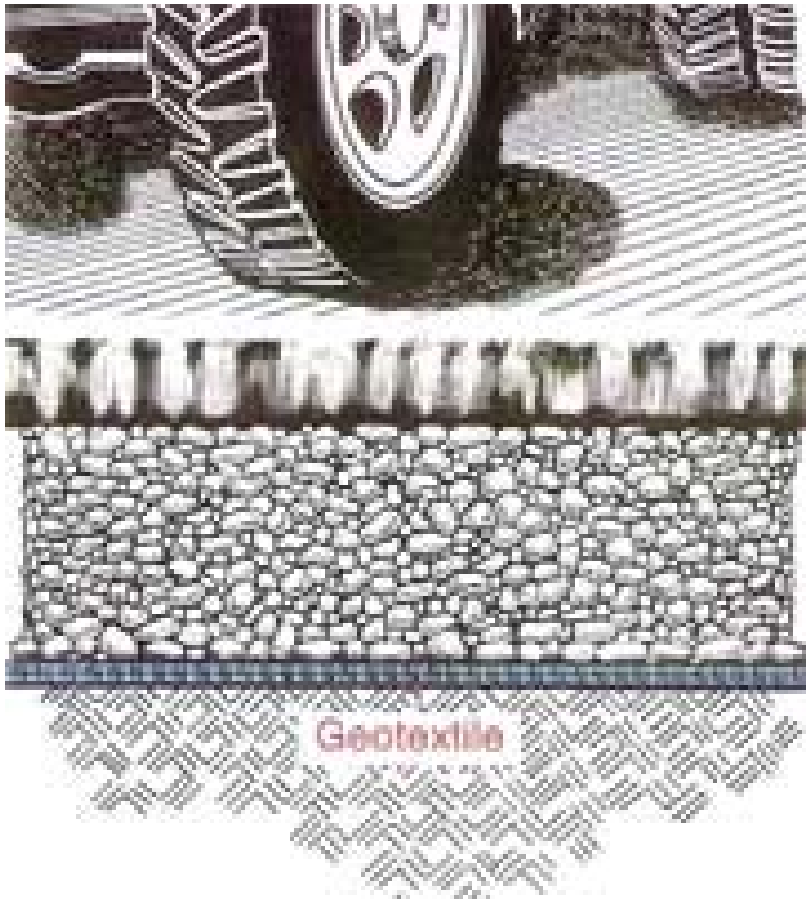
- Reinforcement
- Separation
- Filtration
- Drainage
- Moisture barrier



Applications

- **Foundation for motorways, airports, railroads, sports fields, parking lots, storage capacities**
- **Slope stability**
- **Confinement**
- **Environmental Concerns**
- **Dams and Embankments**
- **Low cost housing**

Applications of Geosynthetics



Improved subgrade or roadbase performance

Applications of Geosynthetics

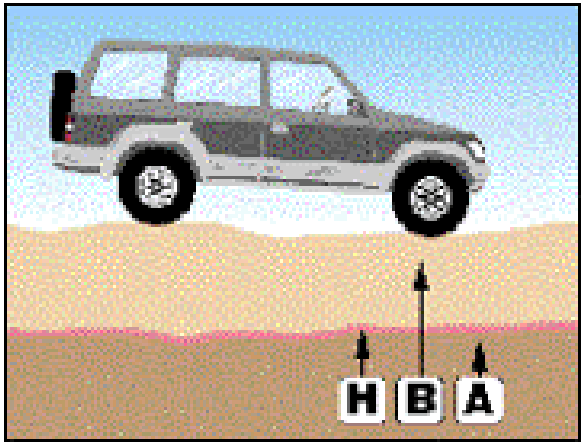
Reinforcement of soils by Geotextiles

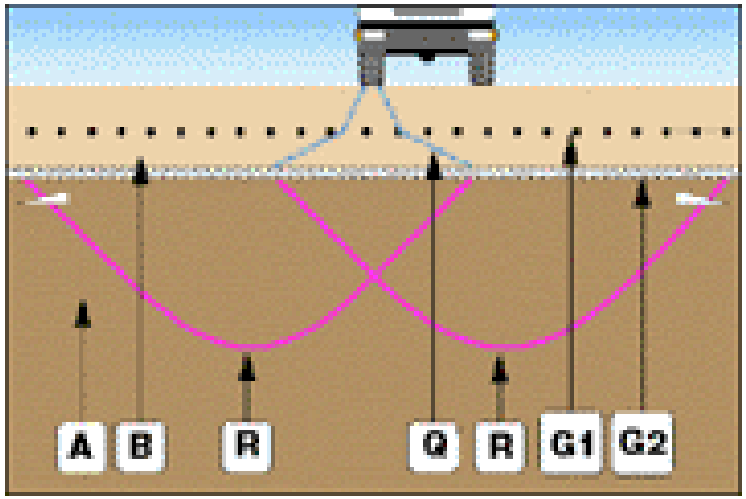
Applications of Geosynthetics

Railroad stabilization by Geogrids

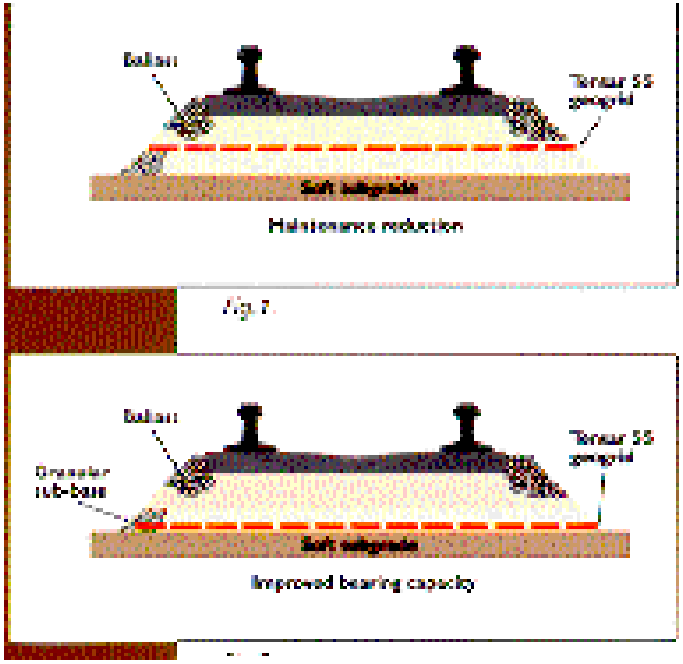
Load Transfer Mechanism of Geosynthetic-Reinforced Soil

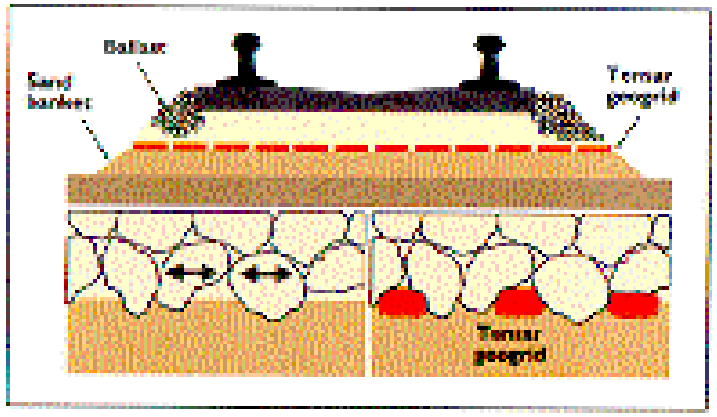
- Interfacial shear mobilization effects
- Membrane effect of the reinforcement
- Confinement effect of the reinforcement
- Reinforcement effect of the fill
- Separation effect of the fill and the soft soil





A - Soft Soil
B - Granular fill
R - Failure planes
H - Deformed profile
M - Soil cracking
Q - Stress distribution
G1 Tensar grid
G2 - Geomembrane





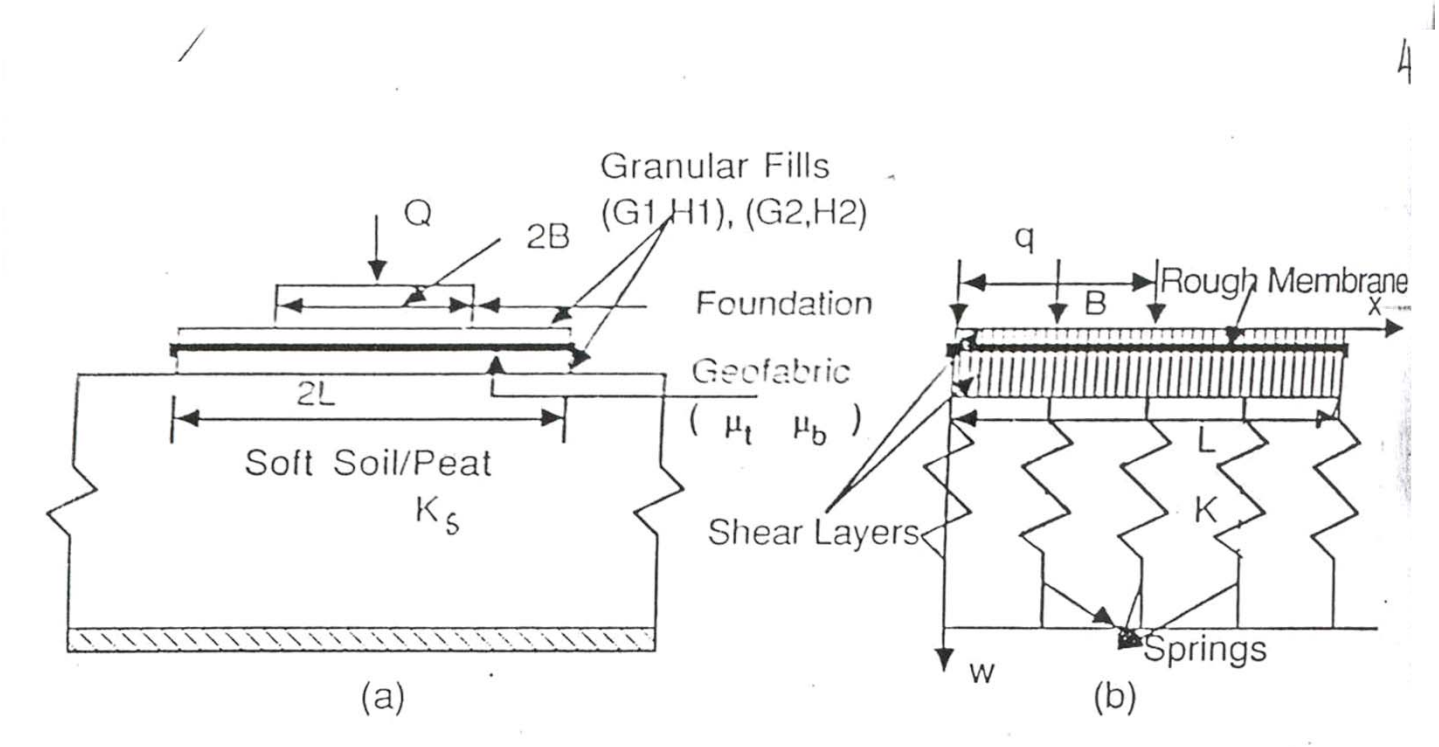
The interlock between the grid and the ballast limits lateral movement of particles even when dynamic loading is applied. In practice this means that the settlement rate of the ballast is reduced.

Use of Geotextiles for foundation Bangkok Highway project

Modelling of reinforced Granular Beds

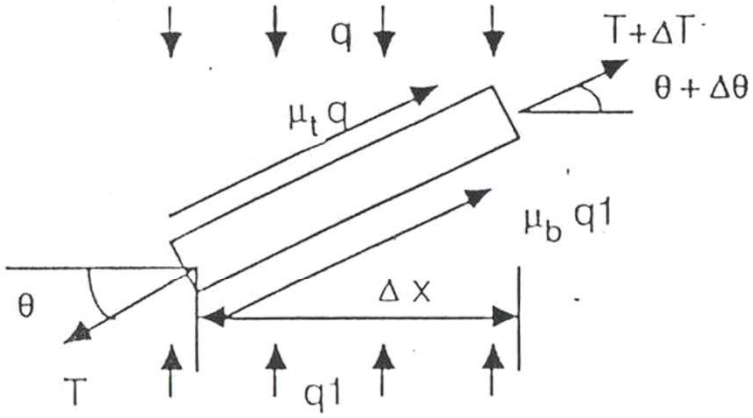
- Assumptions
  - Geosynthetic reinforcement is linearly elastic, rough enough to prevent slippage at the soil interface and has no shear resistance, and thickness of reinforcement is neglected
  - Spring constant has constant value irrespective of depth and time
  - The rotation of reinforcement is small

Madhav and Poorooshasb (1988)

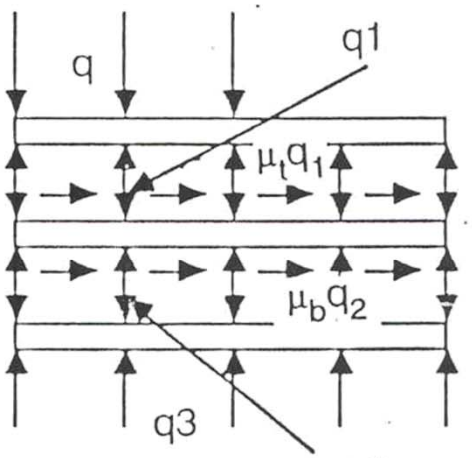


Definition Sketch

Proposed Model



(c)



(d)

Free Body Diagram

Equations for the proposed model:

$$q = kw - (G_t H_t + T \cos\theta + G_b H_b)\frac{d^2 w}{dx^2}$$

and

$$\frac{dT}{dx} = -(\mu_t \sec\theta + \sin\theta)\left(q + G_t H_t \frac{d^2 w}{dx^2}\right) - (\mu_b \sec\theta - \sin\theta)\left(kw - G_b H_b \frac{d^2 w}{dx^2}\right)$$

Boundary conditions:

For an unstretched membrane at x=L: T=0 and the shear stress = 0.

For uniform load of intensity q, from symmetry, at x = 0, dw/dx = 0.

- Settlement Response of a Reinforced Shallow earth bed by C. Ghosh and M.R. Madhav (1994)- Membrane effect of Reinforced layer, Non-linear response of the granular layer and soft soil, plane strain condition.

- Reinforced Granular Fill-Soft Soil system: Confinement Effect by C. Ghosh & M.R. Madhav (1994) -Quantified in terms of average increase in confining pressure due to modified shear stiffness of the granular soil surrounding the reinforcement.

Madhav and Poorooshasb (1989)

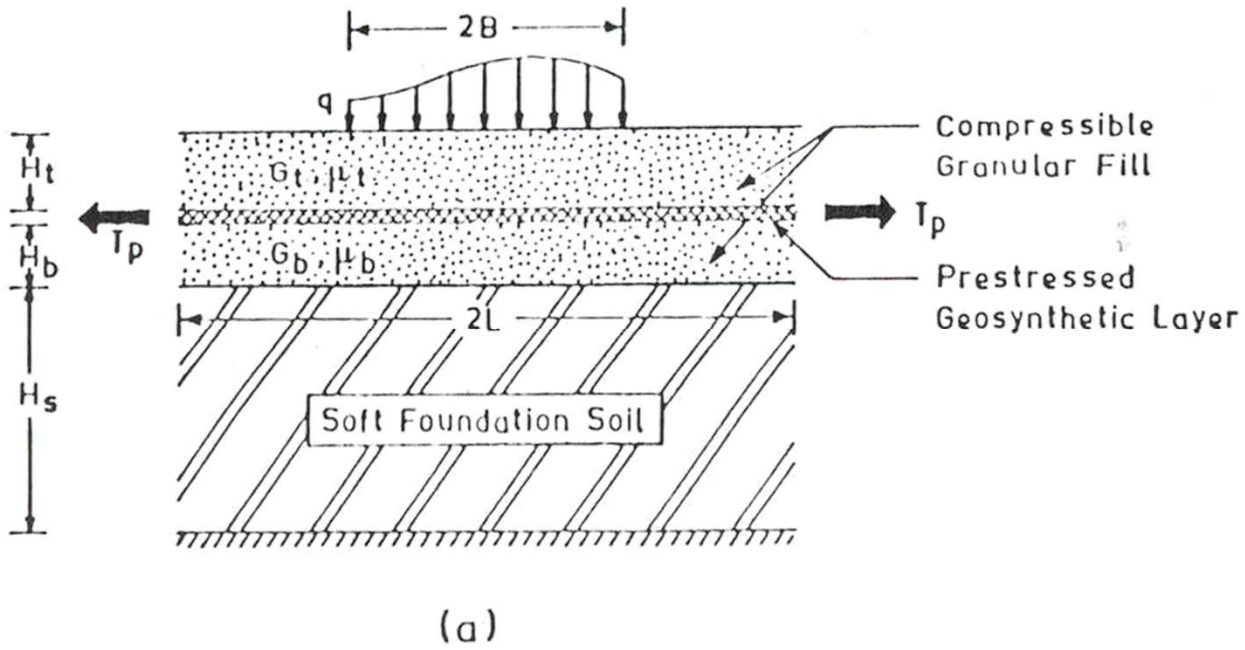
Modifications: To study the influence of the membrane in increasing the lateral stress in the former model some modifications have been made.

Effect of compaction of the Granular layer

Interlocking of stresses on compaction - similar to over consolidated clay behaviour

Shukla and Chandra (1995)

Pretensioning the Reinforcement Layer

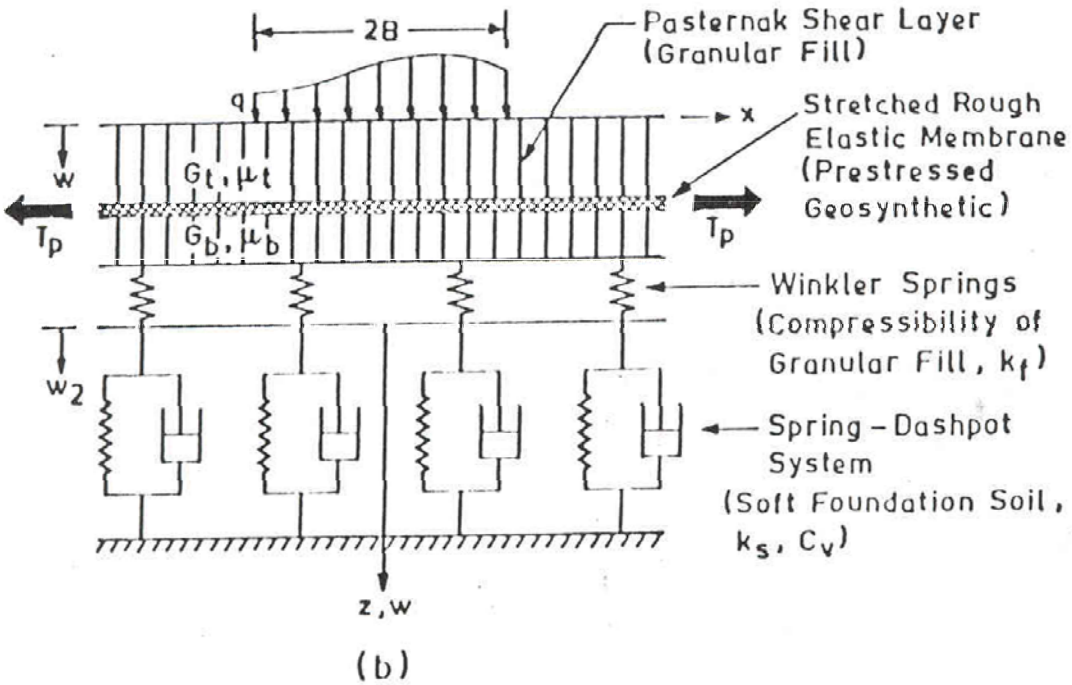


Definition Sketch

Compressibility of Granular fill

Pasternak Shear layer for
Granular material

Time dependent behaviour of soft clay



(b)

Proposed Model

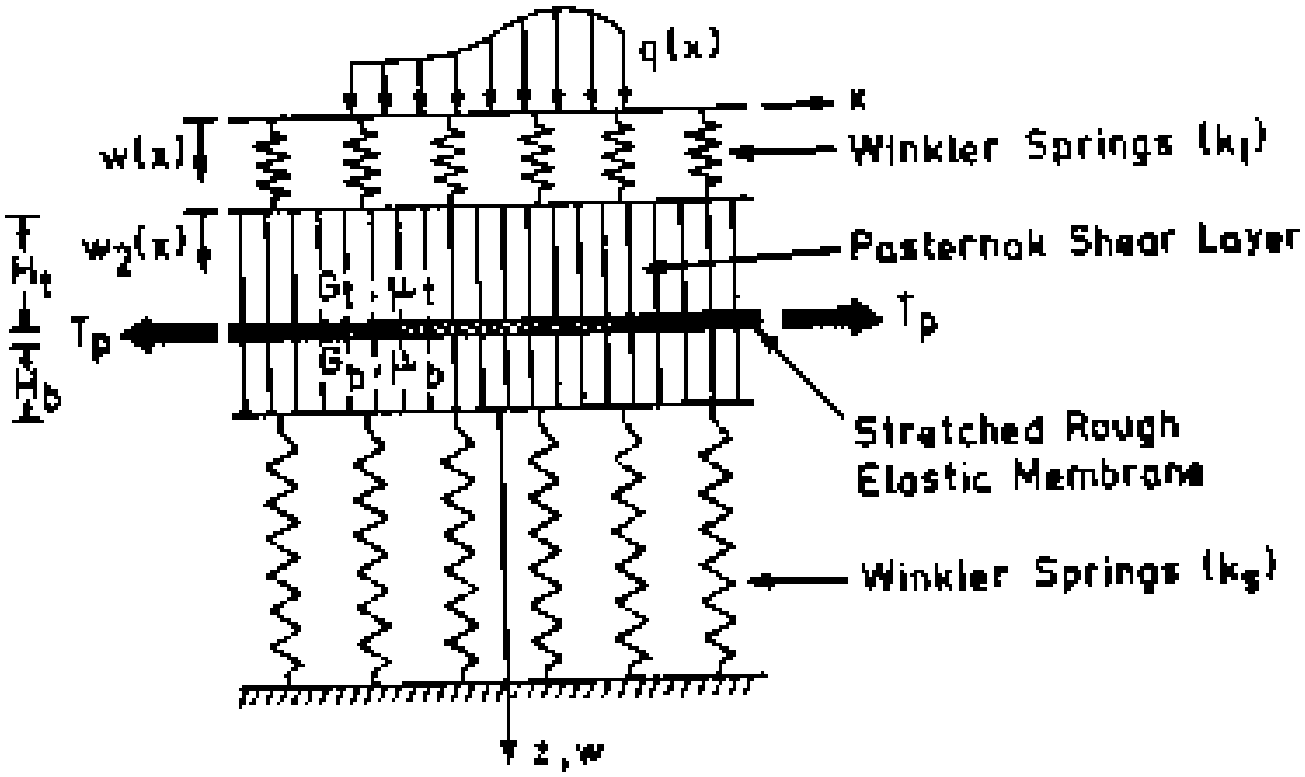


Figure 1. Definition sketch — proposed foundation model

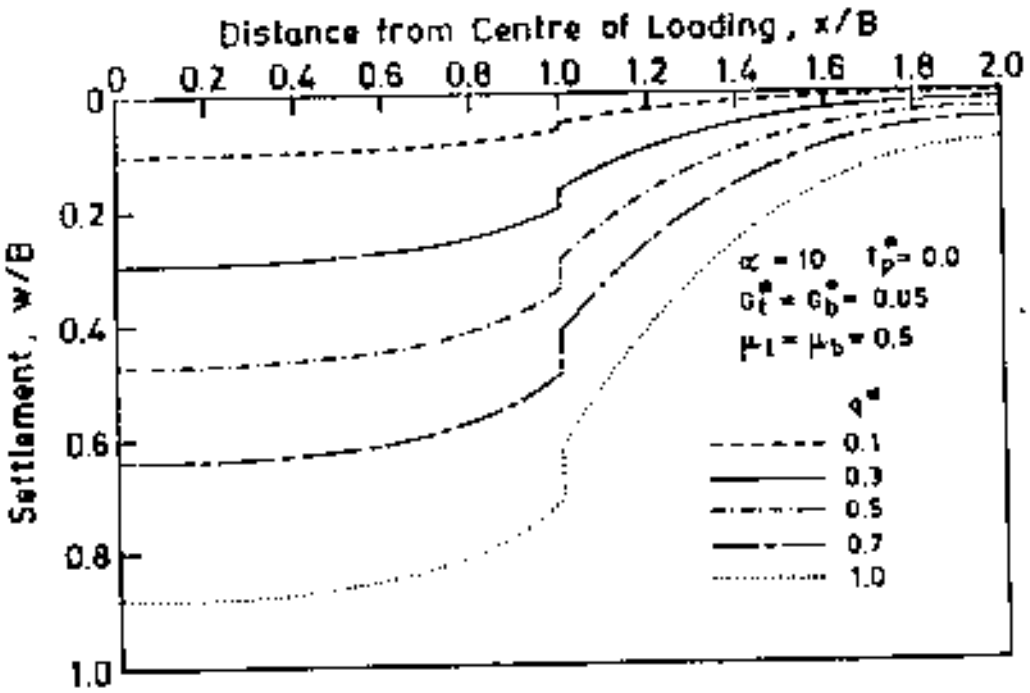
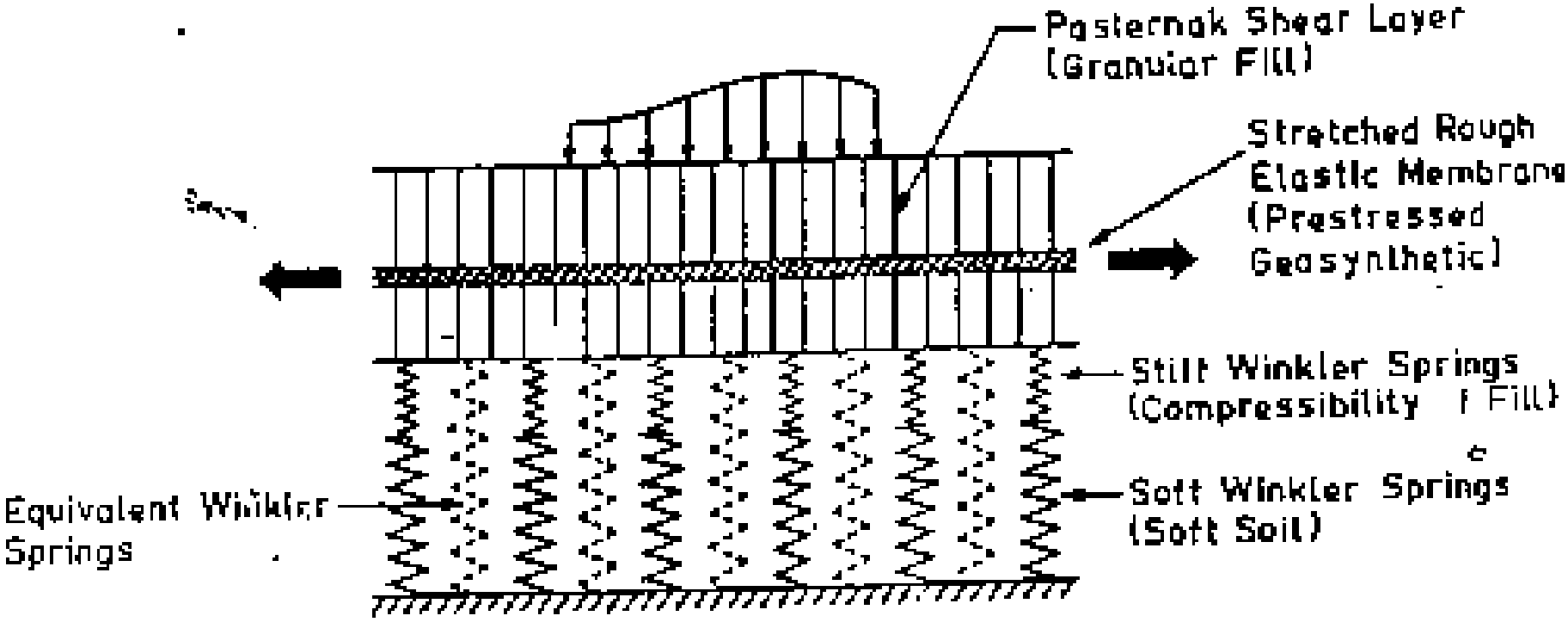


Figure 5. Settlement–distance profiles — effect of load intensity


Pasternak Shear Layer
(Granular Fill)
Stretched Rough
Elastic Membrane
(Prestressed
Geosynthetic)
Stiff Winkler Springs
(Compressibility of Fill)
Soft Winkler Springs
(Soft Soil)
Equivalent Winkler
Springs

Thank You.