


# NLO unitarity and stability bounds in extended Higgs sectors

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# Outline

- Perturbative Unitarity at Next-to-Leading Order
- Vaccum Stability
- Models with Higgs Triplets
- Constraints on the Parameter Space

# Introductions

- Standard Model (SM) of particle physics is a framework $\longrightarrow$ good agreement with the collider data

  Open questions : Neutrino masses , Dark Matter , Baryon asymmetry of the Universe (BAU) , Electroweak Phase Transitions (EWPT)

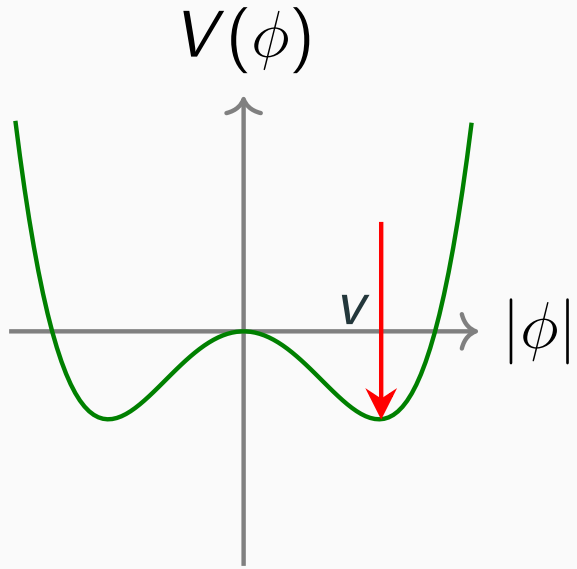
- SM might be a simplified version of a more complicated model

- Do the LHC data preclude the existence of additional multiplets in the scalar sector of the SM ?

# Prior to the Higgs Discovery

Standard Model

$$V(\phi) = -\mu'^2|\phi|^2 + \lambda|\phi|^4$$




Higgs mass : $m_h = \sqrt{2\lambda}\, v$

$\mu'^2 > 0, \quad \lambda > 0$

$m_h \rightarrow$ free parameter

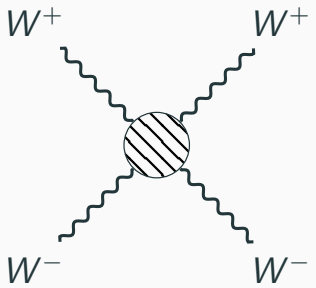
*This is a potential at tree-level*

Theoretical considerations were used to place bound on the Higgs mass

*... such as perturbative unitarity bounds*

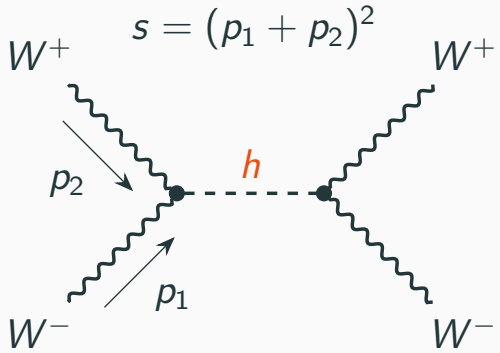
# Does $WW$ scattering make sense?

- Without a Higgs boson




$$\mathcal{M} \sim -g_W^2 \frac{E^2}{4m_W^2}$$

- With a Higgs boson




$$\mathcal{M} \sim g_{hWW}^2 \left( \frac{E^2}{4m_W^4} - \frac{m_h^2}{2m_W^4} \right)$$

for $\sqrt{s} \gg m_h, m_W$

$$g_{hWW} = g_W m_W$$

# Perturbative Unitarity

- Unitarity of the $S$-matrix, $S^\dagger S = I$,

$$\left| a_\ell^{2\to 2} - \tfrac{1}{2} i \right|^2 + \sum_{k>2} \left| a_\ell^{2\to k} \right|^2 = \tfrac{1}{4} \,. \tag{1}$$

  $\to$ $a_\ell$'s are the $\ell$-th partial-wave amplitudes

- For a given $2 \to 2$ process, the unitarity bounds :

$$\left| a_\ell - \tfrac{1}{2} i \right|^2 \leq \tfrac{1}{4} \tag{2}$$

- At tree-level, $a_\ell \in \mathbb{R}$, leads to a strong bound, $|\mathrm{Re}(a_\ell)| \leq \frac{1}{2}$.
- Beyond tree-level, $a_\ell \notin \mathbb{R}$, hence above bound gets weaker.

# One-loop corrections to $S$-matrix

- For a given process, $i \to f$, arXiv: 1512.04567, 1702.08511

$$(a_0)_{i,f}(s) = -\frac{1}{16\pi} b_0 + \frac{1}{256\pi^3}\left(b_1^R + i b_1^I\right), \qquad (3)$$

or,

$$256\pi^3 (\mathbf{a_0^{Q,Y}})_{\text{NLO}} = -16\pi^2 \mathbf{b_0} + (i\pi - 1)\mathbf{b_0}\cdot\mathbf{b_0} + 3\beta_{\mathbf{b_0}}\,.$$

– $\mathbf{b_0}$ is the tree-level $S$-matrix and $\beta_{\mathbf{b_0}}$ represents the matrix involving beta functions.

– For example, if $b_0 = a\lambda_i + b\lambda_j$, then $\beta_{b_0} = a\beta_{\lambda_i} + b\beta_{\lambda_j}$, $\forall a, b \in \mathbb{R}$.

*Does it hold beyond next-to-leading order?*

# Theoretical Bounds in Weakly Interacting Theories

- Standard Model (SM) : Prior to the Higgs discovery

LO unitarity : $\lambda \leq \frac{8\pi}{3}$ [Lee, Quigg, Thacker '77]

NLO unitarity : $\lambda \leq 2 - 2.5$ [Dawson, Eillenbrock '89; Durand, Johnson, Lopez'92]

No revised limit @2-loop [Durand, Maher, Riesselmann, 92]

} Weakly interacting SM Higgs scenario

- Two-Higgs Doublet Model : SM with additional Higgs doublet

arXiv: 1512.04567, 1609.01290

In principle, unitarity can hold up to $\lambda_i(s) \approx 15$

$$R_1 = \frac{\left|a_0^{\rm NLO}\right|}{\left|a_0^{\rm LO} + a_0^{\rm NLO}\right|}\,, \qquad R_1' = \frac{\left|a_0^{\rm NLO}\right|}{\left|a_0^{\rm LO}\right|}\,,$$

$R_1 = 1$, *or* $R_1' = 1$, perturbativity is violated when $\lambda(s) \sim 4.3$

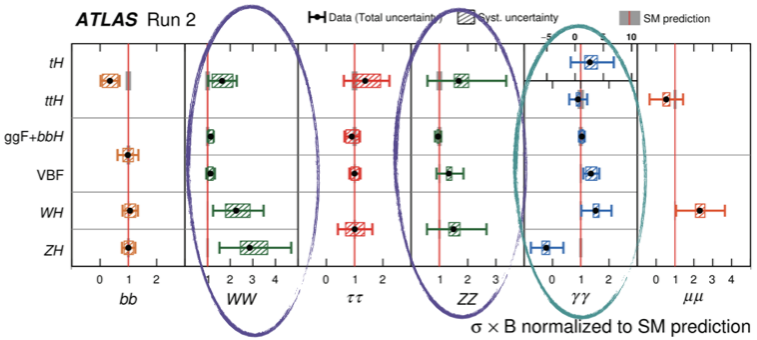
# Models with Higgs Triplets

- Both ATLAS and CMS suggest an enhanced rate of $WW$ and $ZZ$ relative to the SM

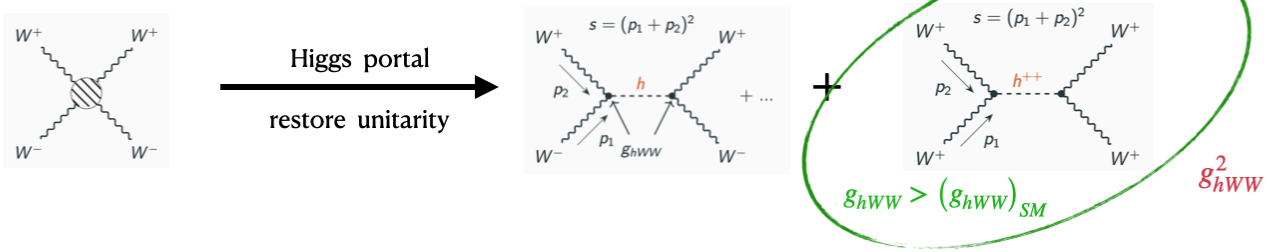
The questions we ask is :

If $WW$ and $ZZ$ rates are enhanced,
how far beyond the SM one must to describe them?

Additional charged Higgs ?




**Recap :** $WW$ scatterings are weakly interacting or strongly interacting ?




- Additional doubly charged Higgs boson plays an important role to restore unitarity at high energies

**Search for BSM @ forefront of particle physics research**

# Models with Higgs Triplets

- Custodial Symmetry : $M_W^2 = M_Z^2 \cos^2\theta_W$

  Rho-parameter, $\rho = \dfrac{M_W^2}{M_Z^2 \cos^2\theta_W}$

  At tree-level, $\rho = 1$

- Triplet extended Higgs sector with Custodial Symmetry :

  SM doublet $\phi$ $(T = 1/2, Y = 1/2)$ + Real triplet $\xi$ $(T = 1, Y = 0)$ + Complex triplet $\chi$ $(T = 1, Y = 1)$

  $\langle\phi\rangle = v_\phi\,, \langle\xi\rangle = v_\xi\,, \langle\chi\rangle = v_\chi$ → $\rho = 1$ in tree-level if $v_\chi = v_\xi$

  * Georgi-Machacek (GM) model :

    Equality of triplet VEVs is preserved by the Higgs potential [Georgi, Machacek '85; Chanowitz, Golden '85]

    Interactions among the Higgs fields maintained $SU(2)_L \times SU(2)_R$ symmetry

  * extended Georgi-Machacek (eGM) model :

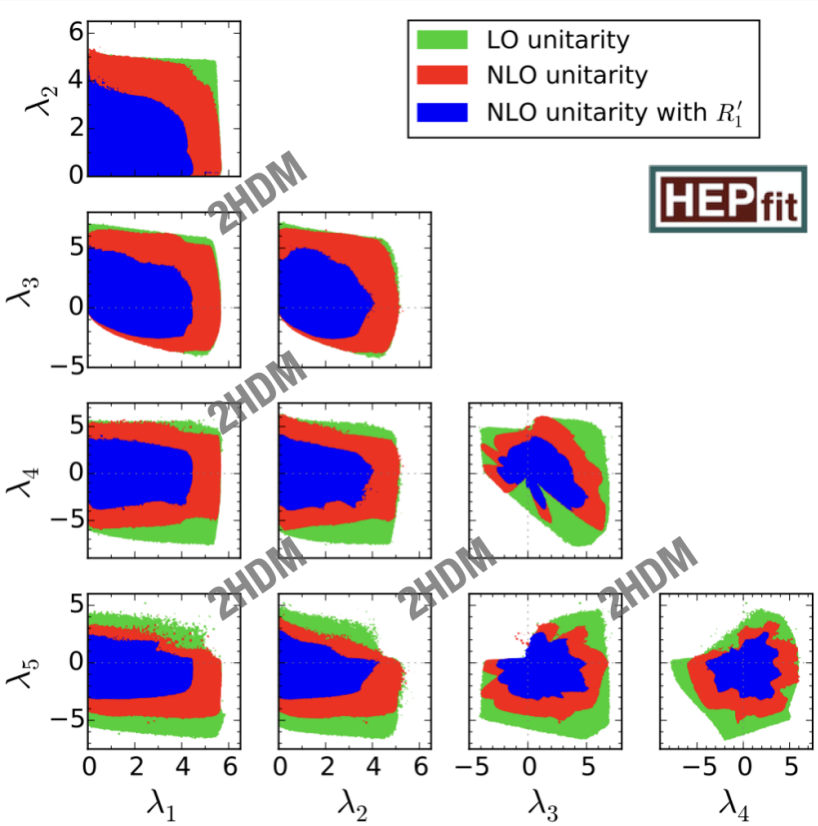
    Equality of triplet VEVs is obtained by tuning the potential parameters @ tree-level [Kundu, Mondal, Pal '21]

    Higgs interactions does not maintained $SU(2)_L \times SU(2)_R$ symmetry

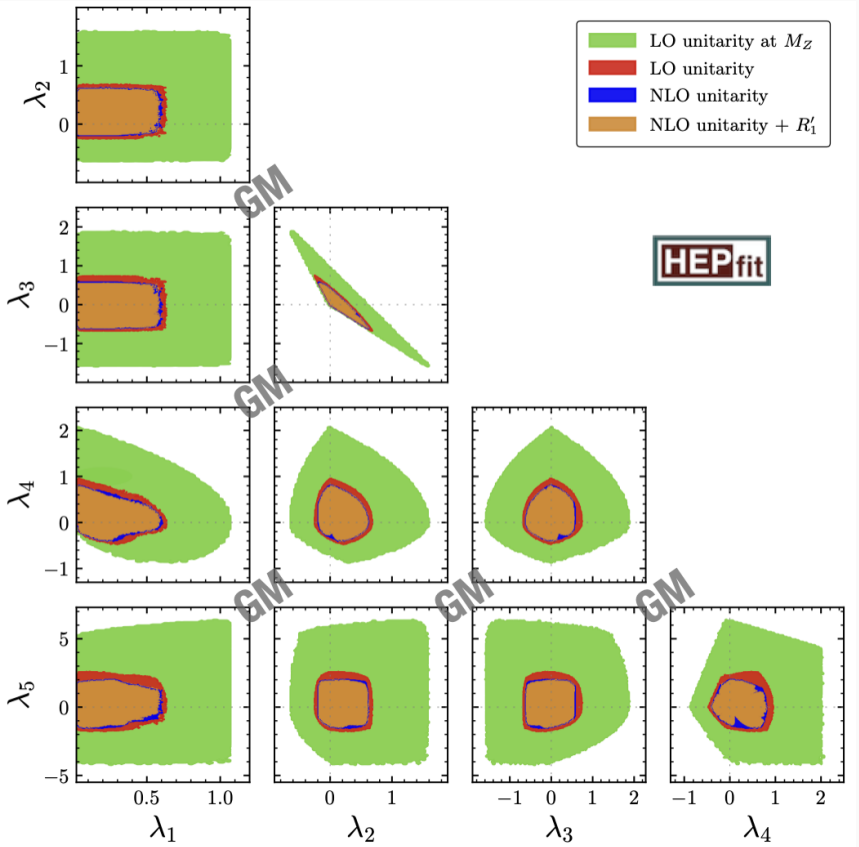
    ...Similar interactions were considered in 2HDM with softly broken $Z_2$ symmetry

**GM and eGM models are classified based on their symmetry, with identical field contents**

# Perturbative Unitarity at One-Loop (NLO)




Cacchio, Chowdhury, Eberhardt, Murphy '16 (arXiv: 1609.01290)

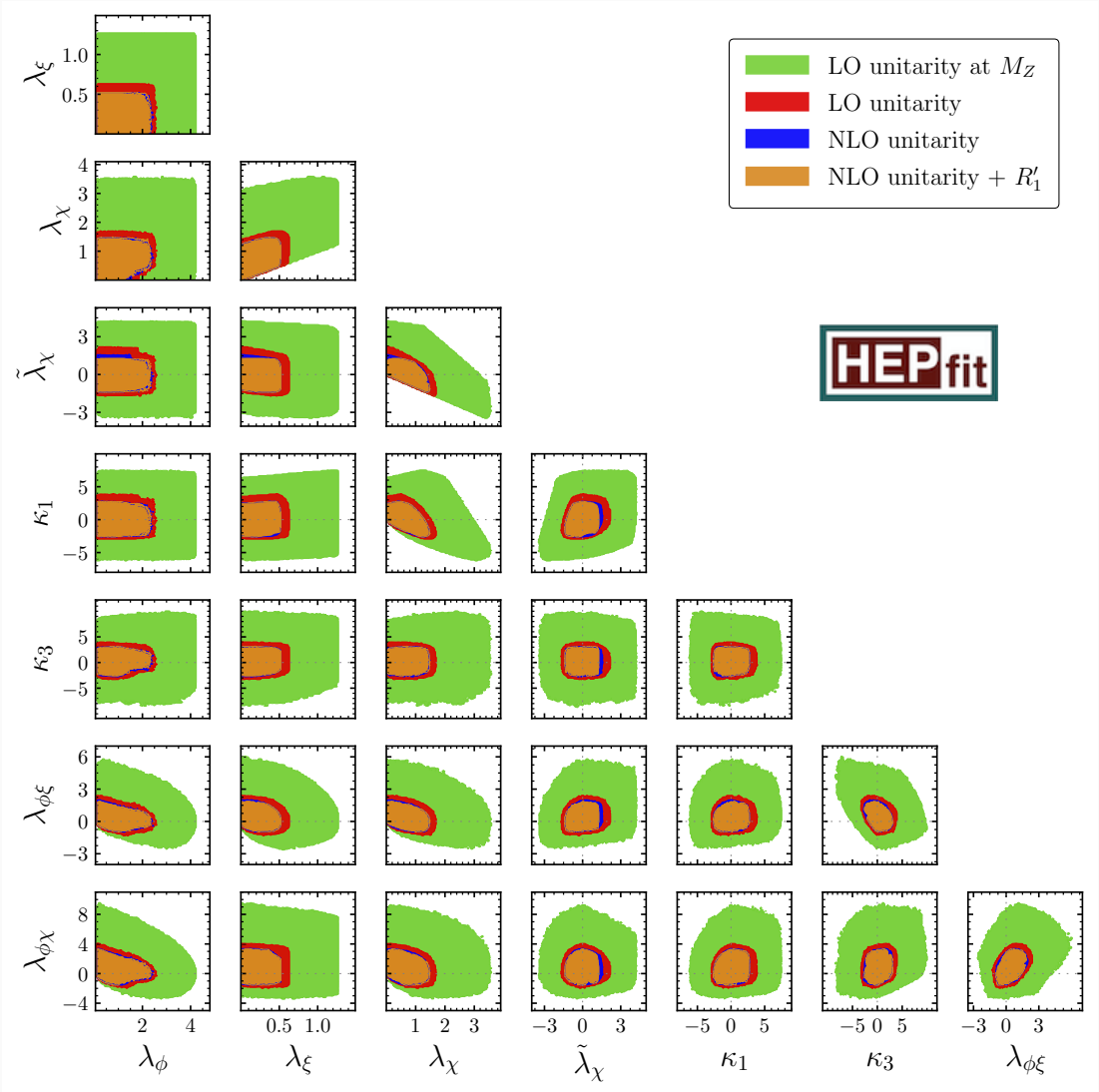



Chowdhury, Mondal, **SS** '24 (arXiv: 2404.18996)

**NLO unitarity significantly refine the parameter space**

arXiv: 1512.04567, 1609.01290, 2404.18996

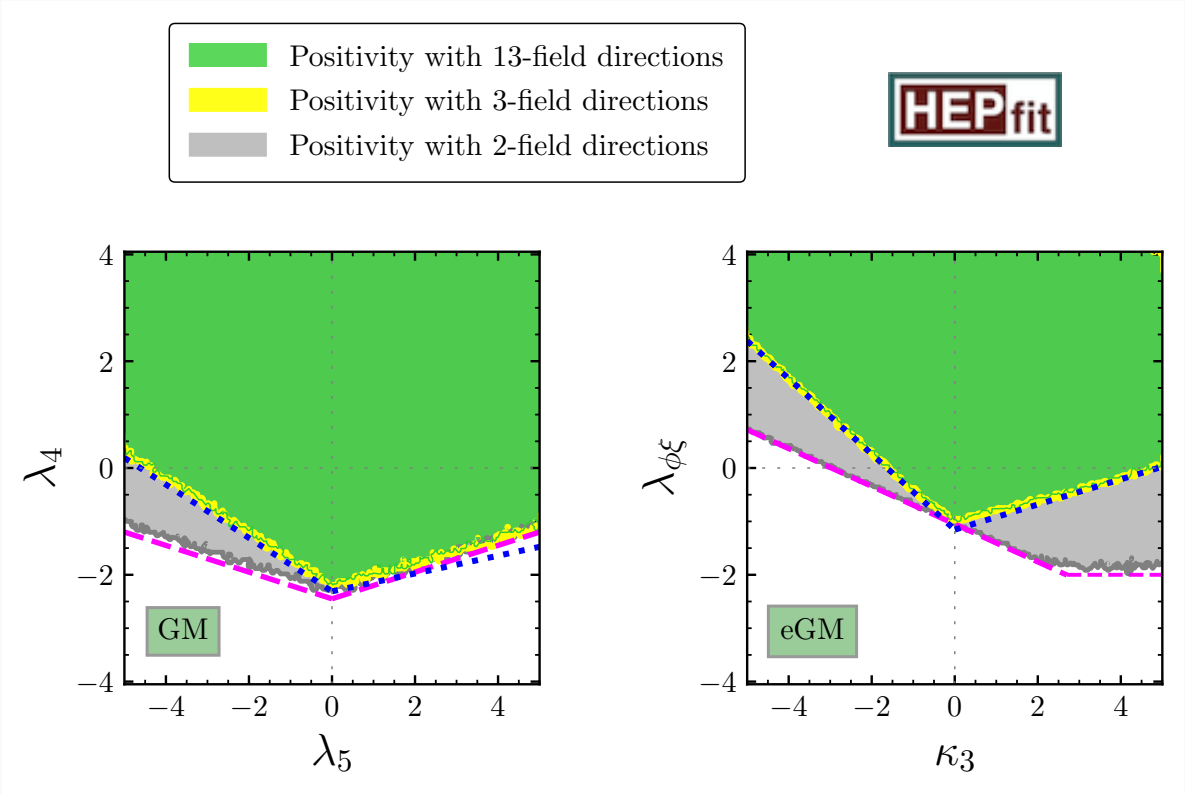
# Extended GM model




Chowdhury, Mondal, **SS '24 (arXiv: 2404.18996)**

# Positivity Conditions

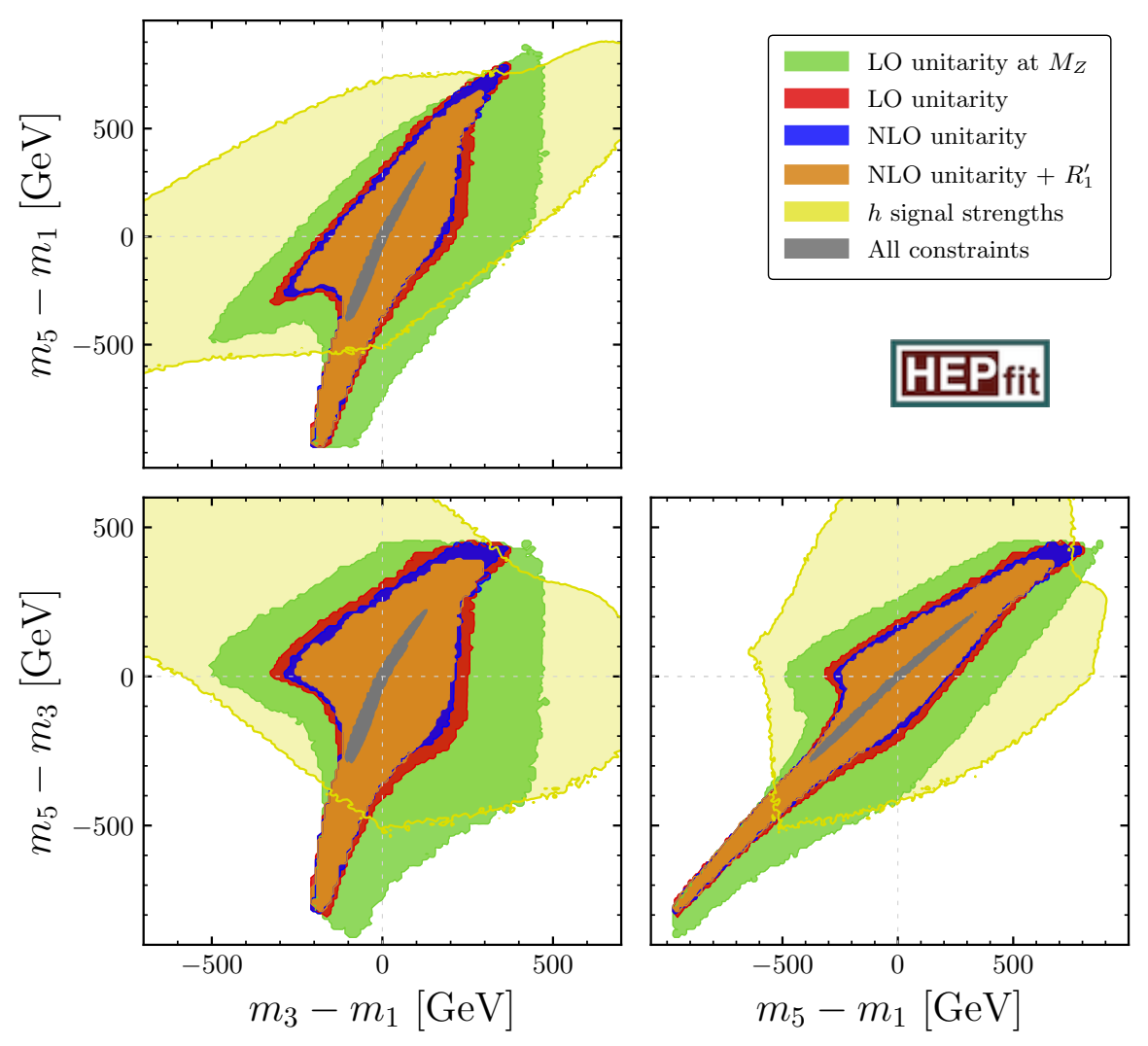
- Ensure that boundedness of the potential in any directions of field space




- These 3-field direction BFB conditions are faster numerically

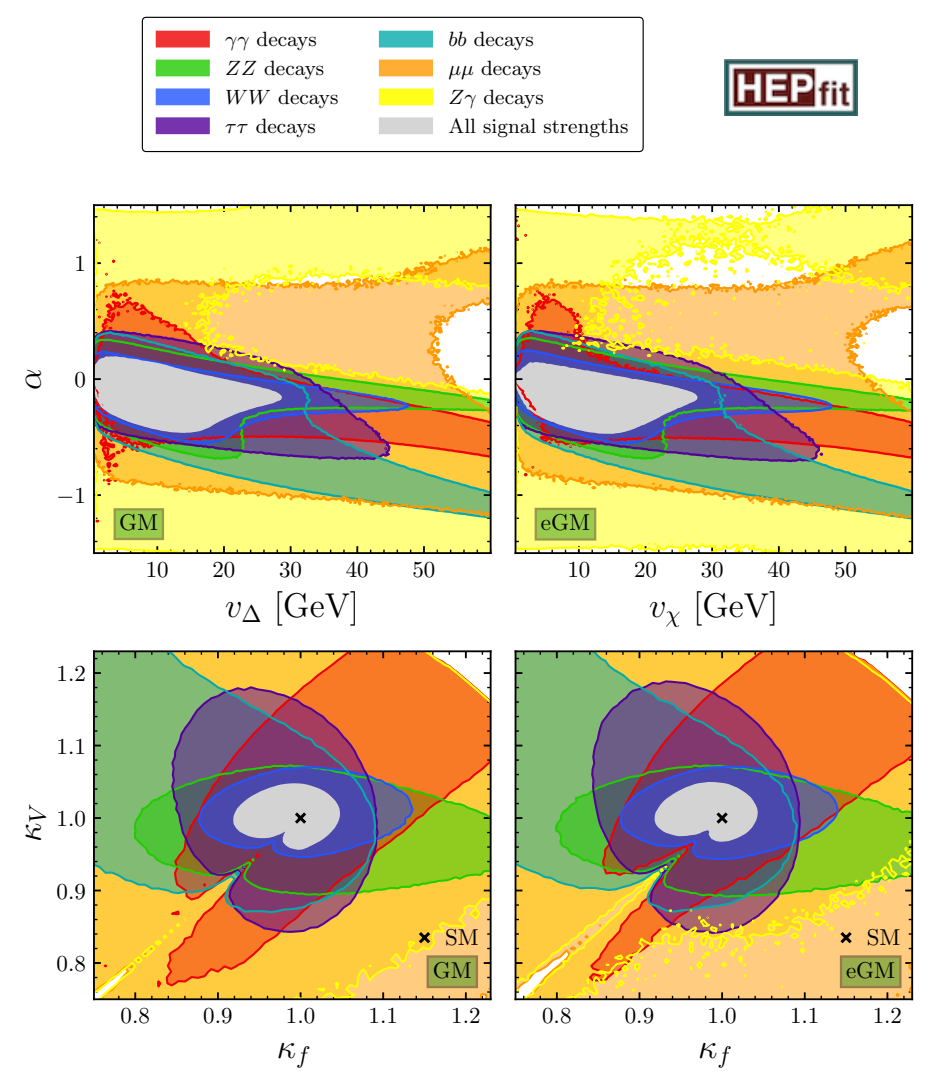
Chowdhury, Mondal, **SS '24 (arXiv: 2404.18996)**

# GM model: combined global fits




Chowdhury, Mondal, **SS '24 (arXiv: 2404.18996)**

# Results from fit to Higgs signal strength data




Chowdhury, Mondal, **SS '24 (arXiv: 2404.18996)**

# Take away message

- Improved theoretical constraints (NLO unitarity with positivity) significantly refine the parameter space of the BSM models.
- Extending the SM with Higgs triplets provides an enhanced tree-level rate for $h \to WW$ or $ZZ$ compared to the SM.
- Triplet VEVs gets more and more constrained from the LHC data.
- Regions where $|\kappa_V| > 1.05$ are disfavoured by the latest LHC data.

$$\kappa_V = \frac{g_{hVV}^{\mathrm{GM}}}{g_{hVV}^{\mathrm{SM}}}$$

# Thank You !

# Questions?