

**Comparing Efficiency across State Transport Undertakings:  
A Production Frontier Approach\***

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## **Comparing Efficiency across State Transport Undertakings:**

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#### **Abstract**

Currently, in India, there are sixty-four State Transport Undertakings (STUs) operating in various states which play a major role in providing short as well as medium distance passenger mobility. The regulated environment in which STUs operate imposes many qualitative as well as quantitative constraints on their production. Therefore, the STUs have relatively few incentives to produce efficiently. This study attempts to answer the question: “How efficient are the STUs?”. In this study, an attempt has been made to quantify the technical efficiency (productive efficiency) of twenty-three major Indian State Transport Undertakings mainly providing rural and inter-city passenger transport services for the year 2000-01. This is done by the estimation of stochastic frontier production function using the method of maximum likelihood. We found that there is huge disparity in technical efficiency across STUs ranging from 56.15% for Madhya Pradesh State Road Transport Undertaking to 98.99% for Tamilnadu State Transport Corpn. Ltd. (Kumbakonam Division II). Average of technical efficiency scores of sample STUs was found to be 84.22%. The main conclusion in our analysis is that given the size distribution of the sample STUs and their working environment, the potential gain in productive efficiency for most of them is very high.

**Key Words:** production frontier models, technical efficiency, State Transport Undertakings

**JEL Classification:** L32, L92, O30, R40

# Comparing Efficiency across State Transport Undertakings:

## A Production Frontier Approach

### 1. Introduction

Buses are playing major role in providing short as well as medium distance passenger mobility in India. Bus transport services in India are provided by both Private Bus Operators (henceforth, PBOs) as well as publicly owned State Transport Undertakings (henceforth, STUs). Indian bus industry, particularly stage carriage services, is dominated by STUs. Currently, there are 64 STUs in India operating in various states of the country. Due to a variety of reasons, most of the STUs over the years have accumulated deficits and have not been able to meet the increasing travel needs of the public. The state governments control the STUs' fares and hence to a large extent hinder the ability of these firms to supply the optimum level of output both in terms of quality as well as quantity. Therefore, the STUs have relatively few incentives to run their business efficiently. The question may be asked how efficient are the STUs? i.e., given the input factor quantities, a comparison is made between the actual output with the maximum possible one that can be produced from these inputs. What is their level of production in comparison to a fully efficient firm having comparable input values?

This study attempts to provide answers to these questions. We may begin by recalling that Farrell (1957) proposed a measure of the efficiency of a firm that consists of two components: *technical efficiency*, which reflects the ability of a firm to obtain maximal output from a given set of inputs, and *allocative efficiency*, which reflects the ability of a firm to use the inputs in optimal proportions, given their respective prices. To estimate the allocative efficiency in STUs is beyond the scope of this study. We will primarily focus on

the levels of technical (in)efficiency (also called productive efficiency) in Indian STUs. To examine the level of technical (in)efficiency in STUs, we estimate a stochastic frontier production function by using the method of maximum likelihood. Annual data for a sample of 23 STUs for the year 2000-01 are used for the purpose of estimation. The statistical program *FRONTIER Version 4.1* is used for this.

The remainder of this study is organized as follows: Section 2 describes the sample STUs and the data and section 3 discusses the methodology adopted for the study and results of estimation of technical efficiency in STUs. Finally, section 4 summarizes and concludes this study.

## **2. Sample STUs and the data**

### ***2.1. Sample STUs***

This study is based on a sample of 23 major STUs in India. Sample STUs mainly provide rural and inter-city services to the public. To make the comparison meaningful we include in the sample to only those STUs, which are having fleet strength more than 700 buses. Here is the list of sample STUs along with their fleet strength i.e., number of buses held during the year 2000-01:

1. Andhra Pradesh State Road Transport Corporation (APSRTC) – 18946
2. Maharashtra State Road Transport Corporation (MSRTC) – 16916
3. Gujarat State Road Transport Corporation (GSRTC) – 9847
4. Uttar Pradesh State Road Transport Corporation (UPSRTC) – 7801
5. Rajasthan State Road Transport Corporation (RSRTC) – 4754
6. Karnataka State Road Transport Corporation (KnSRTC) – 6128
7. Kerala State Road Transport Corporation (KSRTC) – 4478

8. North West Karnataka State Transport Corporation (NWKnRTC) – 3477
9. State Transport Haryana (STHAR) – 3470
10. State Transport Punjab (STPJB) – 2369
11. Madhya Pradesh State Road Transport Corporation (MPSRTC) – 2525
12. Tamilnadu State Transport Corpn. Ltd. (Coimbatore Div. I&III) (CBE-I&III) – 1461
13. Tamilnadu State Transport Corpn. Ltd. (Villupuram Div. I) (VPM-I) – 1035
14. Pepsu Road Transport Corporation (PRTC) – 1156
15. Tamilnadu State Transport Corpn. Ltd. (Kumbakonam Div. II) (KUM-II) – 895
16. Tamilnadu State Transport Corpn. Ltd. (Kumbakonam Div. I) (KUM-I) – 923
17. Tamilnadu State Transport Corpn. Ltd. (Salem Div. I) (SLM-I) – 945
18. Tamilnadu State Transport Corpn. Ltd. (Coimbatore Div. II) (CBE-II) – 916
19. Tamilnadu State Transport Corpn. Ltd. (Madurai Div. I) (MDU-I) – 921
20. Tamilnadu State Transport Corpn. Ltd. (Villupuram Div. II) (VPM-II) – 837
21. Tamilnadu State Transport Corpn. Ltd. (Madurai Div. II) (MDU-II) – 835
22. Tamilnadu State Transport Corpn. Ltd. (Villupuram Div. III) (VPM-III) – 752
23. Tamilnadu State Transport Corpn. Ltd. (Madurai Div. IV) (MDU-IV) – 730

Table 1 presents descriptive statistics of sample undertakings during 2000-01. The size of the undertakings, as measured by effective bus-kilometers (BKm) in 2000-01, ranges from 1069 lakh BKm for VPM-III to 21781 lakh BKm for APSRTC. Fleet strength of sample STUs also varies drastically, from 730 buses for MDU-IV to 18946 buses for APSRTC. As far as economic profitability (defined as a ratio of traffic revenue to operating cost where operating cost does not include tax component of costs) of sample STUs is concerned, it varies from 0.81 for MDU-II to 1.11 for STHAR during 2000-01. Ten out of twenty-three

STUs have managed to raise their traffic revenue sufficient enough to at least recover their respective operating cost. However, majority of the sample undertakings are making huge financial losses.

**Table 1. Descriptive statistics of the sample undertakings during 2000-01**

STUs	Bus-Km (Lakh)	Traffic Rev. (Rs. Lakh)	No. of employees	Economic Profitability <sup>1</sup>
APSRTC	<b>21781</b>	<b>244843</b>	<b>128796</b>	1.03
MSRTC	17944	226435	112116	1.00
GSRTC	11517	119840	61189	0.87
UPSRTC	6895	67745	47369	0.82
RSRTC	5241	56552	25030	0.96
KnSRTC	5971	66964	24117	0.98
KSRTC	3625	55975	34335	0.87
NWKnSRTC	4096	44102	20820	0.99
STHAR	3840	47794	19587	<b>1.11</b>
STPJB	1895	21675	11736	0.96
MPSRTC	2120	27219	19260	0.90
CBE-I&III	1914	22054	11436	0.86
VPM-I	1664	19688	7612	1.06
PRTC	1095	14558	<b>5133</b>	1.06
KUM-II	1492	17463	6589	1.05
KUM-I	1385	16647	6782	1.03
SLM-I	1426	16497	6708	1.02
CBE-II	1451	16521	6599	1.04
MDU-I	1107	14934	6793	0.94
VPM-II	1217	13890	6109	1.00
MDU-II	1149	12570	6468	<b>0.81</b>
VPM-III	<b>1069</b>	12472	5954	0.96
MDU-IV	1071	<b>11980</b>	5507	0.94

<sup>1</sup> Economic profitability is defined as a ratio of traffic revenue to operating cost where operating cost is total cost minus taxes.

## **2.2. Data source**

A cross-section of 23 STUs during 2000-01 forms the primary data base for this study. The annual data were compiled mainly from *State Transport Undertakings: Profile and Performance 2000-2001* published for the ASSOCIATION OF STATE ROAD TRANSPORT UNDERTAKINGS, NEW DELHI by the CENTRAL INSTITUTE OF ROAD TRANSPORT, PUNE, INDIA.

## **3. Technical efficiency in state transport undertakings: analysis and results**

### *3.1 The stochastic frontier approach*

A number of methods for measuring efficiency have been proposed over the last decade, all of which have in common the concept of the frontier; efficient units are those operating on the cost or production frontier, while inefficient ones operate either below the frontier (in case of the production frontier) or above the frontier (in the case of the cost frontier). The literature on frontier models begins with Farrell (1957) who suggested a useful and subsequently widely accepted framework for analyzing economic efficiency in terms of realized deviations from an idealized frontier isoquant.

The stochastic frontier approach postulates that some firms fail to achieve the production (cost) frontier. That is, inefficiencies exist, and these inefficiencies cannot be fully explained by measurable variables. Thus, a one-sided error term, in addition to the traditional symmetric noise term, is incorporated in the model to capture inefficiencies which can not be explicitly explained.

In line with the works of Aigner, Lovell and Schmidt (1977), and Meeusen and van den Broeck (1977) on estimation of inefficiency by using a stochastic frontier approach, we specify stochastic frontier production function for cross-sectional data as:

$$\ln(Y_i) = X_i\beta + (V_i - U_i); \quad i = 1, 2, \dots, N. \quad (1)$$

where,

$\ln(Y_i)$  denotes logarithm of the output for the  $i^{\text{th}}$  firm;

$X_i$  represents a  $(K+1)$  – row vector, whose first element is “1” and the remaining elements are the logarithms of the  $K$ -input quantities used by the  $i^{\text{th}}$  firm;

$\beta$  is a  $(K+1)$  – column vector of unknown parameters to be estimated;

the random error,  $V_i$ , accounts for measurement error and other random factors, such as the effect of strikes, economic activities in the region, luck etc., on the value of the output variable, together with the combined effects of unspecified input variables in the production function. They are assumed to be independent and identically distributed (i.i.d.) normal random variables with mean zero and constant variance,  $\sigma_v^2$ , independent of the  $U_i$ s; and

the  $U_i$ s are non-negative random variables (with standard deviation  $\sigma_u$ ) associated with the inefficiency of the firm  $i$ .

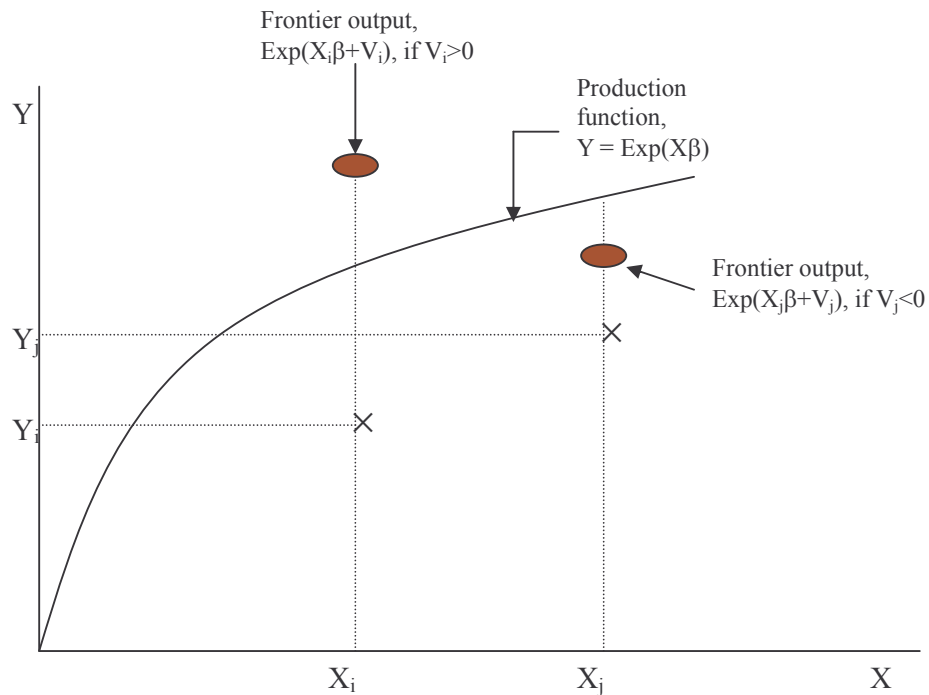
In other words,  $X_i\beta + V_i$  is the stochastic frontier while  $U_i$  is the measure of deviation from the frontier for the  $i^{\text{th}}$  firm. The random error,  $V_i$ , can be positive or negative and so the stochastic frontier outputs vary about the deterministic part of the frontier model,  $X_i\beta$ . The condition,  $U_i$  is non-negative, ensures that all observations lie on or below the production frontier.

The basic features of the stochastic frontier model are illustrated in two dimensions in Figure 1. The inputs are represented on the horizontal axis and the outputs on the vertical axis. The deterministic component of the frontier model,  $Y = \text{Exp}(X\beta)$  is drawn assuming that diminishing returns to scale apply. The observed outputs and inputs for two firms  $i$  and  $j$  are presented on the graph. The  $i^{\text{th}}$  firm uses the level of inputs  $X_i$  to produce the output  $Y_i$ . The observed input-output value is indicated by the point marked with  $\times$  above the value of



$X_i$ . The value of the stochastic frontier output  $Y_i^* \equiv \text{Exp}(X_i\beta + V_i)$ , is marked by the shaded oval point above the production function because the random error  $V_i$  is positive. However, in the case of the  $j^{\text{th}}$  firm, the frontier output  $Y_j^* \equiv \text{Exp}(X_j\beta + V_j)$  is below the production function because the random error  $V_j$  is negative. Of course, the stochastic frontier output  $Y_i^*$  and  $Y_j^*$  are not observed because the random errors  $V_i$  and  $V_j$  are not observable. However, the deterministic part of the stochastic frontier model is seen to lie between the stochastic frontier outputs. The observed outputs may be greater than the deterministic part of the frontier if the corresponding random errors are greater than the corresponding inefficiency effects (i.e.,  $Y_i > \text{Exp}(X_i\beta)$  if  $V_i > U_i$ ).

**Figure 1. The Stochastic Frontier Production Function**



The parameters of the stochastic frontier production function, defined by equation **(1)**, can be estimated using either the maximum-likelihood (ML) method or using a variant of the corrected ordinary least squares (COLS) method, suggested by Richmond (1974). The COLS approach is not as computationally demanding as the ML method, which requires numerical maximization of the likelihood function. This distinction, however, has lessened in recent years with the availability of sophisticated econometric software, such as Frontier Version 4.1, Limdep Version 7.0 etc.

The ML estimator is asymptotically more efficient than COLS estimator but the properties of the two estimators in finite samples can be analytically determined. The finite sample properties of the half-normal frontier model were investigated in a Monte Carlo experiment by Coelli (1995), in which the ML estimator was found to be significantly better than the COLS estimator when contribution of the technical inefficiency effects to the total variance term is large. Given this result and the availability of automated ML routines, the ML estimator should be used in preference to the COLS estimator whenever possible.<sup>2</sup>

We will now discuss the basic elements of obtaining ML estimators for the parameters of the stochastic frontier model. This discussion deals with the case of the half-normal distribution for the technical inefficiency effects, because it has been most frequently assumed in empirical applications. Aigner, Lovell and Schmidt (1977) derived the log-likelihood function for the model, defined by equation **(1)**, in which  $U_i$ s are assumed to be i.i.d. truncations (at zero) of a  $N(0, \sigma_u^2)$  random variable, independent of the  $V_i$ s which are assumed to be i.i.d.  $N(0, \sigma_v^2)$ . Aigner, Lovell and Schmidt (1977) expressed the likelihood function in terms of the two variance parameters,  $\sigma^2 \equiv \sigma_u^2 + \sigma_v^2$  and  $\lambda = \sigma_u/\sigma_v$ . Battese and

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<sup>2</sup> For detailed discussion about COLS estimation, please see Coelli (1995).

Corra (1977) suggested that the parameter,  $\gamma \equiv \sigma_u^2/\sigma^2$ , be used because it has a value between zero and one, whereas the  $\lambda$ -parameter could be any non-negative value. A value of  $\gamma$  of zero indicates that the deviations from the frontier are due entirely to noise, while a value of one would indicate that all deviations from the frontier are due to technical inefficiency. It should be stressed, however, that  $\gamma$  is not equal to the ratio of the variance of the technical inefficiency effects to the total residual variance. This is because the variance of  $U_i$  is equal to  $[(\pi-2)/\pi]\sigma_u^2$  not  $\sigma_u^2$ . The relative contribution of the inefficiency effect to the total variance term ( $\gamma^*$ ) is equal to  $\gamma^* = \gamma/[\gamma+(1-\gamma)\pi/(\pi-2)]$ .<sup>3</sup> One should note that  $\gamma$ -parameterization has advantages in seeking to obtain the ML estimates because the parameter space for  $\gamma$  can be searched for a suitable starting value for the iterative maximization algorithm involved.<sup>4</sup> Battese and Corra (1977) showed that the log-likelihood function, in terms of this parameterization is equal to:

$$\ln(L) = -\frac{N}{2} \ln(\pi/2) - \frac{N}{2} \ln(\sigma^2) + \sum_{i=1}^N \ln[1 - \phi(z_i)] - \frac{1}{2\sigma^2} \sum_{i=1}^N (\ln Y_i - X_i \beta)^2 \quad (2)$$

where  $z_i = \frac{(\ln Y_i - X_i \beta)}{\sigma} \sqrt{\frac{\gamma}{1-\gamma}}$ ; and  $\phi(\cdot)$  is the distribution function of the standard normal variable.

The ML estimates of  $\beta$ ,  $\sigma^2$  and  $\gamma$  are obtained by finding the maximum of the log-likelihood function, defined in equation (2). One should note that the ML estimators are consistent and asymptotically efficient (Aigner, Lovell and Schmidt (1977), p. 28).

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<sup>3</sup> For more details, please see Coelli (1995).

<sup>4</sup> The  $\gamma$ -parameterization also has advantages in COLS estimation, as indicated in Coelli (1995).

The technical efficiency of the  $i^{\text{th}}$  firm is defined by  $TE_i = \exp(-U_i)$  which involves the technical inefficiency effect,  $U_i$ , which is unobservable. Even if the true value of the parameter vector,  $\beta$ , in the stochastic frontier model **(1)** was known, only the difference,  $E_i \equiv V_i - U_i$ , could be observed. The best predictor for  $U_i$  is the conditional expectation of  $U_i$ , given the value of  $V_i - U_i$ . This is related to the Rao-Blackwell theorem (see, Rao (1973, p.121)). This result was first recognized and applied in the stochastic frontier model by Jondrow et al. (1982), who derived the result as follows:

$$E[U_i | E_i] = -\gamma E_i + \sigma_A \left\{ \frac{\phi(\gamma E_i / \sigma_A)}{1 - \phi(\gamma E_i / \sigma_A)} \right\} \quad (3)$$

where  $\sigma_A = \sqrt{\gamma(1-\gamma)\sigma^2}$ ;  $E_i = \ln(Y_i) - X_i\beta$ ; and  $\phi(\cdot)$  is the density function of a standard normal random variable.

An operational predictor of  $U_i$  involves replacing the unknown parameters in equation **(3)** with ML (or COLS) estimators. Jondrow et al. (1982) suggested that the technical efficiency of the  $i^{\text{th}}$  firm be predicted using  $1 - E[U_i | E_i]$ . The rationale for this predictor is that  $1 - U_i$  is a first-order approximation to the infinite-series,  $\exp(-U_i) = 1 - U_i + \frac{U_i^2}{2!} - \frac{U_i^3}{3!} + \dots$ . Other researchers predicted the technical efficiency,  $\exp(-U_i)$ , by substituting  $U_i$  with the predictor associated with the equation **(3)**. Battese and Coelli (1988) point out that the best predictor of  $\exp(-U_i)$  is obtained by using the following formula:

$$E[\exp(-U_i) | E_i] = \frac{1 - \phi(\sigma_A + \gamma E_i / \sigma_A)}{1 - \phi(\gamma E_i / \sigma_A)} \exp(\gamma E_i + \sigma_A^2 / 2) \quad (4)$$

This predictor provides a different value from that which uses equation (3) to predict  $U_i$  in  $\exp(-U_i)$ . This is a special case of the general result that the expectation of a non-linear function of a random variable is not equal to the function of the expectation of the random variable {i.e.,  $E[g(x)] \neq g(E[x])$  for a non-linear function,  $g(\cdot)$ }. The technical efficiency predictor used for this study is obtained by replacing the unknown parameters in equation (4) with their ML estimates (the same is implemented in the Frontier computer program).

For the frontier model defined by equation (1) the null hypothesis that there are no technical inefficiency effects in the model can be conducted by testing the null and alternative hypothesis  $H_0: \gamma = 0$  vs.  $H_1: \gamma > 0$ . We may use the Wald statistic to test this hypothesis. For the Wald test, the ratio of the estimate for  $\gamma$  to its estimated standard error is calculated. If  $H_0: \gamma = 0$  is true, this statistic is asymptotically distributed as a standard normal random variable. However, the test must be performed as a one-sided test because  $\gamma$  can not take negative values.

It was found that the Wald test has very poor size (i.e., probability of Type-I error) properties. Hence Coelli (1995) suggested that the generalized likelihood-ratio test should be performed when ML estimation is involved because this test has the correct size. The generalized likelihood-ratio test requires the estimation of the model under both the null and alternate hypotheses. Under the null hypothesis,  $H_0: \gamma = 0$ , the model is equivalent to the traditional average response function, without the technical inefficiency effect,  $U_i$ . The test statistic is calculated as:

$$LR = -2\{\ln[L(H_0)] - \ln[L(H_1)]\} \quad (5)$$

where  $L(H_0)$  and  $L(H_1)$  are the values of the likelihood function under the null and alternative hypotheses,  $H_0$  and  $H_1$  respectively.

If  $H_0$  is true, this test statistic is usually assumed to be asymptotically as a chi-square random variable with degrees of freedom equal to the number of restrictions involved (in this instance one). However, difficulties arise in testing  $H_0: \gamma = 0$  because  $\gamma = 0$  lies on the boundary of the parameter space for  $\gamma$ . In this case if  $H_0: \gamma = 0$  is true, the generalized likelihood-ratio statistic, LR, has asymptotic distribution which is a mixture of chi-square distributions, namely  $\frac{1}{2} \chi_0^2 + \frac{1}{2} \chi_1^2$ , (Coelli 1995). The critical value for a test of size  $\alpha$  is equal to the value,  $\chi_1^2(2\alpha)$ , where this is the value which is exceeded by the  $\chi_1^2$  random variable with probability equal to  $2\alpha$ . Thus, the one-sided generalized likelihood ratio test of size  $\alpha$  is: “Reject  $H_0: \gamma = 0$  in favor of  $H_1: \gamma > 0$  if LR exceeds  $\chi_1^2(2\alpha)$ ”. Thus the critical value for a test of size,  $\alpha = 0.05$ , is 2.706 rather than 3.842 (see, Table 1 of Kodde and Palm (1986)).

A number of applied studies on stochastic frontier production functions have tested the null hypothesis that the simpler half-normal model is an adequate representation of the data, given the specifications of the generalized truncated normal model. This is done by testing the null hypothesis,  $H_0: \mu = 0$ . This can be easily conducted using either a Wald or a generalized likelihood ratio test.

### *3.2 Definition of variables*

It is argued that the productivity of a bus transport undertaking depends on the efficient use of labor and capital. Productivity measurement of Undertakings, therefore, is a means of quantifying the efficiency with which these two resources are utilized. As a measure of the output of STUs, effective bus-kilometers (BKm) has been considered for this study. Inputs are total number of employees and total number of buses held by the STUs. We did not take

total quantity of fuel consumed as an input since our main concern is to find out the inefficiency in utilizing the labor and capital in different STUs.

### *3.3 Model specification and assumptions*

The estimation of relative efficiency of STUs is conducted by assuming the appropriateness of the log-linear Cobb-Douglas case. No other specification was tested due to smaller number of observation. The logarithmic stochastic frontier model specified for the STUs is defined as follows:

$$\ln Y_i = \beta_0 + \beta_1 \ln L_i + \beta_2 \ln B_i + (V_i - U_i), \quad i = 1, 2, \dots, 23. \quad (6)$$

where  $Y_i$  represents the output of the  $i^{\text{th}}$  STU which is expressed in terms of Bus-Kilometers (BKm),  $L_i$  is total number of employees for  $i^{\text{th}}$  STU,  $B_i$  is total number of buses held by the  $i^{\text{th}}$  STU, and  $V_i$  and  $U_i$  are as defined earlier.

All the estimations were made by using the maximum likelihood methods from the statistical program “FRONTIER Version 4.1” (see Coelli 1994). Total two models were estimated. In Model 1, output is assumed to be BKm and  $U_i$  is half normally distributed whereas Model 2 assumes  $U_i$  to have truncated normal distribution. One should note that the truncated normal distribution is a generalization of the half-normal distribution. It is obtained by the truncation at zero of the normal distribution with mean,  $\mu$ , and variance,  $\sigma_u^2$ . If  $\mu$  is pre-assigned to be zero, then the distribution is the half-normal. The distribution may take a variety of shapes, depending upon the size and sign of  $\mu$ .

### 3.4 Estimation results

The first step in the estimation procedure is to check the sign of the third moment and the skewness of the OLS residuals associated with the sample data (Waldman, 1982). The third moment of the OLS residuals for the model represented by equation (6) is  $-1.177$ . The negative sign implies that the residuals of the sample data possess the correct pattern for the implementation of the MLE procedure. Based on the sample cross-sectional data, the OLS estimates and the MLEs for each of the two assumed distributions of the inefficiency term in the frontier model are shown in Table 2. The estimated OLS coefficients are of limited value but do provide a starting point for the MLE process. The goodness of fit of the estimated regression equation evaluated by  $R^2$  for the least squares method looks reasonably high at 0.967. This implies that the two inputs to the model do satisfactorily explain the model output. In addition the F-statistic of 288.90 shows that the relationship between exogenous and endogenous variables is significant even at the 1% level.

Note that in the above Table, the log-likelihood function for the full stochastic frontier model where inefficiency is assumed to be half-normal is calculated to be 13.10 and the value for the OLS fit of the production function is 8.51, which is less than the full frontier model. This implies that the generalized likelihood-ratio test statistic for testing the absence of the technical inefficiency effects from the frontier is calculated to be 9.18 ( $=2*(13.10-8.51)$ ). This value is significantly higher than the critical value, 2.706 at 5% level of significance, obtained from Kodde and Palm (1986) for the degrees of freedom equal to 1. Hence the null hypothesis of no technical inefficiency effects in STUs production is rejected. Similarly, in case of truncated-normal distribution assumption of inefficiency, the null hypothesis of no technical efficiency effects is rejected. Table 2 also shows that  $\gamma$ -estimate is not significantly different from one, which indicates that the stochastic frontier model may



not be significantly different from the deterministic frontier, in which there are no random errors in the production function. Model 2 of Table 2 is different from Model 1 because it assumes truncated-normal distribution of inefficiency which is a generalization of the half-normal distribution. It is obtained by the truncation at zero of the normal distribution with mean,  $\mu$ , and variance,  $\sigma_u^2$ . If  $\mu$  is pre-assigned to be zero, then the distribution is the half-normal. We performed a generalized likelihood ratio test which do not reject the null hypothesis of  $\mu = 0$ . This shows that Model 2 is not statistically superior to Model 1 since  $\mu$  is not significantly different from zero. Therefore, when bus-km is taken as a measure of output, the half normal model i.e., Model 1 is an adequate representation of the data. Estimated results of the Model 1 reveal that few of the coefficients are not statistically significant as per the t-statistic, however the results of the log likelihood ratio test do not warrant dropping of the same from the model. Therefore, Model 1 in its entirety is the most adequate representation of the data.

**Table 2. Frontier production function of STUs (Output is BKm)**

		Model 1	Model 2
Variables/parameters	OLS	MLE (Half-normal)	MLE (Truncated normal)
Constant	0.203 (0.34)	0.421 (0.72)	0.015 (0.05)
lnL	0.405 (1.88)	0.387 (1.86)	0.578 (4.62)
lnB	0.493 (2.43)	0.511 (2.71)	0.327 (2.71)
Sigma-squared	-	0.057 (2.96)	0.076 (2.28)
Gamma	-	0.995 (26.55)	0.999 (60.95)
Mu	-	-	-0.097 (0.52)
Log-likelihood	8.51	13.10	14.47

Figures in parentheses indicate t-ratios.

Under the assumption of a half-normal distribution for the inefficiency term, the productive efficiency of the STUs is illustrated in Table 3. This Table reveals a marked variation of technical efficiency across STUs, from 98.99% for KUM-II to 56.15% for MPSRTC. This implies that KUM-II has realized 98.99% of the production possible for a

fully efficient firm having comparable input values whereas MPSRTC could do so only to the tune of 56.15%. It was found that, among the sample firms, KUM-II, VPM-I and APSRTC are the three most technically efficient STUs whereas STPJB, KSRTC and MPSRTC are the three least technically efficient STUs. Average of technical efficiency scores of sample STUs is 84.22%. This indicates that even in the existing business environment most of the STUs have ample scope to improve their productive efficiency. Technical efficiency scores for nine of twenty-three sample STUs were below average. For the remaining fourteen firms, which had technical efficiency scores above average, ten STUs had technical efficiency score above 90% out of which five had a score above 95%.

**Table 3. Technical efficiency scores of sample STUs (in percentage).**

STU	Efficiency Score (in %age)	Efficiency Rank	Firm Size (in Mn. BKm)	Size Rank
KUM-II	98.99	1	149.24	14
VPM-I	98.30	2	166.44	13
APSRTC	97.90	3	2178.08	1
CBE-II	97.12	4	145.12	15
GSRTC	96.87	5	1151.72	3
SLM-I	93.54	6	142.56	16
KnSRTC	91.93	7	597.14	5
KUM-I	91.58	8	138.46	17
MSRTC	90.76	9	1794.36	2
RSRTC	90.56	10	524.13	6
NWKnSRTC	89.17	11	409.62	7
VPM-II	88.11	12	121.68	18
MDU-IV	86.57	13	107.10	22
STHAR	85.70	14	384.01	8
VPM-III	82.62	15	106.92	23
CBE-I&III	81.85	16	191.40	11
MDU-II	81.53	17	114.92	19
MDU-I	73.35	18	110.71	20
UPSRTC	72.36	19	689.53	4
PRTC	72.01	20	109.51	21
STPJB	62.75	21	189.45	12
KSRTC	57.28	22	362.50	9
MPSRTC	56.15	23	212.00	10
<b>Average</b>	<b>84.22</b>		<b>672.81</b>	

#### **4. Summary and concluding remarks**

The prime objective of this study was to quantify the technical inefficiency prevailing in STUs. For this, we estimated a stochastic frontier production function for the cross-sectional data by using the method of maximum likelihood. We found that there is huge disparity in technical efficiency across STUs ranging from 56.15% for MPSRTC to 98.99% for KUM-II. Average of technical efficiency scores of sample STUs was found to be 84.22%. Among the sample firms, KUM-II, VPM-I and APSRTC are the three most technically efficient STUs whereas STPJB, KSRTC and MPSRTC are the three least efficient ones. The main conclusion in our analysis is that given the size distribution of the sample STUs and their working environment, the potential gain in productive efficiency for most of them is very high.

## References

- (1) Aigner D., Lovell C. A. and Schmidt P. (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models", *Journal of Econometrics* 6: 21-27.
- (2) Battese G. E. and Coelli T. J. (1988), "Prediction of Firm-Level Technical Efficiencies With a Generalised Frontier Production Function and Panel Data", *Journal of Econometrics* 38: 387-399.
- (3) Battese G. E. and Coelli T. J. (1992), "Frontier Production Functions, Technical Efficiency and Panel Data with Applications to Paddy Farmers in India", *Journal of Productivity Analysis* 3: 152-169.
- (4) Battese G. E. and Coelli T. J. (1995), "A Model for Technical Inefficiency Effects in a Stochastic Frontier Production for Panel Data", *Empirical Economics* 20: 325-332.
- (5) Battese G. E. and Corra G. S. (1977), "Estimation of a Production Frontier Model: With Application to the Pastoral Zone of Eastern Australia", *Australian Journal of Agricultural Economics* 21: 169-179.
- (6) Coelli T. J. (1995), "Estimators and Hypothesis Tests for a Stochastic Frontier Function: A Monte Carlo Analysis", *Journal of Productivity Analysis* 6: 247-268.
- (7) Coelli T. J. (1994), "A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation", Mimeo, Department of Economics, University of New England.
- (8) Cornwell C., Schmidt P. and Sickles R. E. (1990), "Production Frontiers with Cross-sectional and Time-series Variation in Efficiency Levels", *Journal of Econometrics* 46: 185-200.
- (9) Cullinane K. et al. (2002), "A Stochastic Frontier Model of the Efficiency of Major Container Terminals in Asia: Assessing the Influence of Administrative and Ownership Structures", *Transportation Research Part A* 36: 743-762.
- (10) De Rus G. and Nombela G. (1997), "Privatisation of Urban Bus Services in Spain", *Journal of Transport Economics and Policy* 31(1): 115-129.
- (11) Farrell M. J. (1957), "The Measurement of Productive Efficiency", *Journal of Royal Statistical Society, Series A, CXX*, Part 3, 253-290.
- (12) Finn Jorgensen, Pal Andreas Pedersen and Rolf Volden (1997), "Estimating the Inefficiency in the Norwegian Bus Industry from Stochastic Cost Frontier Models", *Transportation* 24: 421-433.

- (13) Forsund F., Lovell C. A. and Schmidt P. (1980), "A Survey of Frontier Production Functions and of Their Relationship to Efficiency Measurement", *Journal of Econometrics* 13: 5-25.
- (14) Hensher D.A. (1987), "Productive Efficiency and Ownership of Urban Bus Services", *Transportation* 14: 209-225.
- (15) Huang C. J. and Liu J. T. (1994), "Estimation of Non-neutral Stochastic Frontier Production Function", *Journal of Productivity Analysis* 5: 171-180.
- (16) Jha R. and Singh S. K. (2001), "Small is Efficient: A Frontier Approach to Cost Inefficiencies in Indian State Road Transport Undertakings", *International Journal of Transport Economics* XXVIII(1): 95-114.
- (17) Jha R. et al. (1999), "Tax Efficiency in Selected Indian States", *Empirical Economics* 24(4): 641-654.
- (18) Jha R. and Singh S. P. (1994), "Intertemporal and Cross-section Variations in Technical Efficiency in the Indian Railways", *International Journal of Transport Economics* 21: 57-73.
- (19) Jondrow et al. (1982), "On Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model", *Journal of Econometrics* 19: 233-238.
- (20) Kodde D. A. and Palm F. C. (1986), "Wald Criteria for Jointly Testing Equality and Inequality Restrictions", *Econometrica* 54: 1243-1248.
- (21) Meeusen W. and van den Broeck J. (1977), "Efficiency Estimation from Cobb-Douglas Production Function with Composed Error", *International Economic Review* 18(2): 435-444.
- (22) Rao C. R. (1973), "Linear Statistical Inference and Its Applications", Second Edition, Wiley, New York.
- (23) Richmond J. (1974), "Estimating the Efficiency of Production", *International Economic Review* 15: 515-521.
- (24) Singh S. K. (2000), "State Road Transport Undertakings, 1983-84 to 1996-97: A Multilateral Comparison of Total Factor Productivity", *Indian Journal of Transport Management* 24(5): 363-388.
- (25) Waldman D. (1982), "A Stationary Point for the Stochastic Frontier Likelihood", *Journal of Econometrics* 18: 275-279.