GMM Lec.
Additional note

* Univariate Gaussian pdf

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

μ : Mean and σ^2 : Variance

* Multivariate Gaussian pdf

$$f(x) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^t \Sigma^{-1} (x-\mu)\right]$$

x : d component column vector
μ : d component mean vector
Σ : dxd covariance matrix
$|\Sigma|$: Determinant of Σ
Σ^{-1} : Inverse of Σ
$(x-\mu)^t$: Transpose of Matrix $(x-\mu)$

GMM Lecture Additional note

Maximum Likelihood Estimates (ML) for univariate Gaussian pdf

Let ϕ is (μ, σ^2)

Log Likelihood

$$\log f(x_k|\phi) = -\frac{1}{2}\log 2\pi\sigma^2 - \frac{1}{2\sigma^2}(x_k-\mu)^2$$

Partial derivative wrt μ is

$$\frac{\partial}{\partial\mu}\log f(x_k|\phi) = \frac{1}{\sigma^2}(x_k-\mu)$$

also

$$\frac{\partial}{\partial\sigma^2}\log f(x_k|\phi) = -\frac{1}{2\sigma^2} + \frac{(x_k-\mu)^2}{2\sigma^2}$$

By summing over all sample data values x_k and equating to zero we have

$$\sum_{k=1}^{n}\frac{1}{\sigma^2}(x_k-\mu) = 0, \quad -\sum_{k=1}^{n}\frac{1}{\sigma^2} + \sum_{k=1}^{n}\frac{(x_k-\mu)^2}{\sigma^4} = 0$$

$\therefore$ ML Estimates for μ and σ^2

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} x_k, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{k=1}^{n}(x_k-\hat{\mu})^2$$

ML For Multivariate case

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

$$\hat{\Sigma} = \frac{1}{n}\sum_{k=1}^{n}(x_k-\hat{\mu})(x_k-\hat{\mu})^t$$